Print your name: ______________________________

Final Examination — Spring 2008
Econ 505 — Page Total = 4

1. Definitions: 5 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

Whenever possible, offer **all three** of the following: a verbal defintion **and** a mathematical definition **and** an illustrative example. (An illustrative example usually includes graphs and algebra; an example is not a definition!) Be sure to *show* clearly (if possible, with both algebra and graphs) how your example fits the definition.

Def 1. difference quotient (of a function $f : \Re \to \Re$).

Def 2. critical point

Def 3. inflection point

Def 4. determinant (of an $N \times N$ matrix)

2. Very Short Answers: 5 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please clearly label your answers with the question number.) When notation is important to your answer, state precisely what your notation means.

VSA1. Consider the function defined by the rule $f(x) = \ln x$. Find the first, second, third, fourth, and fifth-order derivatives of f. Give a general characterization of the n-th order derivative.

VSA2. Set up and solve the following linear system as a matrix equation, and solve for x by using a matrix inverse. (Show *all* of your work!)

$$2x_1 + 4x_2 = 4 \qquad 5x_1 - 3x_2 = -3$$

VSA3. Explain why the property that $|AB| = |A||B|$ implies that a matrix has an inverse only if it has a non-zero determinant.

VSA4. Given the three matrices

$$A = \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 4 \\ 9 & 10 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & 6 \\ 11 & 12 \end{bmatrix}$$

Calculate the element in the first row and second column of the product ABC. For credit, you must do this intelligently and efficiently. Be sure to explain what you are doing.

VSA5. Suppose p and q are 25×8 matrices of prices and quantities, where $p_{t,\cdot}$ is the price vector prevaling in period t, when the bundle $q_{t,\cdot}$ is purchased. Then pq^τ is

(a) a 25×25 matrix of actual expenditures in each period

(b) a 8×8 matrix of actual expenditures in each period

(c) a 25×25 matrix, where the r, k-th element is the cost of the k-th bundle at period r prices.

(d) a 8×8 matrix, where the r, k-th element is the cost of the k-th bundle at period r prices.

(e) a 8×8 matrix, where the r, k-th element is the cost of the r-th bundle at period k prices.

VSA6. Which of the following NumPy commands produces an array containing the integers from 0 to 100?

(a) `arange(100)`
(b) `arange(101)`
(c) `arange(0,100,1)`
(d) `arange(1,101,1)`
(e) none of the above

VSA7. The Python+Numpy statement `x = mat('2 4 6 8')`

(a) assigns a 4×1 matrix to the variable x.
(b) assigns a 1×4 matrix to the variable x.
(c) assigns a 2×2 matrix to the variable x.
(d) tests whether x is equal to this "material" array of numbers.
(e) is not "legal".

VSA8. The Python+Numpy statement `x = mat('2;4;6;8')`

(a) assigns a 4×1 matrix to the variable x.
(b) assigns a 1×4 matrix to the variable x.
(c) assigns a 2×2 matrix to the variable x.
(d) tests whether x is equal to an array of numbers.
(e) is not "legal".

VSA9. Consider the three vectors $x = [1, 0, 0]$, $y = [1, 1, 0]$, $z = [2, 2, 0]$.

(a) the set $\{x, y\}$ is linearly dependent.
(b) the set $\{x, z\}$ is linearly dependent.
(c) the set $\{y, z\}$ is linearly dependent.
(d) the set $\{x, y, z\}$ is linearly dependent.
(e) c. and d.

VSA10. A *linear combination* is

(a) any weighted sum of vectors.
(b) any *finite* weighted sum of vectors.
(c) any *finite* weighted sum of vectors, with positive weights.
(d) any *finite* weighted sum of vectors, with positive weights that sum to one.
(e) any weighted sum of vectors, with positive weights that sum to one.

VSA11. Suppose you have a continuously compounded annual rate of interest of 5% and an initial investment of \$100. If you leave your investment untouched, at the end of a year you will have

(a) \$100
(b) \$105
(c) $e^{.05} \times \$100$
(d) $e^{0.5} \times \$100$
(e) $e^{5} \times \$100$

VSA12. Suppose your productivity at a particular job increases with experience, so that your effective labor input is $L(t) = 2 - e^{-0.1t}$. Your starting productivity (at time $t = 0$) and maximum possible productivity are

(a) $2, \infty$
(b) $1, \infty$
(c) $1, 2$
(d) $-\infty, 2$
(e) $-\infty, \infty$

VSA13. Which of the following are properties of the matrix inverse?

(a) $(A^{-1})^{-1} = A$
(b) $A^{-1}B^{-1} = (BA)^{-1}$
(c) $(A^{-1})^{\top} = (A^{\top})^{-1}$
(d) a. and b.
(e) all of the above

VSA14. If $f(x) = \log_2 x$ then $f'(x) =$

(a) $\ln x / \ln 2$
(b) $1/x \ln 2$
(c) $1/2x$
(d) $1/x$
(e) none of the above

VSA15. If $f(x) = 10^x$ then $f'(x) =$

(a) $\ln 10 \cdot 10^x$
(b) $10 \cdot 10^x$
(c) $1/x \ln 10$
(d) 10^x
(e) none of the above

3. Short Answer: 10 points each

DO THE FIRST QUESTION PLUS FOUR MORE OF THE FOLLOWING QUESTIONS, FOR A TOTAL OF **FIVE (5)** QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

SA1. For the following functions, find any critical points. (You *must* use the chain rule when appropriate.) Show how to use a first-derivative test to determine whether these are maxima or minima. Show how to use a second-derivative test to determine whether these are maxima or minima.

(a) $f(x) = 10$
(b) $f(x) = 5(2 + x)$
(c) $f(x) = (x - 3)^2$
(d) $f(x) = (x - 1)^3$
(e) $f(x) = -(x - 2)^4$

SA2. A firm faces the downward sloping demand function determined by $Q = 1000 - P/2$. Costs are quadratic: $C(Q) = 15 + 5Q + 3Q^2/2$. Find the quantity Q that maximizes profits.

SA3. Real GDP per capita in Pakistan is about 1/16 of real GDP per capita in the US. Also, real GDP per capita in Pakistan is growing at about 4%/year while real GDP per capita in the US is growing at about 2% per year. (The are continuously compounded growth rates.)

If these growth rates persist, how many years will it be before Pakistan reaches the *current* U.S. standard of living?

If these growth rates persist, how many years will it be before Pakistan "catches up" with the U.S.?

You must use the "law of 70" and **fully explain** your reasoning.

SA4. Suppose quantity supplied is determined by rainfall (R) and price (P) as $Q = P^2 - 3R$ while quantity demanded depends on price and income (I) according to $Q = 20 + 3I - 2P$. The endogenous variables are Q and P. Totally differentiate the two equations and set up a matrix equation. Solve for dP and dQ using the matrix inverse. What is the sign of $\partial P/\partial I$? Does this make economic sense? Why or why not?

SA5. A wine dealer owns a case of fine wine that can be sold for $1,000e^{\sqrt{2t}}$ if she holds onto it for t years. There are no storage costs, and the (continually compounded) interest rate is 10% per annum. At what date t should the dealer sell her wine? (You may express the answer algebraically; you do not need a calculator.)

SA6. A cobweb cycle model produces the recurrence relation

$$P_t = 225 - 0.5P_{t-1}$$

If P has a steady state value, find it, and determine whether it is stable.

4. Long Answer: 15 points each

DO BOTH QUESTIONS, FOR A TOTAL OF TWO (2) QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

LA1. Consider the two vectors $v_1 = [0,1]$ and $v_2 = [1,1]$. Show how to produce the linear combination $2v_1 - v_2$ algebraically and graphically, including a *full explanation* of how you are proceding. (You get credit for a *detailed* explanation, not for the trivial computation.) Suppose you are given an arbitrary vector $v_3 \in \Re^2$: should you be able to find scalars α_1 and α_2 such that $v_3 = \alpha_1 v_1 + \alpha_2 v_2$? (If so, show how to do so.) Explain, emphasizing any properties of v_1 and v_2 that make this possible or impossible.

LA2. Here is a simple model of the money supply. Suppose consumers hold currency as a proportion of their deposits according to $C = cD$, and banks hold reserves as a proportion of their deposits according to $R = rD$. Define the high powered money stock as currency plus reserves: $H = C + R$. Define the money stock as currency plus deposits: $M = C + D$. Given the exogenous H (along with fixed parameters r and c), we can determine the endogenous variables C, R, D, M. Set up the matrix equation for this system. Next solve for M using Cramer's rule. (For credit, you *must* show the full cofactor expansion for each determinant.)

LA3. Suppose your utility function is $U(c_1, c_2) = c_1^{1/2} + c_2^{1/2}$ and you face prices $p_1 = 1$ and $p_2 = 3$. Given wealth of $10,000, what is your optimum consumption bundle? Set up and solve as an *unconstrained* optimization problem.