Print your name: ____________________

Midterm Examination — Fall 2017

Econ 705 — Page Total = 5

1. Comparative Definitions: 10 points each

DO **ALL** QUESTIONS. QUESTIONS ARE EQUALLY WEIGHTED. For each term, provide a mathematical statement, an intutively helpful verbal restatement, and an illustrative *example*. (Be sure to explain why your example is in fact an example.) Then compare and contrast the two terms in each problem, highlighting the points of similarity and difference.

DEF1. topology vs algebra of sets

DEF2. compact set vs closed set

2. Problems: 20 points each

DO **ALL** QUESTIONS. QUESTIONS ARE EQUALLY WEIGHTED.

SA1. Nonlinear Comparative Statics:
Consider the supply and demand system represented by these two "structural" equations:

$$Q = S(P, W) \qquad Q = D(P, P_s)$$

Here Q is quantity, P is market price, W is the wage, and P_s is the price of a substitute good. The endogenous variables are P and Q: determine how they respond to the exogenous variables P_s and W.

You should totally differentiate this *structural* form in order to derive the partial derivatives of the implicit reduced form. Provide intuitve economic reasoning to sign the partial derivatives of the structural model functions, and use this information to sign the complete comparative statics results. Specifically:

- Totally differentiate the two equations. Provide an intuitive verbal interpretation of the differentials.
- Prepare for the comparative statics experiments by setting up a matrix representation of this system.
- Produce the inverse of the coefficient matrix. Prove by matrix multiplication that you actually have an inverse.
- Use your inverse matrix to solve for dP and dQ in terms of the exogenous changes dP_s and dW.
- Provide extra detail for one comparative-statics experiment: find and sign $\partial Q/\partial W$. Explain how you get from the previous step to this result.
- Provide a graphical illustration of this comparative-statics experiment.

SA2. Using elements from $X = \{x, y, z\}$, we have the collection of ordered pairs $R = \{(x, x), (x, y), (y, z), (z, x)\}$.

- R defines a binary relation on X. Explain why. (Carefully define all terms.)
- Is this binary relation reflexive, symmetric, or transitive? (Define each term, and justify your answer by referring explicitly to your definition.)
- Construct a boolean matrix representation of this binary relation. Carefully explain how your matrix relates to the relation.
- Carefully explain how to use boolean matrix operations to produce a representation of the reflexive closure, symmetric closure, and transitive closure of *any* binary relation. (Be sure to define all terms.)

- Use matrix operations to produce a representation of the reflexive closure, symmetric closure, and transitive closure of R. (Show any relevant matrix work. Doing things intelligently is valued; e.g., don't do clearly unnecessary matrix operations.) Provide a simple graph plot of each result.

SA3. Consider the binary relation B on a set X.

(a) For any set $S \subseteq X$, describe the B-maximal set in S.

(b) Provide necessary and sufficient conditions for the existence of a maximal element on every non-empty *finite* subset of X. Prove that your conditions are necessary and sufficient.

(c) State and prove Walker's theorem for maximization on any *compact* set. Make sure your math is rigorous and that you define *all* terms. *Note:* Be sure to give a *detailed* intuitive explanation of each step in your proofs.

(d) In a few sentences, explain why Walker's theorem is important for economists.

3. Multiple Choice: 2 points each

DO ALL OF THE FOLLOWING QUESTIONS. ALL QUESTIONS ARE EQUALLY WEIGHTED. (Please write the question number and answer letter *in your bluebook.*)

MC1. As an experiment, we are going to flip a coin n times, keeping track of the result from each flip. For $k = 0, \ldots, n$, we define events $E_k = \{\omega \in \Omega | \operatorname{total}(\omega) = k\}$, where $\operatorname{total}(\omega)$ is the sum over the zeros and ones that are the elements of the resulting n-tuple. Consider the set $\mathcal{A}$ comprising all possible ways to form unions of these events, plus the empty set.

(a) $\mathcal{A}$ is closed under pairwise union.

(b) $\mathcal{A}$ is closed under complementation.

(c) $\mathcal{A}$ is a partition of the sample space.

(d) * a. and b.

(e) all of the above

MC2. The antiderivative of $\sin(x)$ is

(a) $\cos(x)$

(b) * $-\cos(x)$

(c) $\tan(x)$

(d) $-\tan(x)$

(e) none of the above

MC3. The antiderivative of $\cos(x)$ is

(a) * $\sin(x)$

(b) $-\sin(x)$

(c) $\tan(x)$

(d) $-\tan(x)$

(e) none of the above

The rest of this exam is missing.