1. Problems: 15 points each

DO **ALL** (4) QUESTIONS. QUESTIONS ARE EQUALLY WEIGHTED.

SA1. **REQUIRED:** Carefully explain how to construct a binary matrix to represent a binary relation. (Be completely explicit; e.g., start by providing the *definition* of a binary relation.)
Carefully explain how to use boolean matrix operations to produce a representation of the reflexive closure, symmetric closure, and transitive closure of a binary relation R. (That is, do not just show the operations, but explain why they accomplish the goal.)
Consider the following binary matrices:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

For each of the binary matrices,

- write down the represented binary relation (R). (Call the elements x, y, and z, in that order.) Draw a simple graph plot of each relation. Produce a simple graph plot of each inverse relation.
- Create the reflexive closure, symmetric closure, and transitive closure of R. (Produce any needed transformations with *explicit* matrix operations, explaining what is needed, and showing any relevant matrix work.) Draw the related graphs.
- Create the transitive closure of the reflexive closure. (Use matrix operations, and show any relevant matrix work.)
  Doing things intelligently is valued; e.g., don't do clearly unnecessary matrix operations.

SA2. **REQUIRED:** Nonlinear Comparative Statics:
Consider a very simple representation of a Classical IS-LM model. The structural equations are
IS: $Y = A(r, F)$
LM: $m = L(r + \pi, Y)$
The endogenous variables are the real interest rate (r) and the real money supply (m). The exogenous variables are therefore real income (Y), expected inflation (π), and the fiscal policy stance (F). All function responses (partial derivatives) have the usual signs, with $A_F > 0$.

- Totally differentiate the two equations. Provide an intuitive interpretation of the differentials.
- Prepare for the Classical comparative statics experiments by setting up a matrix representation of this system.
- Produce the inverse of the coefficient matrix. Prove that you have an inverse.
- Use your inverse matrix to solve for dr and dm in terms of the exogenous changes.
- Let us focus on one comparative-statics experiment: find and sign $\partial r/\partial F$. Desribe how you get from the previous step to this result.
- Provide a graphical illustration of this comparative-statics experiment.

SA3. **REQUIRED:** Bivariate Optimization:
Consider the function $f(x, y) = 13 - 6x + x^2 - 4y + y^2$.

- Sketch the partially applied functions, first for $x = 0$, and then for $y = 0$. (This does not need to be a detailed drawing, but it should clearly illustrate these function.)
- For each partially applied function, find the stationary points (if any), and show how to determine whether each stationary point represents a maximum or a minimum.

- Find the stationary points (if any) for the bivariate function. For each stationary point, determine whether you have a minimum or a maximum by examining the Hessian of the function. Be sure to show all your work and to explain what you are doing in each step.
- Explain in detail the relation between your univariate results and your bivariate result. (One paragraph.)

SA4. **REQUIRED:** On a set finite X, consider a transitive binary relation R. (Do not assume any other properties.) Let I denote the symmetric component and P the asymmetric component. Prove the following.

- given any walk $(x_n, \ldots, x_0)$, we know $x_n \, R \, x_0$. (Use induction.)
- R satisfies AAC
- I and P are transitive
- $(x \, P \, y \wedge y \, I \, z) \implies x \, P \, z$
- $(x \, I \, y \wedge y \, P \, z) \implies x \, P \, z$

2. Multiple Choice: 2 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please write the question number and answer letter in your bluebook.)

MC1. The antiderivative of $\sin(x)$ is

(a) $\cos(x)$
(b) $-\cos(x)$
(c) $\tan(x)$
(d) $-\tan(x)$
(e) none of the above

MC2. The antiderivative of $\cos(x)$ is

(a) $\sin(x)$
(b) $-\sin(x)$
(c) $\tan(x)$
(d) $-\tan(x)$
(e) none of the above

MC3. The value of the definite integral $\int_0^1 e^x$ is

(a) $e - 1$
(b) $e^x - 1$
(c) $\ln(1)$
(d) $\ln(x)$
(e) undefined

MC4. The Kolmogorov probability axioms stipulate that a probability measure is

(a) positive semi-definite
(b) normalized
(c) sigma-additive
(d) a. and b.
(e) all of the above

The rest of this exam is missing.