Print your name: ______________________________

Final Examination — Spring 2007

Econ 505 — Page Total = 5

1. Definitions: 5 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

Whenever possible, offer *all three* of the following: a verbal defintion *and* a mathematical definition *and* an illustrative example. (An example is not a definition!) Be sure to show clearly how your example fits the definition.

Def 1. concave function

Def 2. inflection point

Def 3. convex combination (of points)

Def 4. Cramer's rule

Def 5. critical point

Def 6. logarithm

2. Very Short Answers: 5 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please clearly label your answers with the question number.) When notation is important to your answer, state precisely what your notation means.

VSA1. Set up and solve the following linear system as a matrix equation, and solve for x by using a matrix inverse. (Show *all* of your work!)

$$x_1 + 4x_2 = 10 \qquad 5x_1 - 3x_2 = 4$$

VSA2. Explain why the property that $|AB| = |A||B|$ implies that a matrix has an inverse only if it has a non-zero determinant.

VSA3. Given the three matrices

$$A = \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 4 \\ 9 & 10 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & 6 \\ 11 & 12 \end{bmatrix}$$

Calculate the element in the second row and first column of ABC. For credit, you must do this intelligently and efficiently. Be sure to explain what you are doing.

VSA4. Suppose p and q are 25×8 matrices of prices and quantities, where $p_{t,\cdot}$ is the price vector prevaling in period t, when the bundle $q_{t,\cdot}$ is purchased. Then pq^τ is

(a) a 25×25 matrix of actual expenditures in each period

(b) a 8×8 matrix of actual expenditures in each period

(c) a 25×25 matrix, where the r,k-th element is the cost of the k-th bundle at period r prices.

(d) a 8×8 matrix, where the r,k-th element is the cost of the k-th bundle at period r prices.

(e) a 8×8 matrix, where the r,k-th element is the cost of the r-th bundle at period k prices.

3. Short Answer: 10 points each

DO THE FIRST QUESTION PLUS FOUR MORE OF THE FOLLOWING QUESTIONS, FOR A TOTAL OF FIVE (5) QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

SA1. For the following functions, find the critical points. Show how to use a first-derivative test to determine whether these are maxima or minima. Show how to use a second-derivative test to determine whether these are maxima or minima.

- $f(x) = (x-3)^2$
- $f(x) = (x-1)^3$
- $f(x) = -(x-2)^4$
- $f(x) = 10 + 5x - 5x^2$

SA2. Consider the Cobb-Douglas production function $f(K, L) = K^{.4}N^{.6}$. Show that this production function exhibits constant returns to scale. Compute the gradient of this function. Find the elasticity of output with respect to K and with respect to N. Compute the Hessian of this production function.

SA3. Real GDP per capita in Pakistan is about 1/16 of real GDP per capita in the US. Also, real GDP per capita in Pakistan is growing at about 4%/year while real GDP per capita in the US is growing at about 2% per year. (The are continuously compounded growth rates.) If these growth rates persist, how many years will it be before Pakistan reaches the *current* U.S. standard of living? If these growth rates persist, how many years will it be before Pakistan "catches up" with the U.S.?

You must use the "law of 70" and fully explain your reasoning.

SA4. A simple "Keynesian" IS-LM model can be written as

$$y = A(i - \pi, y, f) \qquad\qquad m = L(i, y)$$

where A is the aggregate demand function, L is the money demand function, the endogenous variables ("unknowns") are the interest rate i and real income y, and the exogenous variables are the fiscal stance f, expected inflation π, and real money m. Write the total differential of this model as a matrix equation in the form $Ax = b$, and then solve for the matrix reduced form equation using matrix algebra and showing all your work. Solve for $\partial y/\partial\pi$ and $\partial i/\partial\pi$ and, if possible (using the usual theoretical responses), sign these "partial derivatives" of the reduced form.

SA5. A firm faces the downward sloping demand function determined by $Q = 150 - P/2$. Costs are quadratic: $C(Q) = 15 + 5Q + 5Q^2/2$. Find the quantity Q that maximizes profits. (You can leave it in fractional form.)

SA6. Consider the functions defined as follows:

- $f(x, y) = 2x - y^2$
- $f(x, y) = 2xy$
- $f(x, y) = 2x/y$
- $f(x, y) = 2x^2y^3$

For each function, find the partial derivative with respect to x and the partial derivative with respect to y. For each function, find the differential.

Listing 1: Newton's Method

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define function newton(f,Df,x):
    dx ← −f(x)/Df(x)
    while abs(dx) ≥ 1e−9:
        x ← x+dx
        dx ← −f(x)/Df(x)
    output (x)
```

SA7. Compute the determinant of each of the following matrices. (Show the entire expansion, step by step. Credit if for the solution process, not for the final answer.)

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 4 & -2 \\ -8 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & 2 \\ 1 & 0 & -4 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & -4 \\ 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 100 & 2 & 3 & 4 \\ 0 & 0 & 0 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 5 & 1 \end{bmatrix}$$

SA8. Consider the pseudocode in listing 1. Explain what the algoritm accomplishes and how. (Be precise.) Write down a GAUSS implementation of this algorithm.

SA9. A wine dealer owns a case of fine wine that can be sold for $1,000e^{\sqrt{t/2}}$ if she holds onto it for t years. There are no storage costs, and the (continually compounded) interest rate is 10% per annum. At what date t should the dealer sell her wine? (You may express the answer algebraically; you do not need a calculator.)

4. Long Answer: 15 points each

DO BOTH QUESTIONS, FOR A TOTAL OF TWO (2) QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

LA1. Consider the two vectors $v_1 = [0, 1]$ and $v_2 = [-1, 1]$. Show how to produce the linear combination $2v_1 - v_2$ algebraically and graphically, including a full explanation of how you are proceding. Suppose you are given an arbitrary vector $v_3 \in \Re^2$: should you be able to find scalars α_1 and α_2 such that $v_3 = \alpha_1 v_1 + \alpha_2 v_2$? (If so, show how to do so.) Explain, emphasizing any properties of v_1 and v_2 that make this possible or impossible.

LA2. Suppose your utility function is $U(c_1, c_2) = 4\sqrt{c_1} + 8\sqrt{c_2}$ and you face prices $p = (1, 2)^\top$. Given wealth of \$15,000, what is your optimum consumption bundle? Set up and solve as an *unconstrained* optimization problem. (You may leave your answers in fractional form.) Characterize the first-order necessary condition and provide *economic* intuition for why this condition is necessary for an optimum. Determine whether you have found a maximum.