Lecture Notes 3

The Monetary Approach to Flexible Exchange Rates

International Economics: Finance
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3 The Monetary Approach

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This chapter is our first attempt to understand exchange rate determination. A good model will help us understand the past and anticipate the future. In chapter 2 we saw that many economic decisions are affected by expectations of future exchange rates. Yet exchange rates are notoriously hard to predict. One way economists hope to improve our exchange rate predictions is by discovering the fundamental determinants of exchange rate movements. The monetary approach to flexible exchange rates focuses on domestic and foreign money supply and money demand. Monetary policy is given the central role in exchange rate determination. The determinants of domestic and foreign money demand also prove to be fundamental determinants of the exchange rate.

3.1 Key Ingredients of the Monetary Approach

The monetary approach has two key ingredients: exogeneity of the real exchange rate, and a simple Classical model of price level determination.\(^1\) Exogeneity of the real exchange rate means that inflation at home or abroad will not affect how much foreign goods cost in terms of domestic goods. The Classical model of price determination says roughly that the price level is proportional to the money supply, so that monetary policy is the key determinant of inflation rates.

Eventually, we will explore both of these constituents in some detail. Suffice it to say that as short-run descriptions of real economies, both appear quite unrealistic. However as long-run descriptions, they show somewhat more promise. So the monetary approach to flexible exchange rates is best seen as a description of long-run outcomes. As a description of short-run outcomes, it serves as a reference model that highlights some core concerns in our attempt to understand exchange rate determination.

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\(^1\)Some authors treat uncovered interest parity as central to the monetary approach. We will not need this restriction. We treat the risk premium as exogenous but possibly nonzero.
3.1. KEY INGREDIENTS OF THE MONETARY APPROACH

3.1.1 Exogenous Real Exchange Rates

Let $P$ be the domestic consumer price index and $P^*$ be the foreign consumer price index. For now, we will keep things simple by thinking of each price index as the monetary cost of a fixed consumption basket. Equation (3.1) defines the real exchange rate, $Q$.

$$Q \overset{\text{def}}{=} \frac{SP^*}{P} \quad (3.1)$$

We call $Q$ the real exchange rate because it tells you the rate at which domestic goods must be given up to obtain foreign goods. The monetary approach to flexible exchange rates assumes that $Q$ is exogenous. This exogeneity assumption fits naturally with the Classical model of price determination, which generally treats real variables as exogenous.

Given the real exchange rate, the nominal exchange rate and the relative price level have a determinate relationship given by (3.2).

$$S = Q \frac{P}{P^*} \quad (3.2)$$

Here $S$ is the exchange rate, $P$ is the domestic price index, $P^*$ is the foreign price index, and $Q$ is the exogenous real exchange rate. For any given $Q$, equation (3.2) requires that exchange rate movements offset price level movements so that the rate at which goods actually exchange for each other remains unchanged.

Purchasing Power Parity

Most presentations of the monetary approach to flexible exchange rates assume that the real exchange rate is not only exogenous but that it is invariant. This is called the purchasing power parity assumption.\textsuperscript{2} Constancy of the real exchange rate implies that the exchange

\textsuperscript{2}Given a common base year and identical price index construction, the relative price level, $P/P^*$, is often called the purchasing power parity exchange rate (or sometimes simply the purchasing power parity). See chapter 5 for a detailed discussion of purchasing power parity. You may simplify for now by setting $Q = 1$ and thinking of PPP as an application of the law of one price.
rate is proportional to the relative price level. For example, suppose domestic inflation leads to a doubling of the domestic prices, while foreign inflation is zero. Then a doubling of the exchange rate will leave the real exchange rate unchanged. As a theory of exchange rate determination, this is only a beginning: it does not explain the determination of relative price levels. That is our next project.

### 3.1.2 The Classical Model of Price Determination

In the Classical model of price level determination, the supply of money determines the (perfectly flexible) price level. In the simple Classical model, monetary policy has no influence on real economic activity. The real interest rate and real income are determined in the goods and labor markets independently of monetary policy. This *dichotomy* (i.e., independence of the real sector from the monetary sector) is a helpful simplification when we model monetary phenomena such as the nominal interest rate, the inflation rate, or the exchange rate. It allows us to treat real income, the real interest rate, and even the real exchange rate as exogenous when we are modeling the determination of the price level.

Four key assumptions of the simple Classical model are relevant to the determination of the price level.

- real money demand \((L)\) is a stable function of real income \((Y)\) and the nominal interest rate \((i)\).

- the money market is in continuous equilibrium.

- in addition to real income \((Y)\) and the real interest rate \(r\), the nominal money supply \((H)\) is exogenous.

- the price level \((P)\), *not* the interest rate \((i)\), moves to clear the money market.

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3In fact some economists consider it less than a beginning, since they feel that the relevant price indices \((P\) and \(P^*)\) are not observable Hodrick (1978); MacDonald (1993).
The first two of these assumptions are common to almost all macroeconomic models. The last two assumptions are specific to models in the Classical tradition.

The first assumption is implied by standard theoretical treatments of the demand for money.\(^4\) It embodies the theoretical dependence of liquid transactions balances on the desired level of transactions and the opportunity cost of liquidity. The nominal interest rate represents the opportunity cost of holding money rather than less liquid assets, and real income proxies the real value of monetized transactions in the economy. A rise in the interest rate therefore decreases money demand, while a rise in real income increases money demand. Our real money demand function can therefore be written as \(L(i, Y)\).

The second assumption is also standard to almost all macroeconomic models, and since adjustments can take place very rapidly in asset markets, it is very reasonable. It says that the real money supply is always equal to real money demand:

\[
\frac{H}{P} = L(i, Y) \tag{3.3}
\]

Equation (3.3) is a characterization of money market equilibrium, where \(H\) is the money supply, \(P\) is the price level, \(i\) is the nominal interest rate, \(Y\) is real income, and \(L(\cdot, \cdot)\) is the real money demand function. So (3.3) says that real money supply equals real money demand, where real money demand is a stable function of \(i\) and \(Y\).

The third assumption lists the standard exogeneity assumptions of the simple Classical model. For the moment we will additionally treat the nominal interest rate as exogenous—an assumption we will soon drop. The final assumption, perfect price flexibility, is very strong. It implies that our model of money market equilibrium can be interpreted as a model of price level determination.

We are now ready to model our Classical theory of price determination. The theory states that nominal money supplies and real money demand determine the price level. Specifically,

\(^4\)In empirical work, however, this formulation is used for long-run real money demand. Short-run money demand functions are generally characterized in terms of a partial adjustment to the desired long-run level (\(?\)).
the price level is determined as the ratio of nominal money supply to real money demand, where real money demand is determined by real income and the nominal interest rate. Algebraically, we simply solve (3.3) for the price level, which gives us (3.4) a simple Classical model of price determination.

\[
P = \frac{H}{L(i, Y)} \tag{3.4}
\]

Basic Predictions of the Classical Model

Equation (3.3) suggests some immediate predictions of our simple Classical model of price level determination. Given the interest rate, an exogenous increase in the money supply raises the price level so as to leave the real money supply unchanged.\(^5\) This is the core Classical story about price determination: changes in the money supply simply change the price level proportionately without causing any real changes in the economy. Changes in money demand also affect the price level, of course. An increase in real income raises real money demand, and the price level falls to restore equilibrium. Similarly, an increase in the interest rate lowers real money demand, and the price level rises to restore equilibrium.

3.2 The Crude Monetary Approach Model

The monetary approach applies our simple Classical Model to the determination of the price level in both at home and abroad. The foreign country equivalent of (3.4) is (3.5).

\[
P^* = \frac{H^*}{L^*(i^*, Y^*)} \tag{3.5}
\]

Here an asterisk indicates a foreign value. In each country, the price level is determined by the ratio of the nominal money supply to real money demand. It follows that the relative price level is determined by the ratio of the relative nominal money supply to relative real

\(^5\)Recall that the Classical Model treats \(H\) and \(Y\) as exogenous. Interest rate determination is considered in section 3.3.2.
3.2. THE CRUDE MONETARY APPROACH MODEL

money demand.

\[ \frac{P}{P^*} = \frac{H/H^*}{L(i,Y)/L^*(i^*,Y^*)} \] (3.6)

This is true even if these two economies are completely closed. Our consideration of specifically open-economy considerations begins with the introduction of the exchange rate.

In the monetary approach, the exchange rate is determined directly by the relative price level via purchasing power parity (PPP). We use (3.2) and (3.6) to write the crude monetary approach model to exchange rate determination as (3.7).

\[ S = Q \frac{H/H^*}{L(i,Y)/L^*(i^*,Y^*)} \] (3.7)

We call the model “crude” because it remains incomplete: we have not yet modeled the determination of interest rates. Nevertheless, the crude monetary approach does express the exchange rate in terms of variables for which data are readily available—interest rates, incomes, and money supplies—and such data have been used to test it. (See section 3.2.2.) These variables are often referred to as exchange rate fundamentals, and an implication of the monetary approach is that these exchange rate fundamentals should help us explain and predict the behavior of the exchange rate.

3.2.1 Core Predictions of the Monetary Approach to Flexible Exchange Rates

The core prediction of the monetary approach to exchange rate determination is that relative money supplies affect the exchange rate. Looking at (3.7), we can see that an increase in the relative money supply leads to a depreciation of the exchange rate.

\[ \uparrow \frac{H}{H^*} \rightarrow \uparrow S \] (3.8)
Since the monetary approach to exchange rate determination is based upon the purchasing power parity relationship, this result is naturally expected. A change in the relative money supply changes the exchange rate because it affects the relative price level. A higher relative money supply implies a higher relative price level, and by PPP the exchange rate changes with the relative price level.

The prediction is even more specific than this. If the relative money supply doubles, so does the relative price level If the relative price level doubles, so does the spot rate. So according to the monetary approach to flexible exchange rates, the movement in the spot rate is proportional to the movement in the relative money supply. That is, for each percentage change in the relative money supply, there is a one percent change in the spot rate. We say that the elasticity of the spot rate with respect to the relative money supply is unity.

Symmetrically, an increase in relative money demand leads to an appreciation of the exchange rate.

\[
\uparrow L/L^* \rightarrow \downarrow S
\]  

(3.9)

Once again, this is simply a reflection of the effects of relative money demand on the relative price level. We know that a rise in \(Y\) or a fall in \(i\) will raise domestic money demand, which increases relative money demand. So the monetary approach predicts that a rise in
domestic income or a fall in the domestic interest rate will appreciate the domestic currency. Symmetrically, a rise in $Y^*$ or a fall in $i^*$ will raise foreign money demand, which decreases relative money demand. So the monetary approach predicts that a rise in foreign income or a fall in the foreign interest rate will depreciate the domestic currency.

Of course money demand can change for other reasons, such as deregulation and financial innovation. (As examples of innovation, think of the spread of credit cards or ATM machines.) Many observers believe that such factors destabilized money demand in the 1980s and 1990s. The monetary approach predicts that financial innovation that reduces the demand for money will cause the domestic currency to depreciate.

The exchange rate is the domestic currency price of foreign money, so it is natural that an increase in the supply of foreign money will appreciate the domestic currency (i.e., lower the price of foreign exchange) while an increase in the demand for foreign money will depreciate the domestic currency (i.e., increase the price of foreign exchange). This is just what happens in the monetary approach.

<table>
<thead>
<tr>
<th>1% Increase In:</th>
<th>$H$</th>
<th>$H^*$</th>
<th>$Y$</th>
<th>$Y^*$</th>
<th>$i$</th>
<th>$i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resulting $\Delta S$:</td>
<td>1%</td>
<td>-1%</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.1: Core Predictions: Crude Monetary Approach

**Inflation in Zaire**

We have seen that a core prediction of the monetary approach to flexible exchange rates is that, ceteris paribus, money supply increases cause inflation and corresponding depreciation. Even though other factors are always changing, this theory would certainly predict that very large, sustained increases in the money supply will produce inflation and associated currency depreciation. This makes the experience of high inflation countries relevant to a first assessment of the monetary approach. In particular, hyperinflations seem to offer natural testing grounds for the monetary approach model. After all, one of its core components is the Classical model of price determination, which emphasizes long-run relationships between
the money supply and other nominal variables (such as prices or exchange rates). In a hyperinflation, changes in such nominal variables dwarf all the real changes in the economy. We might therefore hope that, during a hyperinflation, the kinds of relationships emphasized by the Classical model would be more readily exposed to view.

Consider the example of Zaire. In the mid-1990s, Zaire experienced an unusually long and severe hyperinflation. During the late 1980s the country had experienced high inflation, around 70% per year. At the end of 1990, things took a turn for the worse. Haughton (1998) reports that in five years consumer prices rose a total of 6.3 billion percent. Beaugrand (1997) reports inflation of nearly 10,000% in 1994. In the midst of all this was a failed currency reform in 1993. On October 22, 1993, at the rate of one for every 3 million (old) zaire, and was initially fixed at 3nz/$ (Beaugrand, 1997). By 1997, a 1-million-zaire note was worth about $3 in Kinshasa. Figure 3.2 illustrates the simultaneous inflation and depreciation, as predicted by the monetary approach to flexible exchange rates.

Inflation in Zimbabwe

Between early 2006 and early 2007, prices in Zimbabwe rose by more than 1,000%.

The Mugabe government nevertheless refused to devalue the Zimbabwean dollar, which on the black market was worth only 5% of its official value. Gideon Gono, governor of the Reserve Bank, announced plans to reduce broad money supply growth from "over 1,000 percent to between 415 and 500 percent by December 2007 and subsequently to under 65 percent by December 2008." While this showed a good awareness of the source of inflation, the next month Gono offered a change in policy that was parodic: he announced that price increases would be illegal from March through June. This policy was abandoned when shortages quickly emerged. In July 2008, Gono estimated the inflation rate at 2.2 million percent and was immediately challenged by analysts who calculated it at more that 12 million percent. By one report, a loaf of bread rose in price from a billion Zimbabwean dollars to 100

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6In an interesting aside (Haughton, 1998) reports: “The new notes were not accepted by residents of Kasai province, who continue to use only the old notes; inflation in the province is said to be negligible.”
Figure 3.2: Money, Prices, and Exchange Rates

billion in about a day. The new 100 billion dollar note issued in mid-July was not enough to buy a loaf of bread by the end of July. Dollarization, although illegal, became increasingly common.

### 3.2.2 Early Tests of the Monetary Approach Model

Naturally economists wish to determine the extent to which the available data are supportive of the core predictions of their models. Before we can develop specific tests of the monetary approach to flexible exchange rates, we need to find data that appropriately represent the variables in our model. We must also parameterize relationships that we have been describing only in the most general terms, perhaps by specifying a functional form for the money demand function. So before proceeding to the empirical test of the monetary approach, we need to discuss relative real money demand in a bit more detail.

We have been writing our money demand functions as $L(i,Y)$ and $L^*(i^*,Y^*)$. This represents in a general fashion our assumption that money demand is a stable function of the nominal interest rate and real income. Monetary theory tells us that real money demand responds positively to real income, which is a proxy for the level of transactions in the economy, and negatively to the interest rate, which is a proxy for the opportunity cost of holding money. The monetary approach to flexible exchange rates tells us that these money demand responses are also relevant to exchange rate determination: the spot rate responds negatively to real income, due to the implied rise in money demand, and negatively to the interest rate, due to the implied fall in money demand. When we conduct an econometric investigation of the monetary approach to flexible exchange rates, we let the data suggest numerical values for the sizes of these responses. For example, we estimate the elasticity of the spot rate with respect to real income. We would like to learn whether our theoretical predictions match the relationships present in the data. We are therefore interested in the sign and the size of our estimates.

Table 3.2 lists some of the early econometric tests of the monetary approach to flexible
3.2. **THE CRUDE MONETARY APPROACH MODEL**

exchange rates. The first study in the list is by far the most famous: Jacob Frenkel’s study of the post-WWI German hyperinflation. As we have discussed, since nominal changes dwarf real changes during a hyperinflation, we might hope that the kinds relationship between money and exchange rates stressed by the monetary approach will be more readily detected in the data. In the case of the German hyperinflation, Frenkel felt this justified ignoring changes in domestic income and all foreign variables, since the changes in these variables were very small compared to the huge changes in the spot rate and German money supply that took place during the hyperinflation. He estimated the response of the spot rate to only two variables: the German money supply, and the opportunity cost of money.

<table>
<thead>
<tr>
<th>Study:</th>
<th>Sample:</th>
<th>Estimated Response of the Spot Rate to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$H$</td>
</tr>
<tr>
<td>Frenkel (1976)</td>
<td>1921.02–1923.08</td>
<td>0.975</td>
</tr>
<tr>
<td>Bilson (1978b)</td>
<td>1972.01–1976.04</td>
<td>1.0013</td>
</tr>
<tr>
<td>Bilson (1978a)</td>
<td>1970.04–1977.05</td>
<td>1.0026</td>
</tr>
<tr>
<td>P&amp;W (1979)</td>
<td>1972–1977</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Note: The reported “responses” are money supply elasticities, income elasticities, and opportunity-cost semi-elasticities of the spot rate. Opportunity costs of holding money are proxied by interest rates or forward discounts. A dash indicates an omitted variable. A left arrow indicates that the domestic and foreign money demand parameters are constrained to equality.

Table 3.2: Early Tests of the Monetary Approach

Frenkel’s results are summarized in table 3.2. The dashes indicate responses that he did not estimate, and the left arrow indicates that he constrained to equality the response to the domestic and foreign interest rate. His estimates, using monthly German data, are correctly signed, significantly different from zero, and of plausible magnitude. In addition, money supply elasticity of the spot rate differs insignificantly from unity. Many economists viewed Frenkel’s results as providing dramatic support for the monetary approach.

Bilson (1978a; 1978b) offers additional early evidence on the crude monetary approach model. Bilson (1978b) estimated a model of the DEM/GBP exchange rate using monthly

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7We discuss his opportunity cost measure in more detail below.
data for the period 1972.01–1976.04. The results are reported in table 5. Overall the results look fairly good. However the coefficient on his opportunity cost measure is not statistically different from zero, and Bilson required considerable econometric manipulation to arrive at the reported results. The study therefore lends modest support to the crude monetary approach. Bilson (1978a) extends this sample to 1970.04–1977.05. (Note that this includes some years preceding the general float.) After resorting to a complicated estimation procedure, Bilson finds the results reported in table 5. His estimates have the correct sign and reasonable magnitude. All the coefficients are significantly different from zero. Again the coefficients on money supplies are close to unity, the value implied by the monetary approach.

Keep in mind that flexible prices, PPP, and stable money demand lie at the core of the monetary approach. It is therefore rather surprising, in a non-hyperinflationary setting, that the monetary approach performed so well with monthly data. Price flexibility and PPP are most likely to hold in the longer run. Indeed, as we shall see, PPP performs very poorly in the short run, and an important reason appears to be price stickiness. Furthermore, the assumption of a stable money-demand function has become increasingly suspect during the post-Bretton Woods era (?).

Hodrick (1978) and Putnam and Woodbury (1979–80) also offer early empirical tests of (3.7) using monthly data from developed countries. Hodrick (1978, eq.7) examine the USD/DEM exchange rate from April 1973 to September 1975. The first row of table 3.1 reports his estimated responses of the spot rate to money, interest rates, and income. Recall that theory predicts that the spot rate moves proportionally to the relative money supply, or equivalently that it is unit elastic with respect to the relative money supply. In

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8. The reported results are for his equation (45), which reports long-run coefficients calculated from an estimation incorporating a partial adjustment money demand equation. (See problem 8.) He also incorporates prior expectations about the coefficients in his estimation procedure.

9. This is his equation (12). Not reported in the table is his coefficient on a time trend, which suggests a half a percent appreciation of the DEM per month. This adds up: over the period, it makes an important contribution to the model's predicted values.

10. See Hodrick’s article for a discussion of the data and of his variable definitions.
table 3.2, we therefore would hope to find numbers close to 1 in the $H$ column and close to -1 in the $H^*$ column. While we do not find a perfect match, the estimated responses were close enough for Hodrick to conclude that the data did not reject this prediction of the monetary approach. His results are somewhat favorable to the theory, in the sense that his estimated responses generally have the predicted signs, are significantly different than zero, and are of plausible magnitude. Moreover, he reports that exchange rate fundamentals are explaining about two-thirds of the variation in the spot rate. Nevertheless, there are a couple of noticeable problems. German income appears to have a weak influence on the exchange rate. (The response is smaller than expected; in fact, Hodrick finds that it is insignificantly different from zero.) Worse, the spot rate response to the German interest rate is the wrong sign (and differs significantly from zero).\textsuperscript{11} The support for the flexprice monetary approach model offered by Hodrick’s research is modest at best.

Putnam and Woodbury’s results are somewhat more encouraging. Their estimates are also reported in Table 3.2.\textsuperscript{12} They use data from 1972–77 to examine the GBP/USD exchange rate. Table 3.2 reports their results for monthly data; they also report results for quarterly data, which look similar. Fewer coefficients are reported because they reported results when relative money, relative income, and the relative interest rate are the regressors, thus constraining the domestic and foreign responses to be the same. (They report that relaxing these constraints produces comparable results.) All the coefficients have the correct sign and are significantly different from zero. The coefficient on relative income is disappointingly far from its predicted value of zero, but the regression otherwise provides good support for the monetary approach to flexible exchange rates.

\textsuperscript{11}Hodrick suggests that German capital controls adopted in February 1973 might offer an explanation, but in the context of the monetary approach this is not evident.

\textsuperscript{12}The estimated “responses” are money supply elasticities, income elasticities, and interest rate elasticities of the spot rate. They have corrected for first order autocorrelation with a Hildreth-Lu procedure. See their article for a discussion of the data and for variable definitions.
Using the Forward Discount

When we begin to think about empirical applications of the monetary approach, we have to decide what measured quantities will represent each of the variables in our exchange rate equation (3.7). For example, there are many different foreign and domestic interest rates: which are appropriate for our purposes? Rather than choose an interest rate, some researchers have turned to the forward discount on domestic currency.

Use of the forward discount instead of domestic and foreign interest rates requires a special assumption, which is fairly standard in the empirical literature testing the monetary approach to flexible exchange rates. This is the assumption of a common interest rate response in foreign and domestic money demand. Relative real money demand is given a special form:

\[ \frac{L(i,Y)}{L^*(i^*,Y^*)} = \mathcal{L}(i - i^*, Y, Y^*) \] (3.10)

With this assumption, we can write

\[ S = Q \frac{H/H^*}{\mathcal{L}(i - i^*, Y, Y^*)} \] (3.11)

In this version of the crude monetary approach, the foreign and domestic interest rates enter only as an interest rate differential \((i - i^*)\). Recall that chapter 2 developed the covered interest parity condition.

\[ fd = i - i^* \] (3.12)

With high capital mobility, covered interest parity should hold closely for assets that are good substitutes. So one natural way to measure the interest rate differential in the monetary approach model is with the forward discount. This yields (3.13).

\[ S = Q \frac{H/H^*}{\mathcal{L}(fd, Y, Y^*)} \] (3.13)
3.2. **THE CRUDE MONETARY APPROACH MODEL**

Frenkel (1976) used this approach in his study of the post-WWI German hyperinflation.\(^\text{13}\) Bilson 1978a; 1978b also uses the forward discount to proxy the relative opportunity costs of holding domestic and foreign money.

**In-Sample Fit and Out-of-Sample Forecasting**

Later studies proved less supportive of the crude monetary approach to flexible exchange rates. It turns out that the in-sample fit of monetary approach models depends on the choice of exchange rate and the sample period. This is rather discouraging news. In addition, only a small fraction of the actual variation in the exchange rate is explained by the monetary approach model. However in-sample fit is only one criterion for model performance, and arguably it is not the most interesting criterion. Since the mid-1980s, economists have been more interested in evidence of the predictive power of their models. In particular, exchange rate research has concentrated on the out-of-sample forecasting ability of existing models. A model that is capable of improving our exchange rate forecasts can be very useful even if its in-sample fit is disappointing.

The classic study of the out-of-sample forecasting ability of modern exchange rate models is Meese and Rogoff (1983). Meese and Rogoff pit a one-step-ahead random-walk model against the core inflation and other models. Using monthly data (1973.03-1981.06) for several countries, they found that the one-step-ahead random walk model outperforms the structural models in forecasting the exchange rate. However Schinasi and Swamy (1987) argue that the one step ahead forecasts favor the random walk model, which uses the lagged exchange rate. They find much greater forecast error in the multi-step random walk forecasts. And indeed, adding a lagged dependent variable to the structural models greatly improves their forecasting performance.\(^\text{14}\)

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\(^\text{13}\)Frenkel's justification of using the forward discount was somewhat different than that offered here.

\(^\text{14}\)Other studies finding some advantages from including the lagged spot rate include Woo (1985), Somanath (1986), Boughton (1987), Driskill et al. (1992), and MacDonald and Taylor (1994).
3.2.3 Volatility Puzzle

Recall that the crude monetary approach to flexible exchange rates, as represented by (3.7), explains the exchange rate in terms of a small number of exchange rate fundamentals: interest rates, incomes, and money supplies. Although we expect short-run deviations from this relationship, we might hope that such deviations are not large relative to the volatility of the exchange rate. However the exchange rate is surprisingly volatile relative to these fundamentals. In order to highlight this volatility puzzle for the monetary approach, Flood and Rose (1999) aggregate these fundamentals in order to offer a rough comparison of the variability of the exchange rate and the variability of the fundamentals. Two problems emerge. First, for the 1980s and 1990s Flood and Rose (1999) find no link across countries between the volatility of aggregate fundamentals and exchange-rate volatility. Second, they can find no evidence that countries tend to experience periods of volatile exchange rates at the same time as they experience volatility in the aggregate fundamentals. For example, the collapse of the Bretton Woods system of pegged exchange rates produced a tremendous increase in exchange-rate volatility without noticeably increasing the volatility of the aggregate fundamentals. To explain this, it seems we need to discover a new exchange rate fundamental that behaves entirely differently under pegged and floating exchange-rate regimes.

\[ L(i, Y) / L(i^*, Y^*) = L(i - i^*, Y/Y^*) = \frac{Y/Y^*}{\exp i - i^*} \]

and the aggregate fundamentals are therefore

\[ \frac{M/M^*}{Y/Y^*} \exp i - i^* \]  

(3.14)
3.3 Exchange Rates and Monetary Policy

It should be clear at this point that a basic contention of the monetary approach is that monetary policy is an important determinant of the behavior of the exchange rate. We might represent a change in monetary policy by a change in the level of the money supply or by a change in its growth rate. This suggests two monetary policy “thought experiments” we can consider in order to explore the basic predictions of the monetary approach.

3.3.1 Money Supply Shocks

An unanticipated exogenous disturbance of the economy is called a shock. Our first thought experiment concerns a money supply shock: a one-time, permanent, unanticipated increase in the domestic money supply, with no associated change in the expected growth rate of the money supply over time. Keeping in mind this is a Classical style model, we expect the domestic price level to increase proportionally (so that the real money supply is unchanged). Keeping in mind the purchasing power parity condition, the exchange rate must increase in proportion to the price level. So the model predicts a depreciation of the exchange rate proportional to the increase in the money supply. This is easily seen in equation (3.7): if we double \( H \) on the right hand side, we must double \( S \) on the left hand side to maintain the equality.

The behavior of the money supply, the real money supply, the price level, and the spot rate are illustrated in figure 3.3.\(^{16}\) The rise in the nominal money supply, \( H \), takes place at time \( t_0 \). The change in the spot rate and the price level proportional to the change in the nominal money supply, so the real money supply remains unchanged.

This first thought experiment illustrates what would happen in a Classical economy where the money supply followed a “random walk”. This is the situation when each month the money supply is just as likely to rise as to fall. Suppose you are asked to offer a single number

\(^{16}\)Absolute heights are meaningless in the graphs: only relative changes matter. To keep the presentation as simply as possible, we have plotted \( P \) and \( S \) in the same graph. (This may be seen as a harmless normalization, where \( Q = 1 \) and \( P^* = 1 \).)
as your forecast of the next month’s money supply in such circumstances. To be as right as possible on average, you should predict that the money supply will remain at its present level. Next month you will be in the same position. The money supply may be higher or lower than you had expected, but you should predict that it will stay at its new level. In this sense, changes in the level of the money supply are permanent. However, changes in the growth rate of the money supply tell us nothing about the future money supply growth rate, which is always expected to be zero.\footnote{Nothing is changed if the expected growth rate is some other constant, say 0.5%/month, except that we must include this expected growth in our forecasts.}

### 3.3.2 Money Growth Shocks

The next experiment we want to consider is a one-time, permanent, unanticipated change in the growth rate of the money supply. As a preliminary, we need to think a bit more carefully about the role of interest rates in the monetary approach model.

**Interest Rates and Inflation**

The Classical model of interest rate determination begins with the following definition of the real interest rate.

\[
  r \overset{\text{def}}{=} i - \pi^e
\]  

(3.15)

The real interest rate is the expected real rate of return from holding an interest bearing asset. In an inflationary environment, part of the nominal rate of return on an asset simply compensates for rises in the price level. For example, if an individual holds his wealth in
assets paying 10% per year when inflation is 10% per year, then the nominal interest rate of 10% per year simply maintains the purchasing power of wealth but does not augment it. The Classical model treats the real interest rate as exogenous, which turns (3.15) into a theory of interest rate determination.

\[ i = r + \pi^e \]  

Equation (3.16) is often called the *Fisher equation*. It decomposes the nominal interest rate into the real interest rate \( r \) and expected inflation \( \pi^e \). Since the real interest rate is exogenous in the Classical model, variations in expected inflation imply variations in the nominal interest rate. We can use the Fisher equation (3.16) to substitute for the nominal interest rate in our representation of money market equilibrium (3.3). The result is (3.17).

\[ \frac{H}{P} = L(r + \pi^e, Y) \]  

The representation of money market equilibrium in equation (3.17) predicts a decline in real balances when expected inflation is higher. Expected inflation should be high when actual inflation is persistently high, this is a prediction that we should be able to roughly test simply by looking at real money balances in high inflation economies. Consider table 3.3, which is based on data from several hyperinflations. The dates are the first and last months for which the inflation rate exceeded 50%/month. The inflation rates in the third column are the average monthly inflation rates for the period. The final column shows the smallest level of real balances reached during the hyperinflation as a ratio of the level of real balances held at the beginning of the hyperinflation. Despite tremendous growth in the money supply during these hyperinflations, the real money supply is declining. High inflation raises the expected cost of holding money. The table shows that the decline in real balances during hyperinflations is dramatic, which supports the prediction of equation (3.17).
### Table 3.3: Inflation and Real Money Demand

<table>
<thead>
<tr>
<th>Country</th>
<th>Dates</th>
<th>%ΔP/month</th>
<th>((H/P)_{\text{min}}/(H/P)_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>1921.10–1922.08</td>
<td>47</td>
<td>0.350</td>
</tr>
<tr>
<td>Germany</td>
<td>1922.08–1923.11</td>
<td>322</td>
<td>0.030</td>
</tr>
<tr>
<td>Greece</td>
<td>1943.11–1944.11</td>
<td>365</td>
<td>0.007</td>
</tr>
<tr>
<td>Hungary</td>
<td>1923.03–1924.02</td>
<td>46</td>
<td>0.390</td>
</tr>
<tr>
<td>Hungary</td>
<td>1945.08–1946.07</td>
<td>19,800</td>
<td>0.003</td>
</tr>
<tr>
<td>Poland</td>
<td>1923.01–1924.01</td>
<td>81</td>
<td>0.340</td>
</tr>
<tr>
<td>Russia</td>
<td>1921.12–1924.01</td>
<td>57</td>
<td>0.270</td>
</tr>
</tbody>
</table>


### Additional Predictions of the Monetary Approach

While monetary policy is sometimes characterized in terms of changes in the level of the money supply, it is more often characterized in terms of the rate of growth of the money supply. Our next thought experiment determines the predictions of the monetary approach for changes in money supply growth rates.

The simplest case to consider is a one-time, permanent increase in the rate of growth of the money supply. Suppose such a change takes place today. Given our previous work on the monetary approach to flexible exchange rates, we might expect that the rate of inflation immediately adjusts to the new growth rate of the money supply, and that by purchasing power parity this becomes the new rate of depreciation as well. This is basically correct, but there must be an additional one time adjustment of the price level. This is because a higher inflation rate implies a higher nominal interest rate, which lowers real money demand.

Recall the Fisher equation for the domestic interest rate.

\[
i = r + \pi^e
\]

Similarly, the foreign interest rate can be written as

\[
i^* = r^* + \pi^{e*}
\]
Substituting for the nominal interest rates in (3.7) allows us to write our exchange rate
determination equation as (3.18).

$$S = \frac{Q}{L(r + \pi^e, Y)/L^*(r^* + \pi^{e*}, Y^*)}$$

Equation (3.18) is known as the *core-inflation* formulation of the monetary approach to
exchange rate determination because it focuses attention on the determinants of expected
inflation. This equation is particularly useful for developing the more detailed predictions of
the monetary approach to flexible exchange rates.

To see this, apply (3.18) to our monetary policy experiment. Looking at (3.18), it
is clear that unless we say something specific about the behavior of expectations, we will
have little to say about the behavior of the exchange rate. Let expectations adjust very
quickly to the policy change. For any level of inflation expectations, we know from our
previous monetary policy experiment that the price level must rise proportionately to the
money supply increases. It is therefore natural to set expected as well as actual inflation
equal to the new money supply growth rate. The higher expected inflation rate lowers
domestic money demand. Prices must rise not only because the money supply is increasing
but also because real money demand has fallen. Thus we have a brief *magnification effect*:
the inflation rate measured over a period near the time of the policy change will actually
exceed the new money supply growth rate. Since exchange rate movements are determined
by purchasing power parity, the rate of depreciation must display a similar magnification
effect (Frenkel, 1976).

Figure 3.4: Increase $\hat{H}$ at $t_0$
Figure 3.4 illustrates these outcomes. The change in the money supply growth rate takes place at time $t_0$. We plot the time path of the growth rate of the money supply ($\hat{H}$), the rate of inflation ($\hat{P}$), and the rate of depreciation ($\hat{S}$). We also graph the level of real balances over time. Note that although the level of the money supply does not jump, the spot rate and price level do jump. This jump is the source of the “magnification effect”. In contrast with our previous experiment, the real money supply is changed by this experiment. This change derives from the higher interest rate implied by higher expected inflation, which in turn is a result of the change in monetary policy.

3.4 Transitory Shocks

We now consider how the exchange rate responds to transitory shocks, by which we will mean exogenous changes that last for a single period.

Recall that our monetary approach model consists of two key ingredients: purchasing power parity, and the Classical model of price determination.

\[
S = QP/P^*
\]

\[
P/P^* = \frac{H/H^*}{\mathcal{L}}
\]

We have been working with a very simple model of money demand, where $\mathcal{L}$ depends on the interest differential and relative income.

\[
\mathcal{L} = \mathcal{L}(i - i^*, Y/Y^*)
\]

In turn, we recognized that covered interest parity holds under perfect capital mobility, so that

\[
i - i^* = fd = \Delta s^e + rp
\]
3.4. TRANSITORY SHOCKS

So our model of the spot rate involves relative money demand, which depends on expected depreciation. This suggests that any discussion of transitory shocks will require careful attention to the role of expectations. In this section, our approach will be to treat individuals as understanding that they are dealing with a transitory shock, which allows them to form correct expectations about the future. Since individuals know that the exchange-rate will return to its long-run equilibrium level next period, we have

\[ \Delta s^e = (S_{t+1} - S)/S = (\bar{S} - S)/S \]  

(3.23)

This must affect our description of short run equilibrium in the money market, since we must allow for the effects of interest rates on money demand, and in the short run interest rates must move with the spot rate. The relative price level is now determined as

\[ \frac{P}{P^*} = \frac{H/H^*}{\mathcal{L}(\frac{S - \bar{S}}{S}, rp, Y/Y^*)} \]  

(3.24)

and therefore must be negatively related to the current spot rate. This manifests as downward slope to the LM curve, which we label LM\(_{sr}\) to emphasize its dependence on the given level of \(\bar{S}\).

Our monetary approach model to temporary shocks is therefore

\[ S = Q \frac{H/H^*}{\mathcal{L}(\frac{S - \bar{S}}{S}, rp, Y/Y^*)} \]  

(3.25)

3.4.1 Money Supply Shock

Suppose that the economy is initially in a long-run equilibrium, with spot rate \(\bar{S}\), when it receives an unanticipated transitory shock to level of money supply. Since individuals know that the exchange-rate will return to its long-run equilibrium level next period, we have

\[ \Delta s^e = (S_{t+1} - S)/S = (\bar{S} - S)/S \]  

(3.26)
As with a permanent shock, the domestic currency must depreciate in response to the increase in $H$. But individuals anticipate that there will be a subsequent appreciation, which lowers the interest differential and increases the relative demand for money. This moderates the short-run movement of the exchange rate, which is therefore less than in the case of a permanent shock.

### 3.5 Anticipated Monetary Policy

In section 3.3 we considered two changes in monetary policy: a change in the level of the money supply, and a change in its growth rate. In both cases we considered an unanticipated change in policy. We now consider what happens when these changes in policy are anticipated.

#### 3.5.1 An Anticipated Change in the Money Supply

Consider a one-time, permanent, anticipated increase in the level of the money supply. The behavior of the money supply is identical to the first monetary policy experiment we considered in section 3.3. However, the increase in the money supply at time $t_0$ is now known at the earlier time $t_a$. That is, at time $t_a$ we correctly anticipate an increase in the money supply.
supply, which will take place at time $t_0$.

Since the behavior of the money supply is identical to our earlier experiment with an unanticipated change in the money supply, we might think that prices, exchange rates, and real balances should also behave in the same fashion. And indeed, if expected inflation were unaffected by the anticipated increase in the money supply—for example, if expectations were “static” in the sense of being exogenously fixed—the outcomes would in fact be identical. But when the increase in the money supply is anticipated, it is not plausible to treat expectations as static.

When we expect the money supply to increase, we predict an increase in the price level. This must show up immediately in the nominal interest rate, which is the real interest rate plus expected inflation. The rise in the nominal interest rate decreases money demand, and this in turn increases the equilibrium price level. By the purchasing power parity relationship, the increase in the domestic price level shows up as a proportional depreciation of the exchange rate. So an anticipated future money supply increase leads to inflation and depreciation today. This is true no matter how far in the future is the anticipated change, although the farther away it is the smaller today’s response will be.

Figure 3.6 characterizes the outcomes when we anticipate at time $t_a$ the increase in the level of the money supply that does not take place until time $t_0$. We can easily characterize economic outcomes before $t_a$ and after $t_0$. Before $t_a$ there is no expected inflation, corresponding to the constant money supply. In line with this, actual inflation is zero, as is exchange rate depreciation. After $t_0$ expected inflation will be correct if it again matches the zero money supply growth rate. So after $t_0$ we should have expected inflation, actual infla-
tion, and exchange rate depreciation each at 0% per year. The difficulty lies in determining what happens between \( t_a \), the time the policy is anticipated, and \( t_0 \), the time the policy is implemented.

Expectations play a critical role between \( t_a \) and \( t_0 \). To aid us in working through the economic outcomes during this period, we will work with expectations that are very accurate: the actual and anticipated economic outcomes are the same. One crucial key to understanding the outcomes lies in the recognition that there cannot be any anticipated jumps in the price level.

Here is why. Suppose you expected an upward jump in the price level next Friday. This jump will lower the value of any money balances you are holding, so you would like to get rid of your money balances the day before. But so would everyone else, and knowing this you expect money demand to fall on Thursday, with the implication that you must expect the jump in the price level to take place on Thursday instead of Friday. Of course you face the same difficulty in expecting the price level to jump on Thursday. Or Wednesday. Or any future day. The conclusion is that if there is to be a jump in the price level, it must be unanticipated. So any jump in the price level must take place at \( t_a \), as soon as the policy change is anticipated.

One implication of this analysis is that, in the case of anticipated changes in the money supply, the contemporaneous link between expected inflation and expected changes in the money supply is broken. Consider price determination in the Classical model, solving (3.17) for (3.27). A rise in expected inflation causes an *immediate* jump in the price level and the exchange rate.

\[
P = \frac{H}{L(r + \pi^e, Y)} \tag{3.27}
\]

Now we have enough information to predict the economic outcomes between \( t_a \) and \( t_0 \). At \( t_a \), the price level rises. Since the money supply has not yet changed, this determines an initial fall in real balances. The lower real balances are compatible with money market equilibrium because of the increase in expected inflation. Remember, expected inflation
tracks actual inflation. But since the money supply has not yet increased, the rise in prices implies a fall in real balances. Inflation expectations must rise over time so as to maintain money market equilibrium at the falling level of real balances. Until the money supply actually increases, the price level and the exchange rate keep rising. Correspondingly, real balances keep falling.

In the middle graph you can see the changes in the price level: the actual inflation. In the last graph, you can see the changes in real balances, which are due to changes in expected inflation. Since actual and expected inflation are tied together by our assumption that individuals form very accurated expectations, we need the inflation implied by the middle graph to be consistent with the expected inflation implied by the last graph.

If individuals are very good at predicting the money supply increase, prices should have risen just enough by the time of this increase that no further changes are necessary to restore equilibrium in the money market. That is, the total change in the price level over the period $t_a$ to $t_0$ will be just proportional to the increase in the money supply that takes place at time $t_0$. At this new price level, the increased money supply is just adequate to raise real balances back to their old level, which is the equilibrium level of real balances when expected inflation is zero.

### 3.5.2 An Anticipated Change in Money Growth

In section 3.3.2, we characterized changes in monetary policy in terms of the rate of growth of the money supply. Our final monetary policy thought experiment determines the predictions of the monetary approach for anticipated changes in money supply growth rates.

Suppose at time $t_a$ we correctly anticipate a one-time, permanent, increase in the rate of growth of the money supply, which will take place at time $t_0$. Suppose annual money growth will rise from zero to ten per cent. Once again, the price level and the inflation rate must respond immediately to our expectations of a future policy change. Correspondingly, via the purchasing power parity condition, the exchange rate also responds. Recall that increases
in expected inflation immediately increase the exchange rate (by reducing relative money demand and thereby increasing the relative price level).

In figure 3.7, we can once again easily characterize economic outcomes before \( t_a \) and after \( t_0 \). Before \( t_a \) there is constant expected inflation, corresponding to the constant money supply growth rate. Let us say this growth rate is 5% per year. In line with this, actual inflation is 5% per year, as is exchange rate depreciation. After \( t_0 \) expected inflation will be correct if it matches the new money supply growth rate of 10% per year. So after \( t_0 \) we should find expected inflation, actual inflation, and exchange rate depreciation each to be 10% per year. The difficulty once again lies in determining what happens between \( t_a \), the time the policy is anticipated, and \( t_0 \), the time the policy is implemented.

![Figure 3.7: Anticipated Increase in \( \hat{H} \)](image)

As before, we rule out anticipated jumps in the price level. This allows us to predict the economic outcomes between \( t_a \) and \( t_0 \). At \( t_a \), the price level rises. Since the money supply has not yet changed, this determines an initial fall in real balances. The lower real balances are compatible with money market equilibrium because of the increase in expected inflation. Expected and actual inflation rise together over time so as to maintain money market equilibrium. The price level and the exchange rate are rising faster than the money supply. Correspondingly, real balances keep falling. If individuals are very good at predicting the monetary policy change, prices should have risen just enough by the time of the policy change so that no further changes are necessary to restore equilibrium in the money market. That is, the total change in the price level over the period \( t_a \) to \( t_0 \) will reduce the real money supply to the level that is compatible with an expectation of 10% annual inflation. At time \( t_0 \) the money supply starts growing by 10% per year, and the economy is in equilibrium with
10% per year in (actual and expected) inflation and exchange rate depreciation.

3.6 Conclusion

The monetary approach to flexible exchange rates has two key constituents: purchasing power parity and a simple Classical model of price determination. The result is a very simple model of exchange rate determination that, like the Classical model of price determination, should have interest primarily as a long run description of exchange rate determination.

Early empirical tests yielded encouraging support for the monetary approach. Later tests proved much less satisfactory. However much of the empirical work on the monetary approach has been conducted with small samples of monthly data for countries with low average inflation rates. The failure of the monetary approach in such tests does not bear on its usefulness as a description of the fundamental long-run influences on the exchange rate. These can be better tested over long periods of time or in situations, such as the German hyperinflation, where the sheer magnitude of the monetary changes ensure that their influence will be felt even in the short run.

Predictions of the monetary approach include the following. An increase in the domestic money supply leads to a proportional depreciation of the spot exchange rate. An increase in domestic income generates an exchange rate appreciation. An increase in the domestic interest rate causes an exchange rate depreciation, and for this reason there is a “magnification effect” of changes in the money supply growth rate. It is natural to wonder how we might test these predictions, which require a characterization of expected inflation and its links to monetary policy. The next chapter addresses this.

Expansionary monetary policy might be represented either as a change in the level or as a change in the growth rate of the money supply. In each case, the policy change may be a complete surprise or it may be anticipated. This leads to four possible scenarios. A primary lesson is that any effort to model exchange rates must pay careful attention to the role of
expectations.
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real interest rate, see interest rate, real
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Problems for Review

1. Let $S$ be the USD/GBP exchange rate, $P^*$ be the pound cost of a consumption basket in the U.K., and $P$ be the dollar cost of a consumption basket in the U.S. Using the units of $S$, $P^*$, and $P$, find the units of the real exchange rate, $Q = SP^*/P$.

2. Suppose you are a U.S. resident with fifteen thousand dollars (USD 15k) for living expenses. You are contemplating a year abroad. Let the dollar-franc exchange rate be USD/FFR 0.2. Is this enough information to determine your relative material standard of living in the U.S. and France? Why or why not?

3. Given our discussion of the crude monetary approach model, what variables did Frenkel (1976) “dump” into the constant terms of his key exchange rate regression? How about Bilson (1978a)?

4. Consider a French resident who invests FFR 10,000 for one year at an annual interest rate of 10%. What is the nominal value of the investment at the end of the year? Suppose the French CPI rises from 100 to 110 that year. What is the real value of the investment at the beginning of the year? What is the real value of the investment at the end of the year? What is the inflation rate? What is the real rate of return on the investment over the year?

5. Produce graphs similar to those in figure 3.3 to illustrate the effects of a one-time, permanent increase in $Y$.

6. Produce graphs similar to those in figure 3.3 to illustrate the effects of a one-time, permanent increase in $i$. (You may assume this increase is due to a one-time, permanent increase in the exogenous real interest rate.)

7. Produce graphs similar to those in figure 3.4 to illustrate the effects of a one-time, permanent increase in $Y$. (Assume a unitary income elasticity of money demand.)
8. Bilson (1978b) gets reasonable results for the crude monetary approach model only by adopting a partial adjustment formulation for money demand. In this formulation, domestic money market equilibrium obtains when

\[ h - p = \beta_0 + \beta_1 y + \beta_2 i + \beta_3 (h_{t-1} - p_{t-1}) \]

Assume the same formulation for the foreign country, and derive the new spot rate equation. (Specifically, show that the current spot rate depends on its own past value.)

9. Suppose news at time \( t_a \) leads to the expectation that money supply growth will fall to zero at time \( t_0 \). Assuming expectations are correct, graph the behavior of the money supply, real balances, the price level, and the spot rate. [Comment: although the price level falls at \( t_a \), it may end up higher than it started by \( t_0 \).]
Bibliography


.1 Empirical Monetary Approach Models

In this section, we develop the popular log-linear representation of the monetary approach to the determination of flexible exchange rates.

.1.1 Characterizing Money Demand

Recall that our money demand function, \( L(i, Y) \), represents in a general fashion the dependence of money demand on the interest rate and income. When we move to empirical considerations, we will need to be more specific about the form of these influences. A popular and convenient functional form for money demand is

\[
L(i, Y) = Y^{\phi}e^{-\lambda i}
\]  

(28)

where \( e \) is the “natural base” (see the appendix to chapter 2). There are two parameters: \( \phi \) is the income elasticity of money demand, and \(-\lambda\) is the interest rate semi-elasticity of money demand. These determine the response of real money demand to real income and to the interest rate. The most important thing to note is that \( \phi \) and \( \lambda \) are positive, so that...
real money demand is increasing in real income and decreasing in the nominal interest rate. Imposing money market equilibrium yields

\[ \frac{H}{P} = Y^\phi e^{-\lambda i} \]  

(29)

Taking logarithms of both sides of (29), we get (30).

\[ h - p = \phi y - \lambda i \]  

(30)

Here \( p = \ln P \) is the logarithm of the price level, \( h = \ln H \) is the logarithm of the money supply, and \( y = \ln Y \) is the logarithm of income.

Price level determination as represented by (3.4) can now be restated as (31).

\[ p = h - \phi y + \lambda i \]  

(31)

We will adopt the functional form of equation (28) for both the domestic and foreign country. So the foreign price level is determined as

\[ p^* = h^* - \phi^* y^* + \lambda^* i^* \]  

(32)

There are two new parameters: \( \phi^* \) is the income elasticity of foreign money demand, and \( \lambda^* \) is the interest rate semi-elasticity of money demand.

**Relative Money Demand**

Relative money demand becomes

\[ \frac{L}{L^*} = \frac{Y^\phi e^{-\lambda i}}{Y^* \phi^* e^{-\lambda^* i^*}} \]  

(33)
With this specification of relative money demand, if we take the logarithm of (3.7), we get the following restatement of the crude monetary approach.

\[ s_t = q + h_t - h^*_t - (\phi y_t - \phi^* y^*_t) + (\lambda i_t - \lambda^* i^*_t) \]  

(34)

Here \( p^* = \ln P^* \) is the logarithm of the foreign price level, \( h^* = \ln H^* \) is the logarithm of the foreign money supply, \( y^* = \ln Y^* \) is the logarithm of foreign income, and \( i^* \) is the foreign interest rate.

### 1.2 Testing the Monetary Approach

Of course we do not expect (34) to hold exactly. When we test the monetary approach, we allow for temporary deviations from purchasing power parity and from our representation of relative money demand to add an “error term” to (34). This lack of perfect fit forces us to consider statistically based tests of the monetary approach, such as the early regression tests of Hodrick (1978) and Putnam and Woodbury (1979–80) discussed in section 3.2.2. Using monthly data from April 1973 to September 1975, Hodrick (1978, eq. 7) found the following for the USD/DEM exchange rate.

\[ s = 7.85 + 1.52h - 1.39h^* + 2.53i + 1.93i^* - 2.23y + 0.073y^* \]  

(35)

In section 3.2.2, we also noted that the monetary approach is usually developed under the further assumption that the foreign and domestic countries have identical interest rate semi-elasticities. In our log-linear specification, this implies that \( \lambda = \lambda^* \), yielding (36).\(^{18}\)

\[ s_t = q + h_t - h^*_t - (\phi y_t - \phi^* y^*_t) + \lambda (i_t - i^*_t) \]  

(36)

\(^{18}\)We also noted that some work assumes identical income elasticities \( \phi = \phi^* \) as well as identical interest rate semi-elasticities. Here is a quick way to develop the crude monetary approach under these assumptions. Note that by taking the logarithm of (3.2), purchasing power parity can be characterized by

\[ s_t = q + p_t - p^*_t \]
In this version of the crude monetary approach, the foreign and domestic interest rates enter only as an interest rate differential \((i_t - i_t^*)\), allowing us to substitute the forward discount from interest parity condition \(fd = i - i^*\). This yields (37).

\[
s_t = q + h_t - h_t^* - (\phi y_t - \phi^* y_t^*) + \lambda fd_t
\]

The numbers reported in table 5 are the estimated coefficients from equation 37, as we see more explicitly in table 4.

<table>
<thead>
<tr>
<th>Study:</th>
<th>Sample:</th>
<th>Estimated Coefficient on</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(fd) (h) (h^<em>) (y) (y^</em>)</td>
</tr>
<tr>
<td>Frenkel (1976)</td>
<td>1921.02–1923.08</td>
<td>0.591 0.975 – – –</td>
</tr>
<tr>
<td>Bilson (1978b)</td>
<td>1972.01–1976.04</td>
<td>0.0228 1.0013 -1.0081 -1.0184 0.9990</td>
</tr>
<tr>
<td>Bilson (1978a)</td>
<td>1970.04–1977.05</td>
<td>1.3853 1.0026 -0.9846 -0.9009 1.0183</td>
</tr>
</tbody>
</table>

Table 4: Germany/U.K.: Early Estimates of Equation (37)

Implicitly, Frenkel dumped domestic income and all foreign variables into the constant term. His equation \((4'')\) is

\[
s = -5.135 + 0.975h + 0.591fd
\]

As discussed in section 3.2.2, all the estimated coefficients are statistically significant, and the coefficient on the money supply differs insignificantly from unity. This was viewed as dramatic support for the monetary approach. The table reports equation (45) of Bilson (1978b), which estimates a model of the DEM/GBP exchange rate using monthly data for

When domestic and foreign money demands have identical parameters, we have

\[
\begin{align*}
    h_t - p_t &= \phi y_t - \lambda i_t \\
    h_t^* - p_t^* &= \phi y_t^* - \lambda i_t^*
\end{align*}
\]

Solving these money market equilibrium conditions for prices and substituting into the PPP equation yields

\[
s_t = q + h_t - h_t^* - \phi (y_t - y_t^*) + \lambda (i_t - i_t^*)
\]
the period 1972.01–1976.04. Despite considerable econometric manipulation to arrive at the reported results, Bilson found the coefficient on \( fd \) not to be statistically different from zero. Bilson (1978a) extends this sample to 1970.04–1977.05. The results reported in table 4 are from his equation (12), which includes a time trend intended to capture trends in relative money demand. His equation (12) is

\[
s = -1.3280 + 1.0026h - 0.9846h^* + 1.3853fd - 0.9009y_t + 1.0183y_t^* - 0.0049t
\]

Although the estimated coefficient of -0.0049 on \( t \) appears small, a half a percent appreciation of the DEM per month adds up. As discussed previously, all the coefficients are statistically significant, and the coefficients on money supplies are close to unity.
.2 Partial Adjustment of Money Demand

In our discussions of the monetary approach, we have treated the current value of real money demand as a simple function of current income and interest rates (see chapter 3). However, applied work on money demand generally uses this functional form only for “long-run” money demand. For example, we might write

$$(h - p)_{t}^{lr} = \phi_{t}^{lr} y_{t} - \lambda_{t}^{lr} i_{t}$$

Since the work of Goldfeld (1973), short run, money demand is generally assumed to adjust only partially to its long-run level $h_{t}^{lr}$.

$$(h_{t} - p_{t}) - (h_{t-1} - p_{t-1}) = (1 - \alpha)\{(h_{t}^{lr} - p_{t}) - (h_{t-1} - p_{t-1})\}$$

or

$$h_{t} - p_{t} = (1 - \alpha)(h_{t}^{lr} - p_{t}) + \alpha(h_{t-1} - p_{t-1})$$

Substituting for long run real money demand yields

$$h_{t} - p_{t} = \phi y_{t} - \lambda i_{t} + \alpha(h_{t-1} - p_{t-1})$$

where $\phi = (1 - \alpha)\phi_{t}^{lr}$ is the short-run income elasticity of money demand and $\lambda = (1 - \alpha)\lambda_{t}^{lr}$ is the short-run interest rate semi-elasticity of money demand. As Bilson (1978b) and Woo (1985) emphasize, this change of functional form has implications for the monetary approach. Money market equilibrium is now characterized by

$$(h_{t} - h_{t}^{*}) - (p_{t} - p_{t}^{*}) = \phi(y_{t} - y_{t}^{*}) - \lambda(i_{t} - i_{t}^{*}) + \alpha[(h_{t-1} - h_{t-1}^{*}) - (p_{t-1} - p_{t-1}^{*})]$$

When combined with purchasing power parity, we then have a new exchange rate equation

$$s_{t} = q_{t} + (h_{t} - h_{t}^{*}) - \phi(y_{t} - y_{t}^{*}) + \lambda(i_{t} - i_{t}^{*}) - \alpha(h_{t-1} - h_{t-1}^{*}) + \alpha s_{t-1} - \alpha q_{t-1}$$
2. PARTIAL ADJUSTMENT OF MONEY DEMAND

Of course the inflation rate depends on many variables in the short run. In the monetary approach model, however, attention centers on the rate of growth of the money supply, and money supply growth rates ($\Delta h$ and $\Delta h^*$) are treated as exogenously set by the central bank. The expected inflation differential is therefore treated as deriving from perceived monetary policy. Recalling that for simplicity we are holding income constant, this implies that when money growth rates are expected to be constant $\pi^e - \pi^{e*} = \Delta h^e - \Delta h^{e*}$. Under such conditions, we can write our exchange rate determination equation as

$$S = \frac{H/H^*}{\mathcal{L}(\Delta h^e - \Delta h^{e*}, Y/Y^*)}$$

(38)

However, when changes in monetary policy are anticipated, we cannot use (38).

We might try to avoid choosing an interest rate by turning to the Fisher equation, which allows us to rewrite this as

$$s_t = \lambda(r_t - r_t^*) + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\pi_t^e - \pi_t^{e*})$$

If we treat the real interest rate differential as fixed, we no longer have to worry about picking the right interest rate. But now we must worry about finding proxies for unobserved expectations.

Capital Mobility

Suppose you are considering the purchase of an asset denominated in foreign currency. For example, you might be a U.S. citizen contemplating the purchase of a Canadian bond. If you expect the U.S. dollar to lose value (depreciate) against the Canadian dollar while you hold the bond, this will raise your expected dollar returns from holding the Canadian bond.

Let us calculate the real returns, from the domestic perspective, to holding the domestic
and foreign interest bearing asset. The real return from holding the domestic interest bearing asset is \( i - \pi \). The real return from holding the domestic interest bearing asset is \( i^* + \Delta s - \pi \).

If the two assets are expected to have the same real return, then

\[
    i = i^* + \Delta s^e
\]

This is “uncovered interest parity” (UIP). When UIP holds, the interest differential equals the expected rate of depreciation.

\[
    i - i^* = \Delta s^e
\]

The difference in the nominal rates of return is offset by an expected change in the price of foreign exchange.

Uncovered interest parity is not implied by covered interest parity. Ensuring UIP requires more than just frictionless financial markets. However, we can imagine special conditions under which UIP should hold, so that the assets of two different currency denominations should be expected to yield the same real returns. Suppose for example that investors are indifferent as to the currency denomination of their assets as long as they have the same expected return. That is, although investors care about their average returns they do not care about the volatility of these return. In particular, they do not care about “currency risk”. This situation is often referred to as perfect substitutability or risk neutrality, and it implies that the expected real rate of return on the domestic asset equals the expected real rate of return from holding the foreign asset: \( i - \pi^e = i^* + \Delta s^e - \pi^e \). Perfect substitutability implies uncovered interest parity (UIP):

\[
    i - i^* = \Delta s^e
\]

Note the UIP is an equilibrium condition, not a theory of interest rate determination. It is a relationship that must hold when foreign and domestic assets are perfect substitutes.
This is a very strong condition on international capital markets. An even stronger condition arises when we add a condition known as ex ante relative purchasing power parity, which just says that changes in the real exchange rate cannot be anticipated. The idea that real exchange rate changes cannot be anticipated is captured by treating the expected change in the real exchange rate as zero: $\Delta q^e = 0$. From the definition of the real exchange rate, this implies

$$\Delta s^e + \pi^e - \pi^* = 0$$

We will refer to this condition as expected PPP. It says that there is no expected change in the real exchange rate. This may be because the real exchange rate has been constant and is expected to stay that way. More realistically, expected PPP results when any past changes in the real exchange rate are expected to be permanent.

Perfect substitutability and expected PPP together imply a very strong condition on world capital markets. Combining expected (41) with (42), we see that the expected real returns earned by domestic resident on domestic assets will equal the real returns expected by foreign residents on foreign assets.

$$i - \pi^e = i^* - \pi^{*e}$$

This situation is known as real interest parity. Most empirical work on the monetary approach assumes expected PPP and UIP. However, some early tests of the monetary approach took place under weaker assumptions than real interest parity, and we will begin by looking at those.
Real Interest Parity

Risk Neutrality

We now add a fourth standard component of the monetary approach to exchange rate determination: the uncovered interest rate parity condition.

\[ i_t - i_t^* = \Delta s_t^e \]

Substituting this into the crude monetary approach solution yields the standard flex-price monetary approach solution for the behavior of the exchange rate:

\[ s_t = h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\Delta s_t^e) \] (43)

Immediately, we see that exchange rate expectations are a crucial determinate of the current exchange rate. Unfortunately, these expectations are not directly observable. Empirical tests of the monetary approach must therefore grapple with the problem of finding a useful proxy for these expectations. We consider three approaches to modeling expectations in our exchange rate models: the core inflation approach (Frenkel 1979), regressive expectations, and rational expectations.

Expected PPP

Note that if purchasing power parity is expected to hold next period, then consistency requires

\[ s_t^e = p_t^e - p_t^{*e} \]
This can be combined with the purchasing power parity condition for the current period to yield a link between expected depreciation and the expected inflation differential.

$$\Delta s_t^e = \pi_t^e - \pi_t^*$$

We can therefore rewrite the standard monetary approach solution as the core inflation approach.

$$s_t = h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\pi_t^e - \pi_t^{*e}) \quad (44)$$

Comparing (44) with (43) makes it clear that this formulation just assumes away the contribution of the real interest differential. Thus if we include a constant term in our empirical work, say because we wish to allow for constant terms in the money demand equations, (44) and (43) are not readily distinguishable.

Comment: some work has been done under the assumption of interest inelastic money demand (e.g., Florentis et al. 1994). This yields the same models as above, with $\lambda = 0$.19

Comment: We have written

$$i_t - i_t^* = \Delta s_t^e = \pi_t^e - \pi_t^{*e}$$

Implying

$$i_t - \pi_t^e = i_t^* - \pi_t^{*e}$$

That is, open interest rate parity plus expected PPP implies real interest rate parity.

---

19The result is identical to that implied by the standard monetary approach when fundamentals follow a random walk. (See the next chapter.)
As discussed in the last chapter, this model is incomplete in that it does not include a characterization of interest rate determination. We therefore introduced the Fisher equation

\[ i_t = r_t + \pi_t^e \]

in order to characterize the interest rate differential as

\[ i_t - i_t^* = r_t - r_t^* + \pi_t^e - \pi_t^{*e} \]  \hspace{1cm} (45)

Combining (.2) and (45) yields (46), the core-inflation characterization of the monetary approach model.

\[ s_t = q + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(r_t - r_t^* + \pi_t^e - \pi_t^{*e}) \]  \hspace{1cm} (46)

Equation (46) highlights some key insights of the monetary approach to flexible exchange rates. Most important is the observation that, ceteris paribus, doubling the domestic money supply doubles the exchange rate.  \(^{20}\)

So far we have simply reviewed the derivation of the core-inflation version of the monetary approach model. We now observe that if PPP holds every period then informed individuals should expect it to hold. That is, the expected depreciation of the exchange rate should equal the expected-inflation differential.

\[ \Delta s^e = \pi^e - \pi^{*e} \]

This condition is known as expected purchasing power parity (EPPP). Under EPPP, the

\(^{20}\)Note that among the ceteris held paribus is expected inflation. Note too that doubling the money supply adds \(\ln 2\) to the log of the money supply.
monetary approach model of flexible exchange rates can be summarized as

\[ s = h - h^* - \phi(y - y^*) + \lambda(s^e - s) \]  

(MAFE)

The key difference in this approach is that the real interest differential is constrained to zero. But if we wish to determine absolute price levels in the two countries as well as their relative price level, we need to explicitly add the Fisher effect and exogenous real interest rates. We also need the assumption that (3.2) is expected to hold every period.

\[ \frac{L(i, Y)}{L^*(i^*, Y^*)} = \mathcal{L}(i - i^*, Y/Y^*) \]

This allows us to rewrite (3.7), the determination of the exchange rate, as (36).

\[ S = Q \frac{H/H^*}{L(i - i^*, Y/Y^*)} \]

(47)

The implied characterization of the foreign price level is given in (48), which is the foreign country equivalent of (31).

\[ p^* = h^* + \phi^* y^* - \lambda^* i^* \]  

(48)

Figure 8: Increase \( \hat{H} \) at \( t_0 \)
\[ \ln X(t) - \ln X(t - 1) = \%\Delta X \]

Application: Let us write real GDP as output per hour, \( A \), times the number of hours worked, \( H \).

\[ Y = AH \quad (49) \]

This allows us to break real GDP growth into productivity growth and growth in hours worked.

\[ \dot{Y} = \dot{A} + \dot{H} \quad (50) \]

For example, conventional wisdom places the potential growth rate of the U.S. economy between 2.0 and 2.5 percent per annum. Labor force growth causes hours worked to rise about 1% per year, and productivity growth makes up the other 1%-1.5% per year. In the postwar period before 1973, productivity grew at an average of 3.2% per annum, which yielded a much higher growth rate of GDP.

We can add some detail, when it is useful Hours worked can be written as the average number of hours worked per worker, \( h \), times the number of workers employed, \( N \).

\[ Y = AhN \quad (51) \]

The number of workers employed is the labor force, \( L \), times the employment rate, \( 1 - u \).

\[ Y = Ah(1 - u)L \quad (52) \]

This allows us to break real GDP growth into productivity growth, labor force growth, and reduction in unemployment.

\[ \dot{Y} = \dot{A} + \dot{h} - \frac{u}{1 - u} \dot{u} + \dot{L} \quad (53) \]

For example, conventional wisdom places the potential growth rate of the U.S. economy between 2.0 and 2.5 percent per annum. The idea is that the labor force grows about 1%
per year, and productivity growth makes up the other 1%–1.5% per year. In the postwar period before 1973, productivity grew at an average of 3.2% per annum, which yielded a much higher growth rate of GDP.

<table>
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<th>( Y )</th>
<th>( Y^* )</th>
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<td>Bilson (1978a)</td>
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Table 5: Germany/U.K.: Early Estimates of Equation (3.13)