1. Short Answer: 15 points each

ANSWER THE FIRST QUESTION PLUS THREE MORE QUESTIONS IN THIS SECTION, FOR A **TOTAL OF FOUR (4)** QUESTIONS.

SA1. Consider a very simple representation of a "Keynesian" model given by
IS: $Y = A(r, Y, F)$
LM: $m = L(r + \pi, Y)$
with the real interest rate (r) and real income (Y) as our endogenous variables, and real money supply (m), expected inflation (pi), and fiscal policy stance (F) as our exogenous variables. All function responses (partial derivatives) have the usual signs. Totally differentiate the two equations, and set up the resulting matrix representation of the Keynesian comparative statics. Use the inverse of the coefficient matrix to solve for dr and dY in terms of the exogenous change dF. Explain how to sign the responses of the endogenous variables to a fiscal expansion ($dF > 0$).

SA2.

$$B = \begin{bmatrix} 3 & 0 & 4 \\ 0 & 10 & 0 \\ 6 & 0 & 1 \end{bmatrix}$$

Define and describe is a characteristic root. Explain why -3 is a characteristic root of B. Define and describe is a characteristic vector. Find a characteristic vector of B associated with the characteristic root -3. Find the characteristic polynomial of B. What are the other characteristic roots of B?

SA3. Given a matrix $A_{N\times N}$, define its determinant and adjugate (adjoint) matrix in terms of its cofactors. Suppose $A_{N\times N}$ is non-singular. Give a detailed explanation of how to construct an inverse for it. Apply your general method to the 3×3 matrix

$$B = \begin{bmatrix} 3 & 0 & 4 \\ 0 & 10 & 0 \\ 6 & 0 & 1 \end{bmatrix}$$

SA4. Consider a binary relation R on a set X. Show that R is transitive iff $R \circ R \subseteq R$. (Rely on definitions, not on previously proved theorems.)

SA5. Consider a dynamical system (S, f) represented by $x_t = f(x_{t-1})$. Suppose for some $x_0, x \in S$ we find that $f^{\circ t}(x_0) \to_{t\to\infty} x$. Prove that if f is continuous at x, then x is a fixed point of f in S. Intuitively, how does continuity get you to this conclusion?

SA6. Two firms supply at *positive* prices substitutable products produced with positive, constant marginal cost. There is a unique initial equilibrium at positive prices. Each raises prices when its marginal cost rises or the competitor's price rises. The reaction functions for firm A and firm B are
$P_A = c_A + \alpha P_B$
$P_B = c_B + \beta P_A$
Set the two reaction functions up as a matrix equation, with endogenous variables P_A and P_B. Solve for the reduced form using a matrix inverse. What is your reduced form equation for P_A? Using this reduced form equation, find and sign the change in P_A that results when c_B increases. Provide careful intution for the sign of this change, supporting your intuition with a careful graph. What details in the description of the problem help you discover this sign?

SA7. What is a topology? What is an algebra of sets? Consider the set $X = \{0, 1, 2, 3, 4\}$. Can you define on X a toplogy that is not an algebra of sets? (Do so if possible. Otherwise, explain why it is not possible.) Can you define on X an algebra of sets that is not a toplogy? (Do so if possible. Otherwise, explain why it is not possible.)

2. Longer Answer: 25 points each

DO ALL OF THE FOLLOWING QUESTIONS.

LA1. Here is a simple model of the money supply. Suppose consumers hold currency as a proportion of their deposits according to $C = cD$, and banks hold reserves as a proportion of their deposits according to $R = rD$. Define the high powered money stock as currency plus reserves: $H = C + R$. Define the money stock as currency plus deposits: $M = C + D$. Given the exogenous H (along with fixed parameters r and c), we can determine the endogenous variables C, R, D, M. Set up the matrix equation for this system.

What is the difference between Gaussian and Gauss-Jordan elimination? Solve the system using Gauss-Jordan elimination. What is an elementary equation operation? Explain what elementary equation operation you are using at each step of your solution. Show *all* of your work.

LA2. Suppose you want to maximize $x^{0.5}y^{0.5}$ subject to $4x + y = k$. (Here k is a fixed parameter.)

Convert this constrained optimization problem into an unconstrained problem, and solve for x and y.

Next, solve the same problem as a constrained optimization problem. Set up a Lagrangian for this problem, and use it to produce first order necessary conditions for a maximum.

Give a detailed explanation of why these first-order conditions are necessary for a maximum.

How do x and y change as k changes? (That is, what is the elasticity of x w.r.t. k, and what is the elasticity of y w.r.t. k?)

LA3. State rove Walker's theorem for maximization on a compact set. Make sure your math is rigorous and that you define *all* terms. (**Note**: you will need to prove a maximization result on finite sets.) Prove in addition that the maximal set is compact. Is it also closed? Explain. Be sure to give a *detailed* intuitive explanation of each step in your proofs. Explain why Walker's theorem is important for ecnonomists.