Print your name: ____________________

Midterm Examination — Fall 2007

Econ 505 — Page Total = 2

1. Definitions: 6 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

Whenever possible, offer *all three* of the following: a verbal defintion *and* a mathematical definition *and* an illustrative example. (An example is not a definition!) Be sure to show clearly how your example fits the definition.

Def 1. exogenous variable

Def 2. logarithm

Def 3. difference quotient

Def 4. convex combination (of points)

Def 5. concave function

Def 6. strictly increasing function (you may consider a real-valued function of a real variable)

Def 7. Law of 70

2. Very Short Answers: 10 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please clearly label your answers with the question number.) When notation is important to your answer, state precisely what your notation means.

VSA1. Set up and solve the following linear system as a matrix equation, and solve for x by using a matrix inverse. (Show *all* of your work!)

$$x_1 + 2x_2 = 15 \qquad x_1 - 3x_2 = -10$$

VSA2. Use the arbitrary points $(x_0, f(x_0))$ and $(x_0 + h, f(x_0 + h))$ to compute the difference quotient for each of the following functions. Use your computed difference quotient to determine whether or not the function is monotone, and explain your reasoning in each case.

$$f(x) = 2000x + 5000$$
$$f(x) = -0.25x + 0.5$$
$$f(x) = -x^2$$

VSA3. Real GDP per capita in Pakistan is about 1/16 of real GDP per capita in the US. Also, real GDP per capita in Pakistan is growing at about 4%/year while real GDP per capita in the US is growing at about 2% per year. (The are continuously compounded growth rates.) If these growth rates persist, how many years will it be before Pakistan reaches the *current* U.S. standard of living? If these growth rates persist, how many years will it be before Pakistan "catches up" with the U.S.?

You must use the "law of 70" and fully explain your reasoning.

VSA4. Consider the following GAUSS program.

```
proc (1) = power(x,n);
  local xn;
  xn = 1;
  for i(1,n,1);
    xn = xn*x;
  endfor;
retp(xn);
endp;
print power(2,12);
```

Provide a comment for *each* line, explaining what that line does. What does the entire program do? (Explain concisely but completely *how* it does this.)

3. Short Answer: 15 points each

DO ALL QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

SA1. Two firms supply at *positive* prices substitutable products produced with positive, constant marginal cost. There is a unique initial equilibrium at positive prices. Each raises prices when its marginal cost rises or the competitor's price rises. The reaction functions for firm A and firm B are
$P_A = c_A + \alpha P_B$
$P_B = c_B + \beta P_A$
Set the two reaction functions up as a matrix equation, with endogenous variables P_A and P_B. Solve for the reduced form by premultiplying *both* sides of the equation by the inverse of the coefficient matrix. (This should take 4 or 5 steps, so that you show your work explicitly and in great detail.) What is your reduced form equation for P_A? Using this reduced form equation, find the change in P_A that results when c_B increases. Provide careful inution for the sign of this change. What details in the initial description of the problem help you discover this sign? Illustrate by sketching the reaction functions in P_A, P_B-space, paying careful attention to the details of the problem description, and showing any shifts and any changes in the equlibrium prices.

SA2. A simple "Classical" IS-LM model can be written as

$$y = A(i - \pi, f) \qquad\qquad m = L(i, y)$$

where A is the aggregate demand function, L is the money demand function, the endogenous variables ("unknowns") are the interest rate i and real money supply m, and the exogenous variables are the fiscal stance f, expected inflation π, and real income y. Write the total differential of this model as a matrix equation in the form $Ax = b$, and then solve for the matrix reduced form equation using matrix algebra and showing all your work. Solve for $\partial i/\partial \pi$ and $\partial m/\partial \pi$ and, if possible, sign these "partial derivatives" of the reduced form.

SA3. Use our definition of strict concavity to prove that $f(x) = -x^2$ defines a strictly concave function.