Print your name: ______________________________

Midterm Examination | Fall 2012
Econ 705 | Page Total = 4

1. Define, Compare, and Contrast: 5 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

Whenever possible, offer *all three* of the following: a mathematical definition (using mathematical notation) *and* a separate, detailed verbal explanation, *and* an illustrative example. (An example is not a definition!) Make sure you *also* (!!!) define any key terms that you introduce as part of a definition. Make sure you *show* that your example is an example: do not just assert that it is. When you compare and contrast, be sure to explain important relationships between the concepts.

D1. acyclicity vs. transitivity (of a binary relation)

D2. equivalence relation vs. weak partial order (define each component property!)

D3. topological space vs. algebra of sets

D4. open set vs. closed set (topological definition)

D5. finite set vs. compact set (topological definition)

2. Multiple Choice: 2 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please write the question number and answer letter in your bluebook.)

MC1. R is a transitive binary relation on X with asymmetric component P. Then it must be true that

(a) $xRy \wedge yPz \implies xPz$
(b) $xPy \wedge yRz \implies xPz$
(c) $xIy \wedge yIz \implies xIz$
(d) a. and b.
(e) all of the above

MC2. R is a transitive and symmetric binary relation on X. Then it must be true that

(a) $x \in \mathrm{Dom}(R) \implies (x,x) \in R$
(b) $x \in \mathrm{Ran}(R) \implies (x,x) \in R$
(c) R is reflexive.
(d) a. and b.
(e) all of the above

MC3. R is a preorder on X. Then it must be true that

(a) $x \in \mathrm{Dom}(R) \implies (x,x) \in R$
(b) $x \in \mathrm{Ran}(R) \implies (x,x) \in R$
(c) R is reflexive.
(d) a. and b.
(e) all of the above

MC4. R is a complete binary relation on the finite set X. Then it must be true that

(a) there exists at least one R-best element in X
(b) there exists at least one R-maximal element in X
(c) every R-maximal element is an R-best element
(d) a. and c.
(e) all of the above

MC5. R is a transitive binary relation on X. x and y are R-best elements, and zRx. Then it must be true that

(a) z is a maximal element
(b) z is a best element
(c) zRy
(d) a. and b.
(e) all of the above

MC6. For arbitrary sets P and Q, we know

(a) $(P \cap Q)^c = (P^c \cup Q^c)$
(b) $(P \cup Q)^c = (P^c \cap Q^c)$
(c) $(P \cup Q)^c = (P^c \cup Q^c)$
(d) a. and b.
(e) all of the above

MC7. Let R be a *complete* binary relation on X. Let $M(X,R)$ be the set of R-maximal elements in X, and let $C(X,R)$ be the set of R-best elements in X. Which of the following are are always true?

(a) $x \in M(X,R) \implies x \in C(X,R)$
(b) $x \in C(X,R) \implies x \in M(X,R)$
(c) $M(X,R) = C(X,R)$
(d) all of the above
(e) none of the above

MC8. Which of the following Mathematica commands produces a list containing the integers from 0 to 100?

(a) `Range[101]`
(b) `Range[0,101,1]`
(c) `Range[0,100,1]`
(d) a. and b.
(e) all of the above

MC9. Suppose we consider $R = \varnothing$ to be a binary relation on the *non-empty* set X. Which of the following are true?

(a) R is transitive.
(b) R is symmetric.
(c) R is reflexive.
(d) a. and b.
(e) all of the above

MC10. The set S is an closed subset of the topological space X. Then

(a) S^c is open
(b) the closure of S equals S
(c) the interior of S equals S
(d) a. and b.
(e) a. and c.

MC11. Using a finite set of n elements, how many sequences of length k can we produce? (Here $k \leq n$.)

(a) $\frac{n!}{k!(n-k)!}$
(b) $n!/(n-k)!$
(c) $n!/k!$
(d) $n!$
(e) none of the above

MC12. Using a finite set of n elements, how many sets of cardinality k can we produce? (Here $k \leq n$.)

(a) $\frac{n!}{k!(n-k)!}$
(b) $n!/(n-k)!$
(c) $n!/k!$
(d) $n!$
(e) none of the above

MC13. Let R_t be the transitive closure of the binary relation R. Then we know

(a) $R \subseteq R_t$
(b) if R is transitive then $R = R_t$
(c) if R is symmetric then so is R_t
(d) a. and b.
(e) all of the above

MC14. 1010000100111111_2 is equal to

(a) $A13F_{16}$
(b) 41279_{10}
(c) 120477_8
(d) all of the above
(e) none of the above

3. Problems: 10 points each

DO THE **FIRST THREE** (3) QUESTIONS AND TWO (2) MORE FOR A TOTAL OF FIVE (**5**) QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED.

SA1. **REQUIRED:** Nonlinear Comparative Statics:
Consider a very simple representation of a "Classical" model given by
IS: $Y = A(r, Y, F)$
LM: $m = L(r + \pi, Y)$
with the real interest rate (r) and real money supply (m) as our endogenous variables, and real income (Y), expected inflation rate (π), and fiscal policy stance (F) as our exogenous variables. All function responses (partial derivatives) have the usual signs. Totally differentiate the two equations, and set up the resulting matrix representation of the Classical comparative statics. Use the inverse of the coefficient matrix to solve for dr and dm in terms of the exogenous change $d\pi$. Find and sign $\partial m/\partial\pi$ and $\partial r/\partial\pi$.

SA2. **REQUIRED:** Optimization:
Suppose you want to maximize $x^{1/3}y^{2/3}$ subject to $x + 2y = k$. (Here k is a fixed parameter.) Set up a Lagrangian for this problem, and use it to produce first order necessary conditions for a maximum. Solve for x and y. How do x and y change as k changes? (That is, what is the elasticity of x w.r.t. k, and what is the *elasticity* of y w.r.t. k?)

SA3. **REQUIRED:** *Given:* A binary relation P (on a set X) with open lower contour sets, and some non-empty compact set $S \subseteq X$.
Prove: if P is acyclic then P can be maximized on S. (I.e., $\exists x \in S$ such that, for all $s \in S$, $\neg sPx$.)
Give a detailed justification of each step in your proof. (This will include a *proof* of the finite case.)
Explain the *economic* interest of the Walker (1977) theorem.

SA4. A binary relation R on a set X has an asymmetric component P. If P is acyclical, must the algorithm found in the following code snippet eventually produce a maximal element for any finite set $S \subseteq X$? (Here `alternatives` is a list of the elements in S, and `beats` is a boolean function of two variables, where `beats(x,y)==True` iff xPy.) Explain *in detail*, and provide an example (where you work through the algorithm, step by step).

```
bestsofar = alternatives[0]
for candidate in alternatives:
  if beats(candidate, bestsofar):
    bestsofar = candidate
```

SA5. Explain how a binary matrix represents a binary relations.
Explain how to use boolean matrix operations to produce a representation of the reflexive closure, symmetric closure, and transitive closure of a binary relation R.
For each of the following binary matrices,

- write down the represented binary relation (R). (Call the elements x, y, and z, in that order.)
- For each matrix, create the reflexive closure, symmetric closure, and transitive closure of R. (Show your matrix work!)
- For each matrix, create the transitive closure of the reflexive closure. (Show your matrix work!)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

SA6. Consider a binary relation R on a set X. Show that R is transitive iff $R \circ R \subseteq R$.

SA7. Let K be a compact set (in a Hausdorff space) and let $p \in K^c$. Prove that K and p can be "separated" in the following sense: there are non-overlapping open sets, one including K and one containing p.