Print your name: ______________________

Final Examination — Fall 2004

Econ 705 — Page Total = 3

1. Definitions: 3 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

Whenever possible, offer *all three* of the following: a verbal defintion *and* a mathematical definition *and* an illustrative example. (An example is not a definition!) Be sure to show clearly how your example fits the definition.

Def 1. null space (kernel)

Def 2. quadratic form

Def 3. leading principal submatrix (of a square matrix)

Def 4. cofactor expansion

Def 5. derivative (of a function)

Def 6. linear space

Def 7. linear operator

Def 8. positive definite matrix

Def 9. bilinear functional

Def 10. (weak) partial order

Def 11. equivalence relation

Def 12. binary relation

Def 13. compact set

Def 14. closed set

Def 15. one-to-one function

2. Very Short Answers: 5 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please clearly label your answers with the question number.) When notation is important to your answer, state precisely what your notation means.

VSA1. Explain why the property that $|AB| = |A||B|$ implies that a matrix has an inverse only if it has a non-zero determinant.

VSA2. Prove Euler's theorem for homogeneous functions.

VSA3. For the following functions, find the critical points and use a derivative test to determine whether these are maxima or minima.

- $f(x) = (x-1)^2$
- $f(x) = (x-2)^3$

- $f(x) = (x-3)^4$
- $f(x) = 10 + 5x - x^2$

VSA4. Suppose R is symmetric and transitive. Then it must be true that

(a) $x \in \text{Dom}(R) \implies (x,x) \in R$

(b) $x \in \text{Ran}(R) \implies (x,x) \in R$

(c) R is reflexive.

(d) *a. and b.

(e) all of the above

VSA5. Consider the function defined by the rule $f(x) = \sin(x)/x$ over its natural domain. At $x = 0$ the function

(a) is not continuous

(b) is not differentiable

(c) is undefined

(d) *all of the above

(e) none of the above

VSA6. Consider the three vectors $x = [1, 0, 0]$, $y = [0, 1, 0]$, $z = [2, 2, 0]$.

(a) y is linearly dependent on x.

(b) z is linearly dependent on y.

(c) x is linearly dependent on y and z.

(d) z is linearly dependent on x and y.

(e) *c. and d.

VSA7. A *linear combination* is

(a) any weighted sum of vectors.

(b) *any *finite* weighted sum of vectors.

(c) a *finite* weighted sum of vectors, with positive weights.

(d) a *finite* weighted sum of vectors, with positive weights that sum to one.

(e) all of the above

3. Short Answer: 10 points each

DO THE FIRST QUESTION PLUS FIVE MORE OF THE FOLLOWING QUESTIONS, FOR A TOTAL OF SIX (6) QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

SA1. Two firms supply at positive prices substitutable products produced with positive, constant marginal cost. There is an initial equilibrium at positive prices. Each raises prices when its marginal cost rises or the competitor's price rises: $P_A = f(c_A, P_B)$ and $P_B = g(c_B, P_A)$. What is the effect on P_A of a rise in c_B? Do the comparative statics algebra, and be sure to carefully explain the signs of the reduced form partial derivatives both intutitively and in terms of the algebra. Draw a graph and refer to it to illustrate your intuitive arguments.

SA2. Give a brief description of how to use "Cramer's rule". Prove "Cramer's rule" using the fact that for certain matrices $|AB| \equiv |A|\,|B|$.

SA3. A certain quadratic from on $\Re^3$ is represented by a diagonal matrix with diagonal $= [1, 0, 1]$. What is the quadratic form? Determine the definiteness of this quadratic form by testing the relevant minors of the matrix. Suppose consider this quadratic form on the subspace of $\Re^3$ given by $x_2 = 0$. Does that affect your evaluation of its definiteness? Explain.

SA4. Does the set of 2×2 orthogonal matrices form a group under matrix multiplication? Does the set of 2×2 permutation matrices form a group under matrix multiplication?

SA5. Describe the three elementary row operations and their associated elementary matrices. Show how to construct the inverse of each type of elementary matrix.

SA6. Consider a consumer whose weak preference relation R is known to be reflexive and transitive. Show that $R \cap R^{-1}$ is an equivalence relation. What aspect of the consumer's preferences does this equvialence relation represent?

SA7. Use cofactor expansion to calculate the determinants of the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 2 & 0 & 4 & 5 \\ 0 & 2 & 0 & 0 & 5 \end{bmatrix}$$

Show all steps.

SA8. Define a *cover* of $S \subseteq X$. Consider a binary relation B on a finite set X. Suppose there is no B-maximal element in the set $S \subseteq X$. For each point $n \in S$, let C_n be the associated lower contour set. Prove that these lower contour sets form a cover of S. Briefly suggest an economic application of your proof.

SA9. Suppose $B = \{\mathbf{b}^1, \ldots, \mathbf{b}^N\}$ is a basis for the vector space V. Prove that any set of $N + 1$ vectors is linearly dependent.

SA10. Define the terms gradient and Hessian. Consider the production function $f(K, L, R) = K^\alpha L^\beta H^{1-\alpha-\beta}$. Show the computations necessary to find the gradient. Show the computations necessary to find the Hessian.

SA11. Suppose your utility function is $U(c_1, c_2) = c_1^{1/2} + c_2^{1/2}$ and you face prices $p = (1, 3)^\top$. Given wealth of \$10,000, what is your optimum consumption bundle? Set up and solve as an unconstrained optimization problem. Characterize the first order necessary condition and explain intuitively why it is necessary. Determine whether this locates a maximum point.