Quick Notes on Growth

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1 Intro

Why do some countries grow rich while others remain poor? For economists, there may be no more fundamental economic question.

Since the industrial revolution, world output has grown faster than world population. But not all countries have grown apace. Diverse growth histories have brought huge differences in average real income.

The typical worker in Ethiopia works a month and a half to earn what a typical U.S. worker earns in a day. Nearly half the world's population live in countries were workers earn less than a tenth of U.S. workers. (Most of this is India and China.)

Real income differences are associated with large differences in nutrition, infant mortality, and life-expectancy. Example: the typical African mother has only a 30% chance of seeing all her children survive to age 5. Example: he life expectancy of a person born in sub-Saharan Africa in 1980 is just 48 years (William Easterly and Ross Levine, 1997).

The long-run effects on welfare of growth dwarf those of macroeconomic fluctuations. Consider two examples from Romer (1996).

Example 1: In the U.S., recessions temporarily depress income by a few percent relative to trend. Compare this to the effect of the 1970s productivity slowdown, which for a couple decades was associated with growth rates about 1%/year below the preceeding trend. Over a couple decades this reduced per capita income by more than 20%.

Example 2: The postwar average annual growth rate of real per capita income in India was about 1.3% for three decades. Growing at this rate, India will reach the current U.S. level in about 200 years. In the 1980s and 1990s, India's per capita real income growth exceeded 3%/year. At this rate it would take less than 100 years. If it could duplicate Japan's postwar growth rate of 5.5%, it will only take 50 years.

2 Stylized Facts

Stylized facts are rough generalizations that are considered to be of enough economic significance that they deserve explanation.

2.1 Kaldor

Kaldor (1958) offered six famous "stylized facts" about the growth of advanced industrialized economies:

1. $\widehat{Y/N} > 0$ relatively constant in LR (exponential growth)

2. $\widehat{K/N} > 0$ and $\hat{K}$ roughly constant

3. $\widehat{Y/K} \approx 0$ in LR: capital output ratio fairly constant

4. $\widehat{\pi/K} \approx 0$ in LR, so the rate of return to capital is trendless. For example, the real interest rate on government debt in the U.S. is trendless.

5. $\hat{Y}$ and $\widehat{Y/N}$ vary a lot across countries

6. high π/Y associated with high I/Y

Comment: 3 and 4 imply π/Y trendless, so the shares of capital and labor are roughly constant, with labor in the U.S. getting about 70%. and 2 and 3 imply I/Y is roughly constant (since $\hat{K}/(Y/K)$ is constant).

2.2 Jones

In addition to Kaldor's stylized facts, Jones (1998, ch.1) offers the following stylized facts:

- Enormous, persistent variation in per capita income across economies. The poorest countries have per capita incomes that are less than 5% of per capita incomes in the richest countries.

- Variations in GDP per worker are equally large and are highly, but not perfectly, correlated with per capita income. (E.g., Japan's high per capita income is supported partly by high labor force participation; it's productivity is below the other industrialized countries.)

- Growth miracles and growth disasters. Kaldor's fifth stylized fact stated that rates of economic growth vary substantially across countries. Jones notes that there are growth miracles (such as the NICs: Hong Kong, Singapore, Taiwan, and South Korea) and growth disasters (Venezuela, Madagascar, Mali, Chad). Note that small differences in growth rates eventually produce enormous differences in per capita incomes.

- Growth rates can vary over time. For example, India grew at 1.3%/yr from 1960–80 but 3.6%/yr from 1980–90. By implication, a country can move from being poor to being rich (and vice versa, like Argentina).

- Growth in output and growth in the volume of international trade are closely related. However most sub-Saharan African countries have high and rising trade intensities—higher than Japan's, which has been falling slightly since the 1960s.

- Both skilled and unskilled workers tend to migrate from poor to rich countries or regions. Note this suggests the real wage for both groups is higher in rich countries, despite the relative scarcity of skilled labor in developing countries.

2.3 International Differences in Growth

Empirical work on the sources of international income differences blossomed after the 1980s. One explanation: lack of data made it hard to compare income levels across a wide range of countries. This has been somewhat ameliorated by the Penn World Tables and other recent data sets. A second reason is more subtle, and turns on a common interpretation of the early theoretical literature. Theoretical work rooted in the mid-1950s suggested that growth was ultimately driven by technical advances: it was widely felt that these advances would be resistant to further analysis.

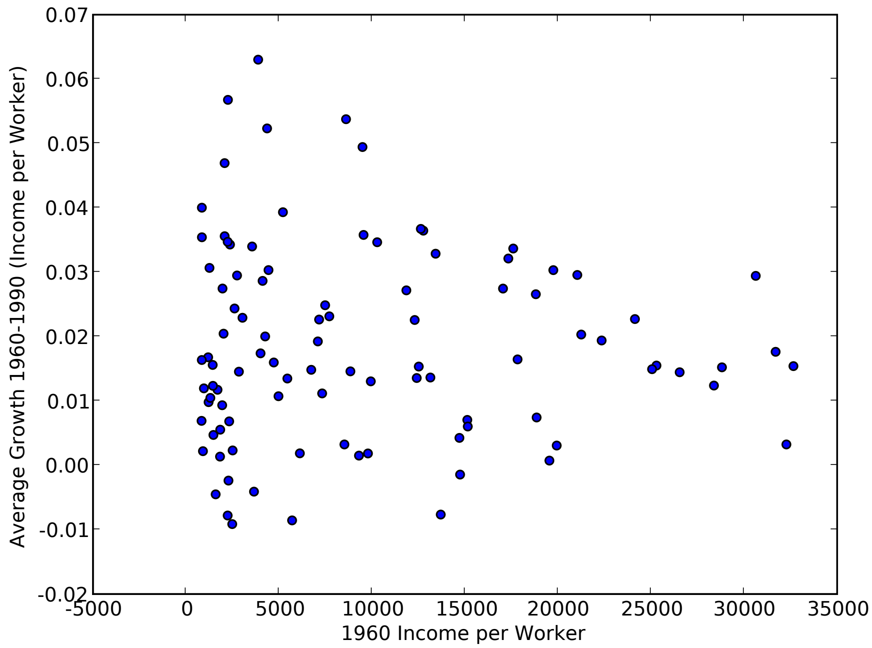
Following, Temple (1999 JEL), let's look at two or our "stylized facts" in a bit more detail.

- Growth Miracles and Disasters
- Large and Persistent Disparities in Per Capita Income

2.3.1 Growth Miracles and Disasters

Plot the average annual growth of real GDP per worker over 1960–90 against its initial level, a familiar diagram in the growth literature. (See, e.g., Temple (1999 JEL) Figure 1.) If countries are converging, one would expect to see a negative slope. Instead we find no general tendency for countries to converge to a common level of per capita income. Only among the already developed countries do we seem to find convergence. (Among countries with 1960 GDP per worker greater than $10,000 in 1985 'international dollars', there has been some convergence.) Within the group of poorer countries, there has been a greater variety of experience: some have done very well and others very badly. (See Sala-I-Martin 1996 EJ.)

These stylized facts lead some economists to think about complex theories that emphasise relative development traps and multiple equilibria. Despite all his emp-



Source: PWT 6.1 (data), based on Temple (1999 JEL, figure 1)

Figure 1: Growth vs. Initial Income p.c.

hasis on the role of incentives, and the role of institutions in creating incentives, Easterly (2001) treats path dependence and relative development traps very seriously. Others suggest simpler explanations, including low levels of factor accumulation among some countries. Still others place emphasis on institutional factors, which received increasing attention in the 1990s.

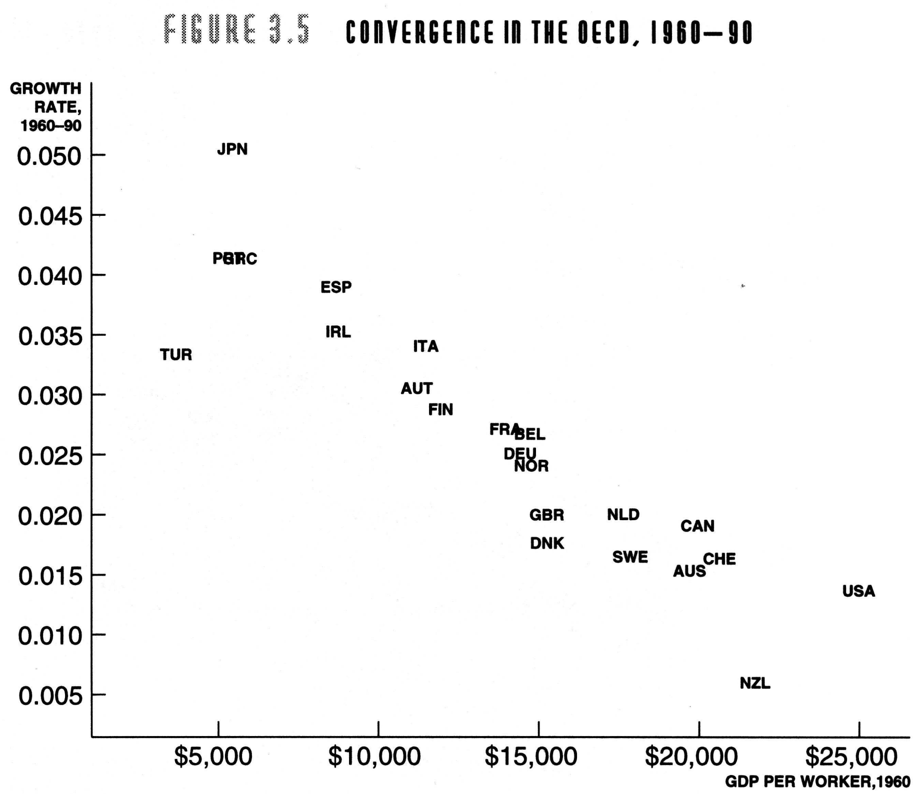


Figure 2: Convergence in the OECD
Source: Jones (1998), figure 3.5.

2.3.2 Large and Persistent Disparities in Per Capita Income

We can extract some other useful information from Table 1. One interesting fact is that most of these large developing countries roughly maintained their position relative to the USA between 1960 and 1990. Since the USA's per capita income grew

Country	POP1990	Y1960	Y1990	Y1995	Y2000
Nigeria	92	8	4	3	3
Indonesia	188	5	11	13	11
Pakistan	113	6	8	8	7
India	841	7	7	7	8
China	1138	3	6	9	12
Iran	57	24	17	19	18
Thailand	55	9	19	24	19
Brazil	151	20	26	24	21
South Africa	37	35	29	25	24
Mexico	84	29	25	23	24
Argentina	33	60	30	36	33
Japan	123	36	82	80	70

Based on Temple (1999 JEL, Table 1), calculated from version 6.1 of the Penn World Table.

Table 1: GDP p.c. Relative to US (1960, 1990, 1995, 2000)

at around 2% a year over this period, this indicates that these countries have grown at a similar rate. There does not appear to be an *absolute* poverty trap.

Be careful in making such comparisons. The obvious method is to value each country's quantities of final goods and services at domestic prices, and then convert these figures into a common monetary unit using the relevant exchange rates.

Picking the relevant exchange rate can be tricky, however. For example, at the turn of the century the EUR fell by 10% against the USD and then rose again by twice as much. Which value is relevant?

Comparisons between developed and developing countries are even trickier. In some sectors, higher relative prices prevail in developed countries (e.g., for services), and these can affect international comparisons. Instead of using exchange rates, incomes should be converted using special currency indexes which are calculated so that one unit will purchase the same bundle of goods across countries. These currency indices are sometimes called purchasing-power-parity-adjusted exchange rates.

To illustrate the difference this makes, Temple (1999) compares the incomes of the US and India. Converting Indian per capita GDP into US dollars at the offical exchange rate suggests that India's average income is just 2% of US income. If we use PPPs, the average income is measured as 5% of the US's. This illustrates the general observation tht comparison using exchange rates tends to overstate the magnitude of income disparities.

Economists now have access to a world table of national accounts with figures that are comparable across space and time and based on PPPs. The United Nations International Comparison Project (ICP) was designed in the 1960s to make such comparisons possible. Over 90 countries have participated in benchmark studies, which are then used to derive an aggregate PPP for each participating economy. These estimates have then been combined with national accounts data to form the Penn World Table (PWT). They are based on the collection of price data for a wide range of goods, which are aggregated to obtain a national PPP.

Two problems (see Heston and Summers 1996).

Data on capital service lives is hard to come by, which makes it difficult to arrive at accurate figures for net investment and capital stocks.

The available data on labor force participation and working hours is low quality. Heston and Summers (1996 p.23) write that participation rates "vary enough to make GDP per capita a very unsatisfactory proxy for GDP per worker". Temple (1999) gives an example: Japan's GDP per capita is around 80% of the USA's according to the PWT, but its GDP per worker is just 60%. This poses a problem for empirical work: growth theories generally model output per worker hour, but the available data on worker hours are weak (especially for developing countries).

Annual growth rates of output per worker

Miracles	Growth	Disasters	Growth
Korea	6.1	Ghana	-0.3
Botswana	5.9	Venezuela	-0.5
Hong Kong	5.8	Mozambique	-0.7
Taiwan	5.8	Nicaragua	-0.7
Singapore	5.4	Mauritania	-0.8
Japan	5.2	Zambia	-0.8
Malta	4.8	Mali	-1.0
Cyprus	4.4	Madagascar	-1.3
Seychelles	4.4	Chad	-1.7
Lesotho	4.4	Guyana	-2.1

Note: figures for Botswana and Malta based on 1960–89.
From Temple (1999 JEL, Table 2), calculated from version 5.6 of the Penn World Table.

Table 2: Growth Miracles and Disasters, 1960–90

2.4 Income vs. Welfare

There are many reasons why income may not be closely associated to welfare. One that has received some attention in the growth literature concerns the difference between income per capita, income per worker, and income per hour. For example, Jones (1997, p.6) notes that while Japan and West Germany had similar GDP per capita in 1990, Japan's higher labor force participation rate (63% vs. 49%) means that Japan had much lower real GDP per worker.

So which is a better measure of welfare? Or even of development? Jones suggests that real GDP per capita is a better welfare measure, while real GDP per worker is a better productivity measure. However he also notes that if real GDP per worker is a good proxy for the productivity of non-market activities, which do not show up in NIA statistics, then real GDP per worker may actually be the better welfare measure.

3 Growth Theories: Old (1950s and 1960s)

In the 1950s, rapid growth of the communist countries caused alarm in the West. It seemed that the USSR would quickly outgrow the Western democracies. What were the sources of this growth? Popular opinion attributed it to the institutional structure of the USSR, that a collectivist, authoritarian state could economically outperform a democratic state. In a way this is correct, as this structure allowed a massive investment in physical capital. It is now accepted that the growth was based entirely on the willingness to sacrifice current consumption for the sake of future production. This has a solid implication: since economic growth based on the expansion of inputs, rather than on the growth in ouput per unit of inputs, is subject to diminishing returns. The alarm was unnecessary: input growth could not continue at its early pace, so growth would inevitably slow down.

More recently, Asian economic institutions—the "Asian system"—have been proposed as models for improving Western growth rates. But the evidence is that Asian growth also seems to be attributable to input growth.[1]

Theorizing about long-run growth revived in the 1980s, after a hiatus of over two decades since the last spurt in the fifties and sixties. We can distinguish two strands: aggregative descriptive models (neoclassical and Post Keynesian), and normative optimizing models.

Barring a few exceptions, in the neoclassical growth models production technology was assumed to exhibit constant returns to scale and in many, though not all models, smooth substitution among inputs with strictly diminishing marginal rates of substitution. Analytical attention was focussed on conditions ensuring the existence and uniqueness of steady state growth paths, along which all inputs and outputs grew at the same rate. The steady state growth rate was the exogenous rate of growth of labor force in efficiency units.

Von Neumann's (1945) model represents a third strand, which we will not spend time on. The Von Neumann model disaggregates production activity and includes no non-produced factors of production such as labor or exhaustible natural resources. The production technology is a set of constant returns to scale activities: inputs are committed at the beginning of each discrete, synchronized production period, and outputs emerge at the end. In the 'primal' version, von Neumann characterized the vector of activity levels that permitted the maximal rate of balanced growth (i.e. growth in which outputs of all commodities grew at the same rate) given that

[1] Lau, Lawrence, "The Sources of the Growth of the East Asian Newly Industrialized Countries," J. of the Japanese and Intl Economies, 1994. Young, Alwyn, "A Tale of Two Cities: Factor Accumulation and Technical Change in Hong Kong and Singapore," NBER Macroeconomics Annual 1992 (MIT Press). Young, Alwyn, "Lessons from the East Asian NICS: A Contrarian View," EER Papers and Proceedings, May 1994. Young, Alwyn, "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience" QJE 110(3), Aug 1995.

the outputs of each period were used as inputs in the next period. In the 'dual' version, a vector of commodity prices and an interest rate were derived which had the properties that the value of output of each activity was no higher than the value of inputs inclusive of interest and that the interest rate was the lowest possible. Under certain assumptions about the technology von Neumann showed that the maximal growth rate of output of the primal was equal to the (minimal) interest rate of the dual.

The first strand is descriptive theory aimed at explaining the stylized facts of long-run growth in industrialized countries (particularly in the United States) such as a steady secular growth of aggregate output, relative constancy of the share of savings, investment, labor and capital income in aggregate output. Harrod (1939 EJ) offered a dynamic extension of the Keynesian model (with its constant marginal propensity to save) that contrasted two growth rates: the *warranted* rate of growth, which is consistent with maintaining the savings-investment equilibrium, and the *natural* growth rate, which is determined by the growth of labor force and technical change. If the warranted rate exceeds the natural rate, there is growing underutilization of capacity. If the natural rate exceeds the warranted rate, there is growing unemployment. Balance growth can happen only by luck accident, when the behavioral and technical parameters keep the economy on the "knife edge" of equality between warranted and natural growth rates.

This knife-edge property, resulting from Harrod's assumption that capital and labor are used in fixed proportions, led Solow to look for growth paths converging to a steady state by replacing Harrod's technology with a neo-classical technology of positive elasticity of substitution between labor and capital. Solow's (1956 QJE, 1957 REStat) celebrated articles and later work by Jorgenson and Griliches (1966) and others are examples of descriptive growth theory and related empirical analysis.

As Stiglitz (1990) remarked, by showing that the long-run steady state growth rate could be unaffected by the rate of savings (and investment) and even in the short run, the rate of growth was mostly accounted for by the rate of labor augmenting technical progress, Solow challenged the then conventional wisdom.

Note that Solow (1956 QJE) himself drew attention to the possibility that a steady state need not even exist and even if one existed it need not be unique. Indeed output per worker could grow indefinitely even in the absence of labor augmenting technical progress, if the marginal product of capital was bounded below by a sufficiently high positive number. Also, there could be multiple steady states some of which were unstable if the production technology exhibited nonconvexities.

A second strand is normative theory which drew its inspiration from Ramsey's (1928) classic paper on optimal saving. Normative models derived time varying savings rates from the optimization of an intertemporal social welfare function. There were mainly two variants of such normative models: one-sector models (Koopmans 1965 and Cass 1965) and two-sector models (Srinivasan 1962, 1964 and Uzawa 1964).

4 Simple Solow Model

> "There are always aspects of economic life that are left out of any simplified model. There will therefore be problems on which it throws no light at all; worse yet, there may be problems on which it appears to throw light, but on which it actually propagates error. It is sometimes difficult to tell one kind of situation from the other. All anyone can do is to try honestly to limit the use of a parable to the domain in which it is not actually misleading, and that is not always knowable in advance." Solow (p.2)

Solow model bottom line:

- accumulation of physical capital cannot account for differences in per capita income longitudinally nor cross sectionally.
- natural resource constraints have not been very important for growth.

In his 1987 Nobel lecture, Solow remarks that in the 1950s he was bothered by aspects of the work of Roy Harrod, Evsey Domar, and Arthur Lewis. He saw Harrod and Domar as arriving by different routes to a common answer to the question: When can the economy achieve a steady state growth path? The answer they derived was that there must be an equilibrium in which the capital stock and the labor force grow at the same rate. Otherwise there would be growing unemployment or excess capacity.

Define $k = K/AN$, and let $I = S = sY$ and $\dot{K} = I - \delta K$. Then

$$\begin{aligned}\hat{k} &= \hat{K} - \hat{A} - \hat{N} \\ &= I/K - \delta - \hat{A} - \hat{N} \\ &= sY/K - n\end{aligned} \tag{1}$$

Here we have defined $n = \hat{N} + \hat{A} + \delta$. We see that the economy will only be in a steady state such that $\hat{k} = 0$ if $sY/K = n$. Does such a steady state exist?

The Harrod-Domar model is usually formulated with the capital-output ratio treated as a fixed constant. Define $v = K/Y$ and we can reformulate equation (1) as

$$\hat{k} = s/v - n \tag{2}$$

We will treat labor force participate rate as roughly constant and the population growth rate as fixed at , which gives us $\hat{N}$ as the labor force growth rate as well. Each year there are new workers, which (cet.par.) decreases the per worker capital stock. (In the absence of investment and depreciation, the per worker capital stock would fall at the rate n.)

So we need to be able to settle down at $s/v = n$ if we are to see a steady state value of k. As characterized by Solow, Harrod and Domar took s, v, and n to be constant "facts of nature." They distinguished between the "natural rate of growth" n and the warranted rate of growth where $S = I$. E.g., if investment is determined by a simple accelerator, $I = v\dot{Y}$, and saving is a fixed fraction of income, $S = sY$, then goods market equilibrium requires $sY = v\dot{Y}$ or $\hat{Y} = s/v$.

Comment: If $K = vY$ determined by a constant capital output ratio, then $\dot{K} = v\dot{Y}$, motivating the accelerator formulation. Alternatively, if $\dot{K} = v\dot{Y}$ then $\dot{K}/K = v(\dot{Y}/Y)(Y/K)$ so that in a steady state (where $\hat{Y} = \hat{K}$) $v = K/Y$, making the link to the Solow's capital output ratio.

The possibility of steady growth then appears to be an unlikely accident of nature. (Hicks's book *The Trade Cycle* added a full employment ceiling and a zero gross investment floor to Harrod's model to generate business cycles.) Solow found this bothersome.

In addition, Solow also found implausible the implication that a transition from slow to rapid growth could be engineered by a sustained rise in the savings rate.

Solow proposed a simple change: allow for substitutability between factors of production and therefore endogeneity of v. "[E]ven if technology itself is not so very flexible for each single good at a given time, aggregate factor-intensity must be much more variable because the economy can choose to focus on capital-intensive or labor-intensive or land-intensive goods."[2]

This simple modification addressed both of Solow's concerns about the Harrod-Domar model: existence of a steady state became much more feasible, and the equilibrium rate of growth turned out to be independent of the saving rate. Recall Kaldor's stylized facts. Denison's book *Why Growth Rates Differ* offers some evidence that most countries show some trend in K/Y, but it rises in some, falls in others, and for some switches from rising to falling if different data sets are used. Solow claims that, all in all, Kaldor's fact 3 is not too bad as a stylized fact.

Similarly, Solow argues (using Matthews and Feinstein's data) that although the profit share has declined over the last century, the rate of profits is hardly changed.

[2]Solow also considers whether variations in n or s might plausibly drive the economy toward a steady state. A Malthusian story might link n positively to wages and wages negatively to U, but Solow finds this an unlikely mechanism for driving advanced capitalist economies toward a steady state. Variations in income distribution can cause s to vary between s_w and s_π, so that we get another possible mechanism if the profit share declines in the face of excess capacity. But if $s_w/v = n$, this would imply labor gets all of Y in the steady state, an implication Solow finds implausible.

4.1 Steady State Predictions of the Solow Model

Concentrate on this material.

Define $y = Y/AN$ and characterize a steady state by $\hat{y} = 0$. Then we can immediately characterize the growth of income and per captia income in a steady state economy: $\hat{Y} = \hat{A} + \hat{N}$ and $(\widehat{Y/N}) = \hat{A}$. The fundamental determinate of per capita income growth is productivity growth.

Recall Solow model: $\dot{k} = sy - nk$ where $n = \hat{N} + \hat{A} + \delta$. Here $k = K/AN$ and $y = Y/AN$. We characterize a steady state by $\dot{k} = 0$. So $sy = nk$ in the steady state, or equivalently $k/y = s/n$. If y can be written as a function of k, this implicitly defines k_{ss} as a function of s/n. If y can be written as a function of k, we expect that the capital output ratio is declining in k. So we will get an important prediction in the steady state: y will be negatively correlated with n and positively correlated with s.

For example, suppose $Y = F(K, AN)$ is homogeneous of degree one, so that we can write $y = f(k)$. Then $s/n = k/f(k)$ implicitly defines $k_{ss} = k_{ss}(s/n)$.

$$d(s/n) = \frac{f - kf'}{f^2} dk = \frac{1-\alpha}{f} dk \tag{3}$$

where α is the elasticity of y w.r.t. k. The elasticity of the steady state capital stock w.r.t. (s/n) is therefore

$$\frac{dk_{ss}}{d(s/n)} \frac{s/n}{k_{ss}} = \frac{1}{1-\alpha} \tag{4}$$

The response of steady state y to a change in (s/n) can therefore be written as

$$\frac{dy_{ss}}{d(s/n)} \frac{s/n}{y_{ss}} = \frac{df}{dk} \frac{dk_{ss}}{d(s/n)} \frac{s/n}{k_{ss}} \frac{k_{ss}}{y_{ss}} = \frac{\alpha}{1-\alpha} \tag{5}$$

So as long as we can write $y = f(k)$ we can say that steady state Y/N evolves roughly according to

$$\ln(Y/N) = a_t + \frac{\alpha}{1-\alpha}\ln s - \frac{\alpha}{1-\alpha}\ln n \tag{6}$$

where $a_t = \ln A_0 + \hat{A}t$. Ceteris paribus, countries with lower population growth rates and higher savings rates have higher per capita income. For example, suppose $Y = K^\alpha(AN)^{1-\alpha}$, so that $y = k^\alpha$. Then $k_{ss} = (s/n)^{1/(1-\alpha)}$, and steady state Y/N evolves according to (6).

We get an intriguing if imperfect fit to Kaldor's stylize facts.

HW: Show.

Note: in order to fulfill stylized fact 6, we need to discuss the elasticity of substitution. Ignore depreciation for the moment:

$$\begin{aligned}
\frac{I}{Y} &= \frac{sY}{Y} = s \\
\frac{\Pi}{Y} &= \frac{rK}{Y} = \frac{r}{q} \\
&= 1 - \frac{\ell q'}{q} \\
&= 1 - \frac{F_N N}{Y} \\
&= 1 - \eta_N
\end{aligned}$$

where $q = F(K,N)/N$, $\ell = N/K$, and $\eta_N = \ell q'/q$.

In the steady state, $q = n/s$, so $\ell = q^{-1}(n/s)$. Thus

$$\frac{d(\Pi/Y)}{ds} = -\frac{d\eta_N}{d\ell}\frac{d\ell}{ds}$$

Since the last term is negative, we need $d\eta_N/d\ell > 0$ to get the desired correlation.

I.e.,

$$\frac{q'}{q} + \frac{\ell q''}{q} - \ell \left(\frac{q'}{q}\right)^2 > 0$$

Recalling that for a linear homogeneous function $F(N, K)$ we have the elasticity of substitution defined as

$$\sigma = \frac{F_N F_K}{F_{NK} F} = \frac{q'(q - \ell q')}{-\ell q'' q} = \frac{\ell (q'/q)^2 - q'/q}{\ell q''/q}$$

we see that we need $\sigma < 1$. Recall that the Cobb-Douglas production function has $\sigma = 1$. Recall

$$\begin{aligned}
\hat{k} &= s/v - n &\quad (7)\\
v &= k/y &\quad (8)\\
\dot{k} &= sy - nk &\quad (9)
\end{aligned}$$

Thinking about $y = Y/AN = F(K, AN)/AN$. Given constant returns to scale we can write $y = F(K/AN, 1) = f(k)$.

4.2 Golden Rule

Consider the response of steady state consumption to changes in the saving rate.

$$\frac{\partial c^\infty}{\partial s} > 0 \iff f'(k) > n$$

As long as $f'' \leq 0$. So set

$$f'(k) = n$$

to maximize c, since $c^\infty = f(k) - nk = (1 - s)f(k)$.

Note that since $f' = n$ as well as $sf = nk$, we find $f'k = sf$, or equivalently

$$F_K K/Y = s$$

The steady state profit share is equal to the savings rate under the golden rule.

Now let $Y = F(AN, K)$ be a homogeneous degree one aggregate production function.[3] This suggests that other factors of production—e.g., natural resources—are relatively unimportant. We can then write the production function in *intensive* form: $y = F(1, k) \equiv f(k)$.

Inada (1964) conditions: sufficient for existence, uniqueness, and stability of a non-trivial steady state in the Solow model:

i. $f(0) = 0$, ii. $f' > 0$, iii. $f" < 0$, iv. $f'(0) = \infty$, v. $f'(\infty) = 0$

HW: Show that $K^{\alpha}(AN)^{1-\alpha}$ satisfies these conditions.

Now $sf(k) = nk$ in the steady state. How well does this description of steady state growth fit Kaldor's stylized facts? [Hint: Assume competitive markets, define $\omega = w/r$, and recall that $\sigma = (dk/d\omega)(\omega/k)$ is the elasticity of substition.]

4.3 Criticisms of the Framework

1. There is no attention to Keynesian problems such as liquidity traps, inflexible wages, or $MP_K = 0$, although Solow suggests these can be introduced as side constraints.

2. Existence of F (reswitching problem)

3. K and C assumed identical, so the Wicksell effect is neutral. But history has shown that current value does diverge from opportunity cost.

[3] That is, for any $c > 0$, $cF(AN, K) = F(cAN, cK)$.

4. Disembodied technical progress is unrealistic
5. Assume ex ante saving equals ex post saving
6. Doesn't address ($s_w \neq s_\pi$) nor $s = s(r)$
7. Perfect foresight only makes sense along a BGP.

5 Dynamic Analysis of the Solow Model

The Solow model implies

$$\dot{k} = sf(k) - nk \tag{10}$$

which is often called the fundamental equation of the Solow model. Note that it is a first-order, nonlinear differential equation in k.

Stability analysis:

$$\frac{d\dot{k}}{dk} = sf' - n \tag{11}$$

For stable dynamic adjustment we need $d\dot{k}/dk < 0$ near the steady state.

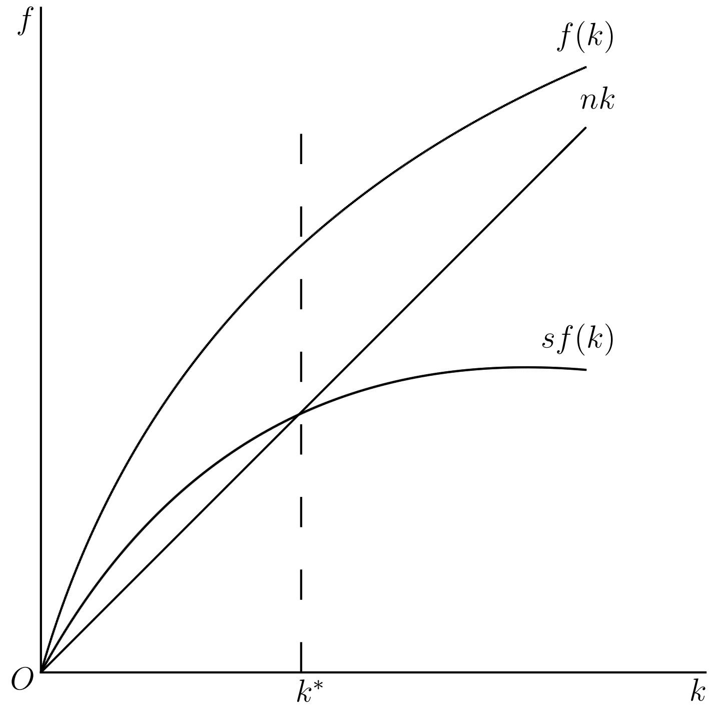
5.1 Cobb-Douglas Production Function

Let us give a more explicit characterization of the production technology:

$$Y = AK^\alpha L^{1-\alpha} \qquad 0 < \alpha < 1$$

which can be written in per capita terms as

$$y = Ak^\alpha \qquad y = Y/L, k = K/L$$



Recalling that $\dot{k} = sf(k) - nk$ we can write

$$\dot{k} = sAk^{\alpha} - nk$$

which is a Bernoulli equation.[4] Bernoulli equations allow explicit solution by a simple transformation of variables. Define

$$z = k^{1-\alpha}$$

so that

$$\dot{z} = (1-\alpha)k^{-\alpha}\dot{k}$$

[4] A Bernoulli equation has the form

$$\dot{x} + f(t)x = h(t)x^{\alpha}$$

This implies

$$\dot{k} = \frac{k^{\alpha}}{1-\alpha}\dot{z}$$

Put this together with our basic equation of motion to get

$$\frac{k^{\alpha}}{1-\alpha}\dot{z} = sAk^{\alpha} - nk \tag{12}$$

$$\dot{z} = \frac{1-\alpha}{k^{\alpha}}(sAk^{\alpha} - nk) \tag{13}$$

$$\dot{z} = (1-\alpha)(sA - nk^{1-\alpha}) \tag{14}$$

$$\dot{z} = (1-\alpha)(sA - nz) \tag{15}$$

We can readily solve the simple linear differential equation in z:

$$z(t) = As/n + (z_0 - As/n)e^{-(1-\alpha)nt} \tag{16}$$

where $z_0 = k_0^{1-\alpha}$. Let's re-express this in terms of k and solve for the explicit time path:

$$k^{1-\alpha} = As/n + (k_0^{1-\alpha} - As/n)e^{-(1-\alpha)nt} \tag{17}$$

$$k(t) = [As/n + (k_0^{1-\alpha} - as/n)e^{-(1-\alpha)nt}]^{1/(1-\alpha)} \tag{18}$$

Recall

$$y = Ak^{\alpha}$$

So we also have a solution for y:

$$y(t) = [As/n + (k_0^{1-\alpha} - As/n)e^{-(1-\alpha)nt}]^{\alpha/(1-\alpha)} \tag{19}$$

5.2 Adjustment Time to the Long Run

Adjustment time in the neoclassical model was first considered by Ryuzo Sato (1963). (Also see R. Sato (1980).) The long-run equilibrium is approached only asymptotically, so we want to know how long it takes to get "close" to it. Let's pick a fraction θ of the distance to equilibrium, and ask what is t_θ such that

$$z(t_\theta) - z_0 = \theta(z_\infty - z_0) \tag{20}$$

Combining (20) and (16) we can write

$$\begin{aligned} &As/n + (z_0 - As/n)e^{-(1-\alpha)nt_\theta} - [As/n + (z_0 - As/n)] \\ &= \theta\{As/n - [As/n + (z_0 - As/n)]\} \end{aligned} \tag{21}$$

$$(z_0 - As/n)e^{-(1-\alpha)nt_\theta} - (z_0 - As/n) = -\theta(z_0 - As/n) \tag{22}$$

$$e^{-(1-\alpha)nt_\theta} - 1 = -\theta \tag{23}$$

$$t_\theta = -\ln(1-\theta)/(1-\alpha)n \tag{24}$$

For example, to make 90% of the adjustment might reasonably take more than half a century.

$$t_{0.9} = -\ln(0.1)/0.7 * (.01 + .02 + .03) \approx 55 \tag{25}$$

(Actually, since $\ln y = \alpha/(1-\alpha)\ln z$, y adjusts (proportionately) a bit faster than z.)

Researchers are interested in the proportionate rate at which regions and countries close the gap between their current positions and their respective steady states. It is common to read claims that this rate is stable at around 2% a year (Barro and Sala-i-Martin 1992, Mankiw 1995, Sala-i-Martin 1996). This implies that returns to physical and human capital diminish only very slowly.

Problem: the 2% estimates are taken from simplistic cross-section regressions. Fixed effects are often ignored or dismissed. Sensitivity to measurement error is rarely addressed. The effects of heterogeneity biases or outliers are underplayed. Subsequent work (both panel data and time series) addresses some of these problems and produces estimates of the rate of convergence between zero and 30% a year.

6 A Problem

Problem: A stylized fact that received increasing attention in the 1980s was that per capita income is not converging. That is, the poorest countries as a group are not "catching up" with the richest countries. Countries seem to be on different "growth paths".

To see why this is a problem, Recall Solow model:

$$\hat{k} = sy/k - n$$
$$\hat{y} = \alpha\hat{k}$$

where $\alpha = (df/dk)(k/y)$ is the elasticity of production with respect to the capital stock. [HW: Show.] As you know, given perfectly competitive factor markets this is capital's share of income, while labor's is $(1 - \alpha)$.

Similarly,

$$\begin{aligned}\hat{y} &= \alpha\hat{k} \\ &= \alpha[sy/k - n] \\ &= \alpha[sY/K - n]\end{aligned} \tag{26}$$

The term in brackets is just the growth of the capital intensity of production, $\hat{k}$.

Assume a constant elasticity of substitution. So if all countries have access to the same technology, differences in $\hat{y}$ (i.e., differences in $\hat{Y/N}$, since $\hat{A}$ is equal) between countries with similar population growth rates must be due to differences in saving and capital intensity, the two items constituting the rate of gross capital formation.

$$\begin{aligned}\hat{y} &= \alpha\hat{k} \\ &= \alpha[sy/k - n] \\ &= \alpha[sy/f^{-1}(y) - n]\end{aligned} \tag{27}$$

For example, suppose $Y = K^{\alpha}(AN)^{1-\alpha}$, so that $y = k^{\alpha}$ and thus $k = y^{1/\alpha}$. Then we can write $y/k = y^{(\alpha-1)/\alpha}$ so that

$$\hat{y} = \alpha[sy^{(\alpha-1)/\alpha} - n]$$

Now Romer (1994 JEP) proposes the following calculation: with competitive factor markets, $1 - \alpha$ is labor's share of GNP. With Romer's benchmark labor share of 0.6, we have $(\alpha - 1)/\alpha = -1.5$. Now consider the US and the Phillipines in 1960, where the US income per capita was about ten times that of the Phillipines. If A is the same and n is the same, for them to have the same growth rate we need $s(Y/N)^{(\alpha-1)/\alpha}$ to also be the same. But Y/N differs by 10x, and $.1^{-1.5} \approx 30$, so it seems the US needs a savings rate 30x that of the Phillipines to grow as fast.[5]

This is implausible. While a cross section of countries does suggest richer countries have a higher investment rate than poor countries, this is an order of magnitude too small to explain the growth differences. If we believe technology is the same across countries, it seems we need α larger—but then we need to explain why capital is paid less than its marginal product.

[5]If $\alpha = 1/3$, as in MRW, then $(\alpha - 1)/\alpha = -2$, and $.1^{-1} = 100$.

7 Some Notes on Human Capital

7.1 Lucas (1990)

Lucas (1990) also notes that capital flows in the wrong direction.

Here is the basic story for capital flows. Start with a Cobb-Douglas production function.

$$Q = AK^{1-\alpha}L^{\alpha} \tag{28}$$

and find the marginal product of capital as

$$\begin{aligned}\frac{\partial Q}{\partial K} &= (1-\alpha)AK^{-\alpha}L^{\alpha} \\ &= (1-\alpha)A^{1/(1-\alpha)}\left(\frac{Q}{L}\right)^{-\alpha/(1-\alpha)}\end{aligned} \tag{29}$$

Now if we take the parameter A and α to be the same across countries then we see that relative marginal product of capital between two countries is linked directly to their relative average productivity. For example, let us follow Lucas in comparing India and the US with a tentative $\alpha - 0.6$. Then we calculate India's relative return to capital as

$$\left(\frac{1}{15}\right)^{-0.6/0.4} = 15^{1.5} \approx 58 \tag{30}$$

Apparently we should expect to see a rapid flow of capital into India from the US, but instead we see plenty of domestic investment in the US and little flow of capital to poor countries. One explanation might lie in differential riskiness of capital investment in the two countries: the differential return is real, but it compensates for risk. Lucas tries a different kind of explanation.

Lucas's first adjustment: recognize that Q/L should really be output per *effective* worker rather than output per worker, and suppose US workers are about 5 times as productive as Indian workers. That lowers what we need to explain to $(1/3)^{-0.6/0.4} \approx 5$. Better, but still we would expect substantial capital flows to India, and we do not see them.

Lucas's second adjustment: let us change the production function to allow for human capital.

$$Q = AK^{1-\alpha}L^{\alpha}h^{\gamma} \tag{31}$$

Now we can rewrite the marginal product of capital as

$$\begin{aligned}\frac{\partial Q}{\partial K} &= (1-\alpha)AK^{-\alpha}L^{\alpha}h^{\gamma} \\ &= (1-\alpha)A^{1/(1-\alpha)}\left(\frac{Q}{L}\right)^{-\alpha/(1-\alpha)} h^{\gamma/(1-\alpha)}\end{aligned} \tag{32}$$

Following Lucas, guess that US workers have 5 times the human capital of Indian workers, and pick a tentative $\gamma = 0.4$. This almost exactly equates the marginal product of capital in the two countries.

7.2 Human Capital as Productivity Differential

This is based on chapter 3 of Jones. Notation differs somewhat!! E.g, we let $y = Y/AH$ not Y/L.

Recall that our simplest Solow model used the Cobb-Douglas technology

$$Y = K^{\alpha}L^{1-\alpha} \tag{33}$$

where L is the input of labor hours. Jones takes the following approach to introducing human capital. First he supposes that the original formulation is mispecified due to a neglect of the skill level of labor input into production. So we move the an alternative production technology:

$$Y = K^{\alpha}(AH)^{1-\alpha} \tag{34}$$

where H is the supply of skilled labor and A represents labor-augmenting technology. We can rewrite this in per-effective-worker terms as

$$y = k^{\alpha} \tag{35}$$

where $y = Y/AH$ and $k = K/AH$.

The description of physical capital accumulation is unchanged:

$$\dot{K} = s_k Y - \delta K \tag{36}$$

We can therefore repeat our previous reasoning:

$$\begin{aligned} \hat{k} &= \frac{\dot{K}}{K} - \hat{A} - \hat{H} \\ &= \frac{s_k Y - \delta K}{K} - \hat{A} - \hat{H} \\ &= s_k y/k - n \end{aligned} \tag{37}$$

where $n = \delta + \hat{A} + \hat{H}$. That is,

$$\dot{k} = sy - nk \tag{38}$$

as before. Once again

$$y/k = n/s \tag{39}$$

in a steady state. But we get a slight difference tracing to our new production function:

$$\frac{y}{k} = k^{\alpha-1} \tag{40}$$

So we solve for steady state values as before:

$$k^{\alpha-1} = \frac{n}{s} \tag{41}$$

$$k = \left(\frac{n}{s}\right)^{1/(\alpha-1)} \tag{42}$$

$$y = \left(\frac{n}{s}\right)^{\alpha/(\alpha-1)} \tag{43}$$

All this looks the same, but keep in mind our changed production function. When we solve for income per worker we get

$$\left(\frac{Y}{L}\right)_{ss} = \left(\frac{n}{s}\right)^{\alpha/(\alpha-1)} Ah \tag{44}$$

where $h = H/L$, which is assumed by Jones to be a country specific constant. If we can motivate country specific difference in h, we can therefore motivate differences in steady state per capita income.

Jones takes the ratio of skilled labor to raw labor input to be a fixed constant in this simple model. This is no suprise since the only change was to add a constant to the production function. So, what is the point? The point is to treat h as *country specific* and therefore a motivation of differences in per capita income.

7.3 Thinking About h

Jones proposes the following technology as a way to follow the Lucas (1988 JME) proposal that human capital is accumulated by spending time learning new skills instead of working. We introduce time spent learning as a country specific constant u, and we parameterize the responsiveness of skilled labor to changes in u as follows:

$$H = e^{\psi u} L \tag{45}$$

so that

$$H/L = e^{\psi u} \tag{46}$$

This means that our production function in per worker terms can be written as

$$y = k^{\alpha}(Ae^{\psi u})^{1-\alpha} \tag{47}$$

Note that

$$\frac{d \log H}{du} = \psi \tag{48}$$

so that ψ might be called the schooling time semi-elasticity of skilled labor. Jones justifies this formulation in terms of a labor-economics literature that finds an additional year of schooling to be worth about a 10% increase in wage. He therefore sets $\psi = 0.10$.

7.4 Empirical Application

Jones asks whether we can ignore technology differences and still get a good fit for the predicted difference in relative income implied by this model. Letting $\psi = 0.10$ and measuring u as the average educational attainment of the labor force (in years), he answers "yes".

For all countries assume: $\alpha = 1/3$, $\hat{A} + \delta = 0.75$, $\psi = 0.10$, and same level of technology.

See his figure 3.1. Data in Appendix B.

Jones then asks whether we can improve the fit by adding technology differences. A key question is how to measure these cross country technology differences. Jones observes that the production function implies a value for A:

$$\frac{y}{k} = k^{\alpha-1} \tag{49}$$

$$\frac{Y}{K} = \frac{K}{AH}^{\alpha-1} \tag{50}$$

$$\frac{Y}{K} = \frac{K}{H}^{\alpha-1} A^{1-\alpha} \tag{51}$$

$$A = (\frac{Y}{K})^{1/(1-\alpha)} \frac{K}{H} \tag{52}$$

$$A = (\frac{Y}{K})^{1/(1-\alpha)} \frac{KY}{YH} \tag{53}$$

$$A = (\frac{Y}{K})^{\alpha/(1-\alpha)} \frac{Y}{H} \tag{54}$$

Jones rewrites this as

$$A = (\frac{Y/L}{K/L})^{\alpha/(1-\alpha)} \frac{Y/L}{h} \tag{55}$$

and (for 1990) uses data on Y/L, K/L, and u to "estimate" A. This improves the "fit". (See his figure 3.2.) So technology level matter. (This is compatible with steady state differences in income as long as there are no long run differences in technology *growth rates*.)

Jones observes that A computed this way combines many factors and may be best referred to as "total factor productivity". (See Hall and Jones (1996 NBER) for more discussion.)

Jones concludes: "[t]he Solow model is extremely successful in helping us to understand the wide variatioln in the wealth of nations." However he notes that the model does not explain the differences in the crucial parameters and, also important, the technology levels.

7.5 Mankiw, Romer, and Weil (1992 QJE)

Concentrate on this material.

Mankiw et al. (1992 QJE) suggest that many shortcomings of the Solow model can be easily overcome by adding human capital. (Knight (1946, p.99) notes the need for a broad concept of capital.) Their model is clearly just the Solow model augmented with human capital and an assumption that countries share the same rate of efficiency growth, where the initial level of efficiency, $A(0)$, is assumed to vary randomly across countries (due to local factors like climate) and this can be used to justify the error term. Their work received a lot of attention for several reasons:

- There had been a tendency to presume that the Solow model "'explains' cross country variation in labor productivity largely by appealing to variations in technologies", but
- Most provocatively, it argued that saving and population growth differences do a good job in explaining differences in per capita income
- Economists were also interested in it as a counterthrust to the endogenous growth literature, which tends to assume constant returns to scale in capital inputs (possibly physical and human capital inputs).
- Finally, it offers an answer to Romer's question of how to reconcile relatively similar growth rates with quite different per capita incomes.

The key move will be to augment the Solow growth model to include the accumulation of human capital.

Predictions: Recall the Solow model predicted

- increased s increases Y/N in the steady state

- increased n decreases Y/N in in the steady state

Data: Real national accounts of Summers and Heston (1988), covering 1960-85. ($s = I/GDP$, n consists of $\hat{N}$, the average growth 1960-85 of 15–64 age group ("working age"), and $\hat{A} + \delta$, set to .05. They claim plausible changes don't affect results.) Subtleties: In practice investment rates are not constant, so MRW average them over the period.

Mankiw et al. show that cross section data support these predictions of the Solow model, explaining 50%+ of the variation in cross section data, but actual correlations in the data are larger than those predicted by the Solow model. (However, don't reject the implication that effects of s and n are of equal magnitude but opposite in sign.)

Note that the model predicts both the signs and the *magnitudes* of s and n on Y/N, since α is capital's share and is known to be about 1/3 (implying $[\alpha/(1-\alpha)] \approx .5$. Their cross section estimates of (6) support the sign prediction, and they also accept the equal size of the coefs on n and s. But yield $\alpha \approx .59$, which is much too large. (And the s.e. is small enough to easily reject $\alpha = 1/3$.)

Mankiw et al. remedy this by adding a proxy for human capital to (6). Their argument is as follows. Let $Y = F(AN, K, H)$ where H is human capital and evolves according to $\dot{H} = s_h Y - \delta H$, or equivalently $\dot{h} = s_h y - nh$ where $h = H/AN$. The model is otherwise unchanged, so we still have $\dot{k} = s_k y - nk$. In the steady state, then, we have $s_h y = nh$ and $s_k y = nk$ or $s_h/n = h/y$ and $s_k/n = k/y$. If h/y and k/y can be written as functions of h and k, then we can solve for the steady state values of h and k. For example, suppose

$$Y = K^\alpha H^\beta (AN)^{1-\alpha-\beta}$$

where $\alpha + \beta < 1$. Then

$$k_{ss} = (s_k^{1-\beta} s_h^{\beta}/n)^{1/(1-\alpha-\beta)}$$

and

$$h_{ss} = (s_k^{\alpha} s_h^{1-\ \alpha}/n)^{1/(1-\alpha-\beta)}$$

Yielding the new steady state evolution of Y/N as

$$\begin{aligned} \ln(Y/N) = a_t + \frac{\alpha}{1-\alpha-\beta} \ln s_k + \frac{\beta}{1-\alpha-\beta} \ln s_h \\ - [(\alpha+\beta)/(1-\alpha-\beta)] \ln n \end{aligned} \tag{56}$$

Comment: note the restriction that the last three coefficients sum to zero.

Comment: suppose $\alpha = \beta = 1/3$, then coef on s_k is unity. So we have a possible explanation of the apparently high coef on s_k that we found in our earlier work even if capital is paid its marginal product. This is because the higher per capita income associated with higher saving also implies a higher steady state level of human capital (even given s_h).

Comment: the coef on n exceeds in size the coef on s_k. This is in fact what MRW found in their original estimates.

Data: Need observations on a new variable: s_h. They proxy this by a measure of the % of working age population in secondary school. Of course, this restricts the notion of human capital (ignoring, e.g., investments in health). Problem: how to observe the investment in education, which has a large component of foregone earnings? How to get data on all the sources of expenditure? How to separate out the consumption component of education? The pragmatically duck all of these questions and hope their variable SCHOOL is proportional to s_h.

7.6 Algebraic Details

Start with $Y = F(K, H, AN) = ANF(K/AN, H/AN, 1) = f(k, h) = k^\alpha h^\beta$. Maintaining the standard accumulation form, with exogenous savings rates, we have

$$s_h k^\alpha h^\beta = nh \quad \Rightarrow \quad k^\alpha h^{\beta-1} = \frac{n}{s_h} \tag{57}$$

$$s_k k^\alpha h^\beta = nk \quad \Rightarrow \quad k^{\alpha-1} h^\beta = \frac{n}{s_k} \tag{58}$$

In log form,

$$\alpha \ln k - (1-\beta)\ln h = \ln(\frac{n}{s_h}) \tag{59}$$

$$\beta \ln h - (1-\alpha)\ln k = \ln(\frac{n}{s_k}) \tag{60}$$

Express this as

$$\begin{bmatrix} \alpha & -(1-\beta) \\ -(1-\alpha) & \beta \end{bmatrix} \begin{bmatrix} \ln k \\ \ln h \end{bmatrix} = \begin{bmatrix} \ln \frac{n}{s_h} \\ \ln \frac{n}{s_k} \end{bmatrix}$$

and solve for $\ln k$ and $\ln h$ as

$$\begin{aligned} \begin{bmatrix} \ln k \\ \ln h \end{bmatrix} &= \frac{1}{\alpha\beta - (1-\alpha-\beta+\alpha\beta)} \begin{bmatrix} \beta & (1-\beta) \\ (1-\alpha) & \alpha \end{bmatrix} \begin{bmatrix} \ln \frac{n}{s_h} \\ \ln \frac{n}{s_k} \end{bmatrix} \\ &= \frac{-1}{(1-\alpha-\beta)} \begin{bmatrix} \beta \ln \frac{n}{s_h} + (1-\beta)\ln \frac{n}{s_k} \\ (1-\alpha)\ln \frac{n}{s_h} + \alpha \ln \frac{n}{s_k} \end{bmatrix} \end{aligned} \tag{61}$$

Since $\ln y = \alpha \ln k + \beta \ln h$, we have

$$\ln y = \frac{-\alpha}{(1-\alpha-\beta)} \left[\beta \ln \frac{n}{s_h} + (1-\beta)\ln \frac{n}{s_k}\right] - \frac{\beta}{(1-\alpha-\beta)} \left[(1-\alpha)\ln \frac{n}{s_h} + \alpha \ln \frac{n}{s_k}\right]$$

which can be rewritten as

$$\ln y = \frac{\beta}{(1-\alpha-\beta)} \ln s_h + \frac{\alpha}{(1-\alpha-\beta)} \ln s_k - \frac{(\alpha+\beta)}{(1-\alpha-\beta)} \ln n$$

7.7 Results

- s_h significant
- increases $\bar{R}^2$ to .78 (but $\bar{R}^2 = .24$ for OECD, and the coeficients are not significant!).
- lowers estimated α to $\sim 1/3$
- does not reject restriction that coefficients sum to zero

Conclusion: adding human capital improves the empirical performance of the Solow model and calls into question the need for the endogenous growth models. (You will cover endogenous growth in 19.712, but read the section in this article now.)

But: somewhat dubious proxies for aggregate human capital. The focus is almost exclusively on schooling rather than training. This is mainly due to data limitations, but less excusable is the attention frequently paid to school enrollment rates. These rates were initially regarded as one of the more robust and satisfactory variables in the growth literature, but it is worth remembering that they are likely to be positively correlated with initial efficiency, so the results could often be spurious. Certainly it has been much harder to find an effect of human capital in panel data studies, although it is also true that too few researchers think carefully about the specification. (Rather optimistically, they tend to expect a change in school enrollments to raise growth almost instantly.) Equally importantly, there are some conceptual difficulties with the use of school enrollment data. Only rarely do these rates correspond well to the human capital variables highlighted in theoretical models. In many empirical growth papers it is not clear whether school enrollment rates are intended to represent a Àow of investment in human capital, or its stock. In practice these rates may be a poor proxy for either, and given that data on average years of schooling in the population or labour force is now available, the continuing use of school enrollment figures has little to recommend it.

7.7.1 Understanding the Change in Results

Recall

$$h_{ss} = (s_k^\alpha s_h^{1-\alpha}/n)^{1/(1-\alpha-\beta)}$$

or

$$(1-\alpha-\beta)\ln h_{ss} = \alpha \ln s_k + (1-\alpha)\ln s_h - \ln n$$

So that

$$\ln s_h = -[(1-\alpha-\beta)/(1-\alpha)]\ln h - \frac{\alpha}{1-\alpha}\ln s_k + \frac{1}{1-\alpha}\ln n$$

is a steady state relationship. Recalling our regression equation (56)

$$\ln(Y/N) = a_t + [\alpha/(1-\alpha-\beta)]\ln s_k + [\beta/(1-\alpha-\beta)]\ln s_h - [(\alpha+\beta)/(1-\alpha-\beta)]\ln n$$

and substituting for s_h yields

$$\ln(Y/N) = a_t + [\alpha/(1-\alpha)]\ln s_k - [\alpha/(1-\alpha)]\ln n + [\beta/(1-\alpha)]\ln h_{ss} \tag{62}$$

Note that (62) is identical to (6) except for the last term. So (6) was estimated with an omitted regressor.

But our steady state solution for h implies that it is positively correlated with s_k and negatively correlated with n. Omitting h would therefore tend to increase the magnitude of the coefficients on the other two regressors. This bias can explain our earlier puzzle about the coefficient magnitudes when we estimate (6).

A focus on potential bias can be problematic for their results as well, however. For example, as noted by Grossman and Helpman (1994), if low Y/N causes high n this can explain their negative coeficient on n. (Similarly if investment is encouraged by growth.)

Problem: If the sample is restricted to only the 22 OECD countries, the adjusted R^2 falls to .28. The differences in n and I/Y between rich and poor countries drives their results.

Problems: There are still some problematic stylized facts. As Lucas (1988) notes, international wage differentials and patterns of migration don't seem to fit the neoclassical framework. To continue Romer's Phillipines example, a worker should not be able to earn a higher wage by migrating to the US. Further, empirically human capital migrates from places where it is scare (and, in a neoclassical setting, should therefore be highly rewarded) to places where it is abundant.

MRW draw attention to one particularly dramatic finding. They claim that around 80% of the international variation in per capita incomes can be explained using just three variables: population growth, and investment rates for physical and human capital. The corollary is that differences in technical efficiency can have only a small role to play in explaining cross-country income variation. However, their approach makes two controversial assumptions, that investment rates are exogenous to the level of income and uncorrelated with efficiency. More recently, Peter J. Klenow and Andrés Rodríguez-Clare (1997) have drawn attention to another underlying problem with the MRW finding. The human capital variable that MRW use only captures variation in secondary schooling. Since it ignores primary schooling, it tends to exaggerate the variation in human capital across countries. When Klenow and Rodríguez-Clare correct for this, they find that MRW's model only explains around half the variation in incomes, leaving a central role for technology differences.

If one follows them in assuming that TFP has grown at a rate of 2% a year in all countries since 1960, what does one make of the many countries which have grown at less than 2% a year over this period? The model implies that unless these countries have received a large negative shock to technology they are converging to their steady state from above. (I.e., such a country must have exceeded its steady state capital stock in the early 1960s and have been running it down subsequently.) This seems unlikely. There are other problems. Economic miracles, such as Japan's post-war growth, are hard to explain by capital accumulation alone. In most frameworks, such a process would have to be accompanied by a steeply falling real interest rate, something which has not been observed. Temple (1999) argues that such problems suggest that the neoclassical revival in growth economics went too far. Understanding the reasons for differences in technical efficiency and TFP growth is essential to the empirical growth project.

Convergence

The MRW model seems to do pretty well in the explanation of steady state income differences. We now consider the ability of the model to explain differences in growth rates.

Background: Gerschenkron (1952) and Abramovitz (1986 JEHist) hypothesize that less developed countries should be able to grow faster than developed countries. We refer to the faster growth of poorer countries as *convergence*.

Broadly speaking: convergence must be false.

Throughout most of human history, almost everyone lived near subsistence. Now some countries are rich, while others remain near subsistence. This is extreme divergence.

Why do economists search for convergence in contemporary data?

- policy concern that poor countries "catch up" with rich countries
- suspicion that technology transfers could close most of the gap between rich countries and poor countries. (These need not be policy based: business and engineering journals are available online.)
- the neoclassical growth model has convergence as a basic prediction,

An early effort to shed empirical light on historical convergence trends was offered by Baumol (1986 AER). Jones (fig 3.3, p.57) plots similar data over the period 1870–1994 for Germany, Japan, the US, and the UK. Appearance: clearly shows narrowing gaps between the countries.

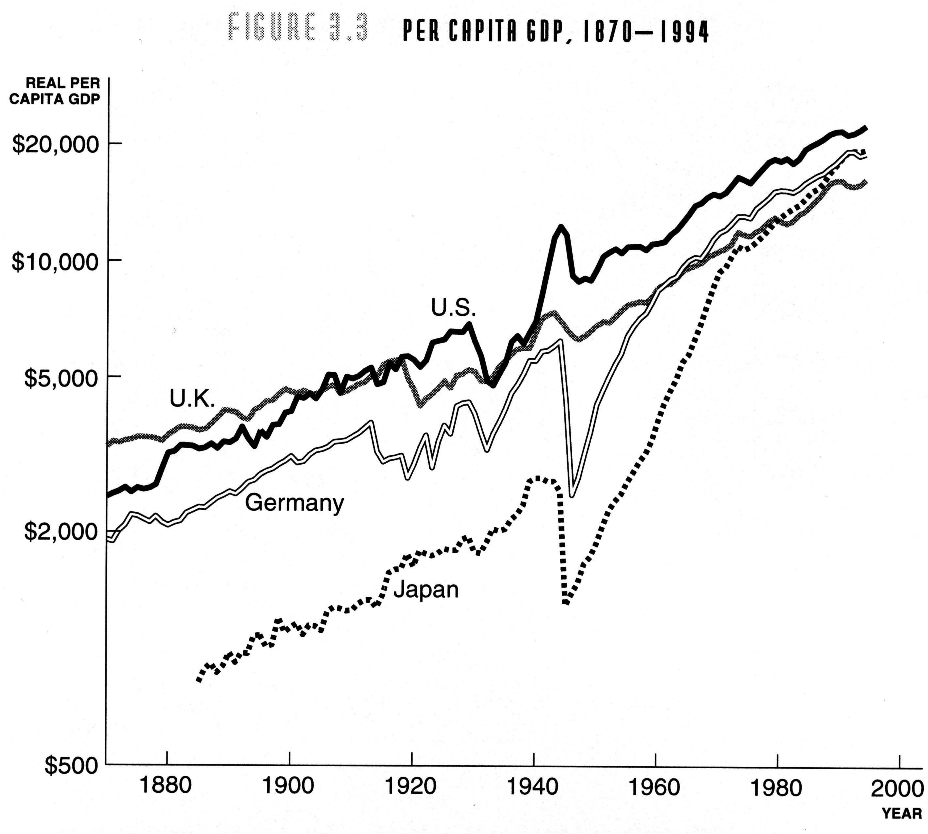


Figure 3: Long-Run Convergence in Four Developing Countries
Source: Jones (1998), figure 3.3.

For the OECD countries (fig 3.5, p.59), he presents an analagous scatter plot for 1960–90. Shows a strong negative correlation.

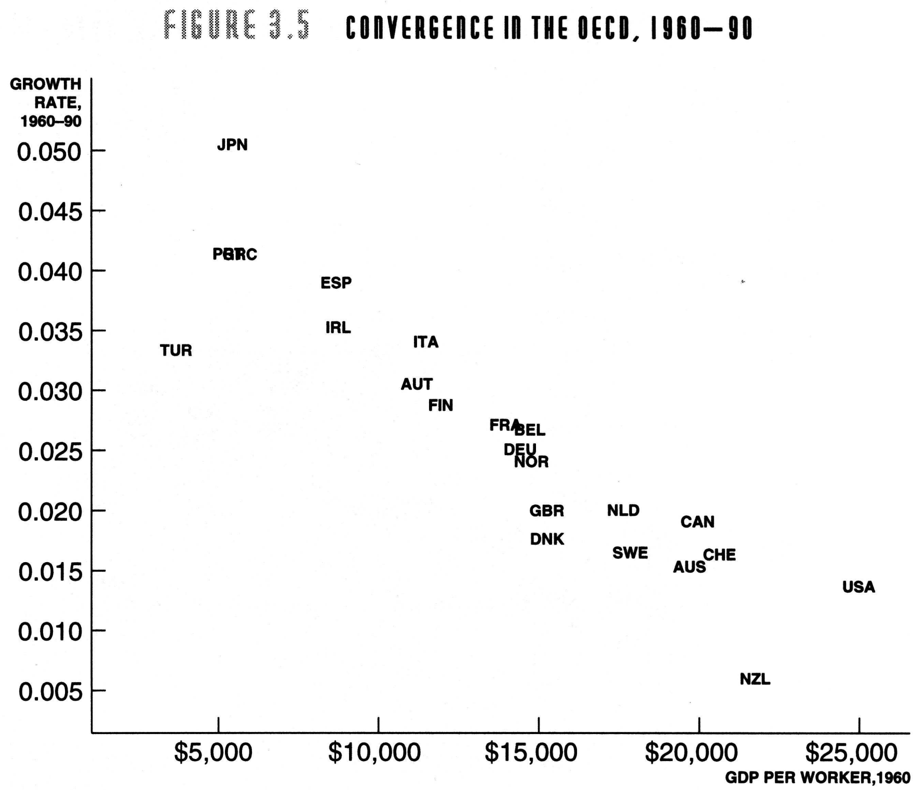


Figure 4: Convergence in the OECD
Source: Jones (1998), figure 3.5.

Using a rather remarkable data set, Jones (fig 3.4, p.58) also presents a scatter plot of average annual growth rates vs. initial income from 1885–1994 for a larger set of industrialized countries. (Similar to your HW, but larger time span.) Again: a strong negative correlation.

On the face of it, a simple convergence hypothesis does remarkably well in this qualitative sense. However, De Long (1988 AER) pointed to an important difficulty with Baumol's results (and all these figures): only countries that were rich at the end of the sample were included. This means that some countries that were relatively rich

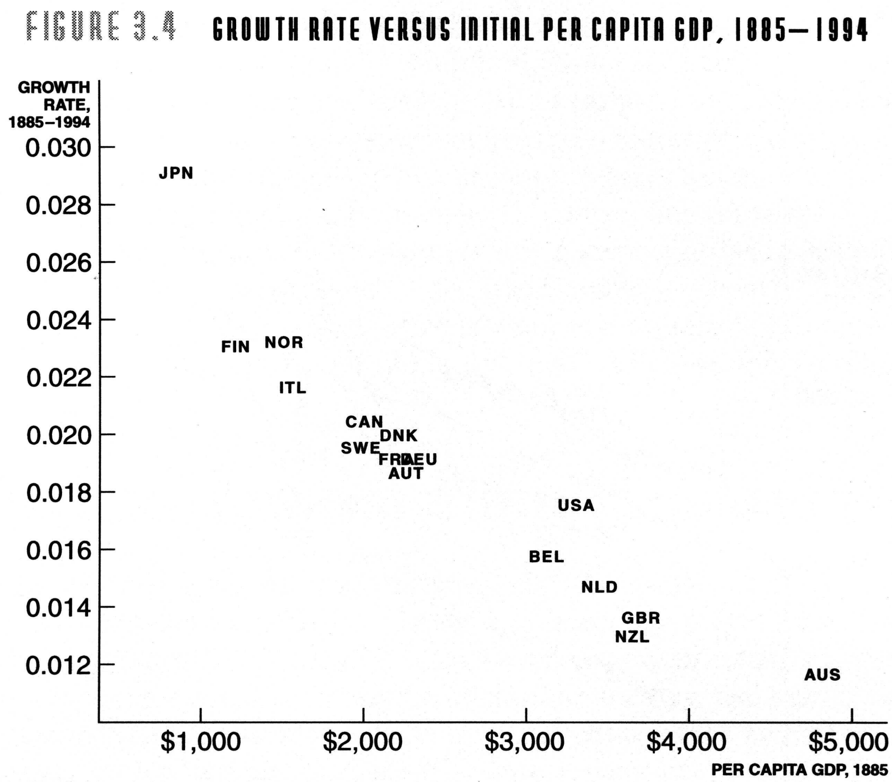


Figure 5: Long-Run Convergence in Developing Countries
Source: Jones (1998), figure 3.4.

at the beginning of the period were not included, biasing the apparent results toward convergence. Jones's OECD data (as he recognizes) has the same problem.

Jones (fig. 3.6, p.60) therefore produces an analagous scatter plot for most countries in the world for 1960–1990. In this graph, there is no clear tendency toward convergence. (Baumol had also reported that in large samples of countries there is no evidence of convergence.)

FIGURE 3.6 THE LACK OF CONVERGENCE FOR THE WORLD, 1960–90

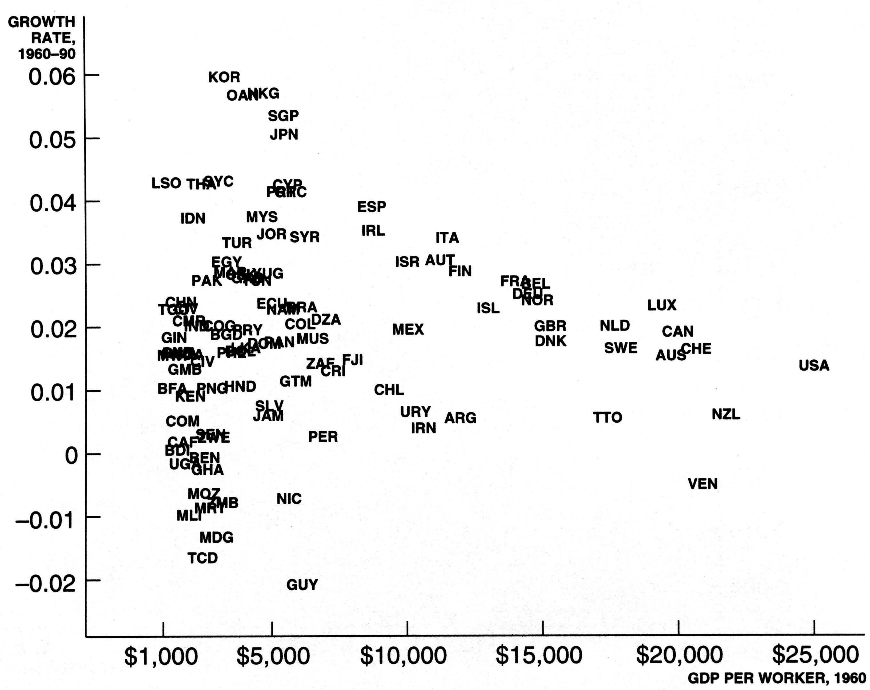


Figure 6: Convergence in the World
Source: Jones (1998), figure 3.6.

Quah (1993) indentified another problem: Galton's fallacy. (You did a HW exploring this.)

7.7.2 Convergence in the MRW Model

The rate of convergence is

$$\lambda = (n + g + \delta)(1 - \alpha - \beta)$$

Note that n and δ will vary across countries, but onventional estimation of the model ignores this subtlety.

Note also that the convergence equation is obtained by taking a Taylor series approximation around a deterministic steady state. Binder and Pesaran (1996) argue that, when growth is stochastic, such linearization is potentially misleading.

Why does initial income affect growth? The reason is 'transitional dynamics'. The relatively poor economy must have lower stocks of physical and human capital. The marginal product of extra capital is correspondingly higher in this economy: for a given rate of investment its growth will be faster. Thus in a regression that controls for the determinants of steady states, like investment ratios, the initial income will take a negative sign.

This is called 'conditional convergence'. It is conditional in that it appears after we control for determinants of the steady state levels of income.

In this sense, conditional convergence does not imply that poorer countries will catch up with rich ones. For example, anything that drives apart investment rates in rich and poor countries will tend to lead to increased income dispersion.

In principle the level of technology should be included in the regression. But this variable is unobserved. Omitted variable problem: the other parameter estimates are biased if one or more regressors are correlated with the level of technology. One proposed solution: panel data methods.

The debate about convergence rates can be linked to another ongoing dispute:

the importance of differences in technology across countries. Recall that MRW show that income and growth differences can be almost completely explained using a model in which technology is a public good, freely available to individuals in all countries. They attribute differences in BGP incomes largely to steady state levels of human and physical capital.

Most development economists, economic historians, and theorists, have not been satisfied with this. They stress technological and institutional factors in the problems of developing countries. Capital accumulation is important *because* it is part of the important task of adopting appropriate foreign technology.

Researchers in this latter tradition emphasise the role of IP protection and trade secrets. I find this somewhat implausible, especially when it comes to patents, which expire after 20 years. Twenty-year old technology is more than adequate to make poor countries rich. Of course there is a learning by doing problem, but the NICs demonstrated that this can be done very swiftly.

8 Romer (1990)

In this section we will take up a version of the Romer (1990) model presented by Jones (1995) and Jones (2002, ch.5). I am going to try to stick very close to your textbook, aside from some minor notational differences.

Three sector model: research sector (producing ideas), intermediate goods (producing capital), and final goods.

Aggregate production function:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha} \tag{63}$$

Features: CRTS in K and L_Y, *but* IRTS if A is also a factor. For now, think of A as "ideas".

Similar to Solow:

$$\dot{K} = s_K Y - dK \qquad \hat{L} = n \tag{64}$$

But now labor is allocated to the production of ideas:

$$\dot{A} = \delta A^{\phi} L_A^{\lambda} \tag{65}$$

or

$$\hat{A} = \delta \frac{L_A^{\lambda}}{A^{1-\phi}} \tag{66}$$

$\phi > 0$ means research become more productive as stock of ideas increases. ("Standing on the shoulders of giants.")

$\phi < 0$ means research become less productive as stock of ideas increases: the stock of available ideas gets "fished out".

Solow case: $\phi = 0$, so that $\hat{A}$ is independent of stock of ideas, and $\lambda = 0$, so that $\hat{A}$ is independent of research effort.

Implications:

Suppose $\phi = 0$ and $\lambda = 1$, so that

$$\hat{A} = \delta L_A / A \tag{67}$$

This means that a constant population economy must experience a declining rate of growth of innovation. In this special case, the growth rate of innovation depends on population growth!

As for the Arts of Delight and Ornament, they are best promoted by

> the greates number of emulators. And it is more likely that one ingenious curious man may rather be found among 4 million than among 400 persons ...
>
> William Petty (cited in Simon 1981 and Jones 2005)

> One can hardly imagine, I think, how poor we would be today were it not for the rapid population growth of the past to which we owe the enormous number of technological advances enjoyed toay. ...If I could re-do the history of the world, halving the population size each year from the beginning of time on some random basis, I would not do it for fear of losing Mozart in the process.
>
> Phelps (1968, pp.511-512) as quoted by Jones (2005)

Context: enormous increase inn world research effort. (See figure 7.)

Why has this not led to a rapid increase in growth rates? Jones (2002) says this is strong evidence that $\phi < 1$. (In contrast with the original Romer model.) But $\phi = 1$ was the basis of policy influences on growth!

8.1 Labor Market

Labor allocated to final goods production and idea production

$$L_Y + L_A = L \tag{68}$$

For now the allocation proportion is fixed: $L_A = s_A L$.

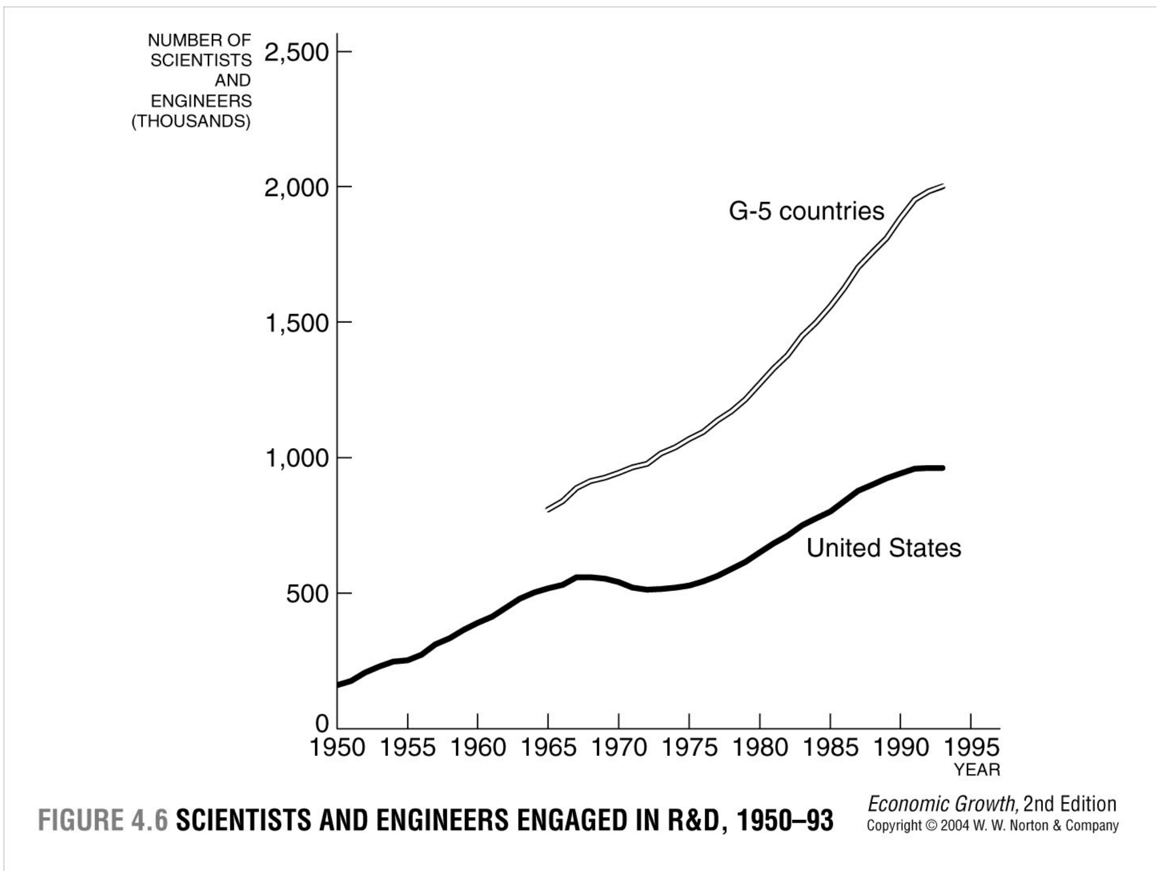


Figure 7: R & D Effort over Time (Source: Jones 2002, Fig 4.6)

8.2 Balanced Growth

Along a balanced growth path, the following are constant:

$$\hat{y} = \hat{k} = \hat{A} \tag{69}$$

(Here $y = Y/L$ and $k = K/L$.)

We can therefore ask, what is the rate of technological change along a balanced growth path. This requires

$$\hat{A} = \delta \frac{L_A^\lambda}{A^{1-\phi}} \tag{70}$$

to be constant, so that

$$\lambda \hat{L_A} = (1 - \phi)\hat{A} \tag{71}$$

or, since $\hat{L_A} = n$ (constant labor allocation)

$$\hat{A} = \lambda n/(1 - \phi) \tag{72}$$

Similarities to Solow: Note: s_K is again irrelevant! (Even research share of labor does not matter.)

8.3 Increase in R & D Intensity

See Figure 8.

Simplifying assumptions: $\lambda = 1, \phi = 0$. (Not qualitatively important.)

Under these assumptions:

$$\hat{A} = \delta L_A/A \qquad \hat{A}_{ss} = n \tag{73}$$

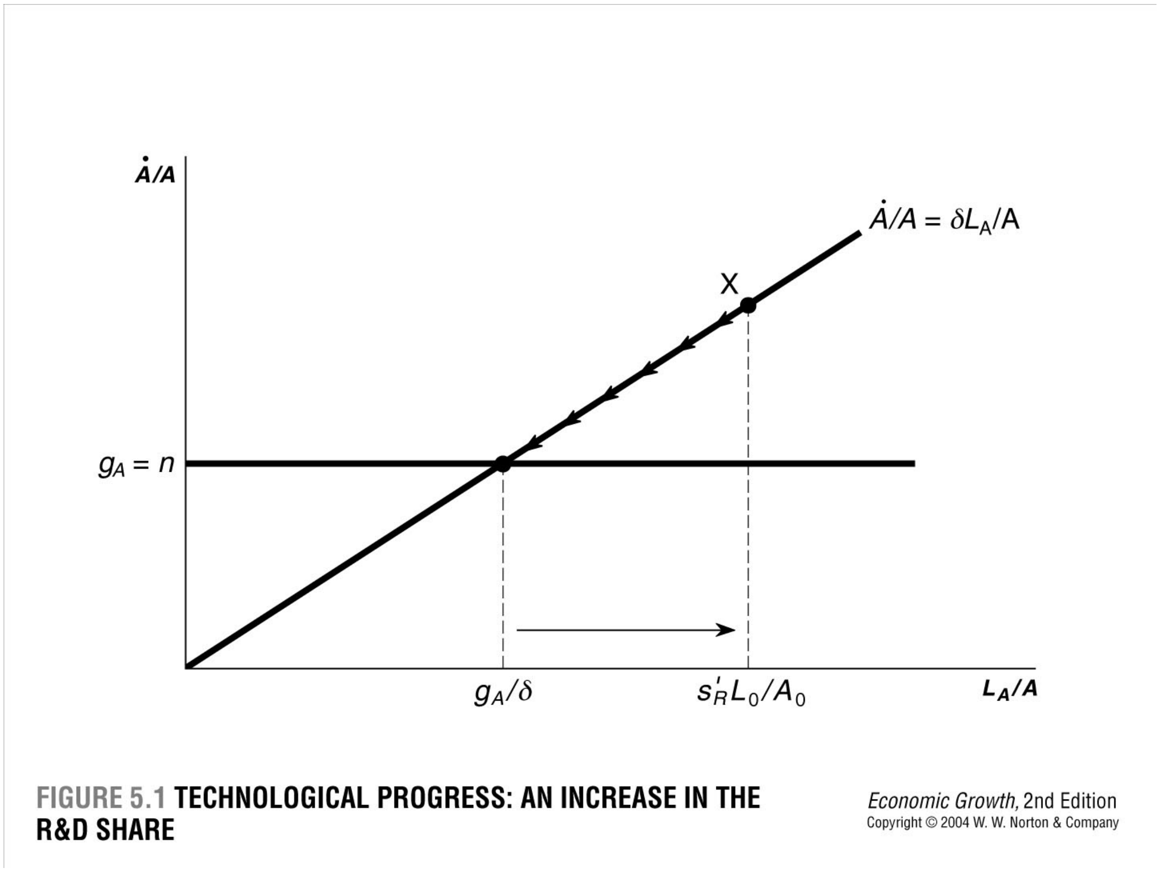


Figure 8: One-Time Increase in s_A

Again, $\hat{A}$ must fall as A grows faster than n (and thus faster than L_A.

Still, the temporary acceleration of A growth does have a *level* effect on A.

Comment: should remind you of effect of an increase in the saving rate in the Solow model.

Steady state output determination look remarkably like the Solow model:

$$\left(\frac{y}{A}\right)_{ss} = \left(\frac{s_K}{\hat{L} + \hat{A} + d}\right)^{\alpha/(1-\alpha)}(1 - s_A) \tag{74}$$

However the description of y along a balanced growth path is a bit more complex, once we substitute for A in (75):

$$y(t)_{bg} = (\frac{s_K}{\hat{L} + \hat{A} + d})^{\alpha/(1-\alpha)}(1 - s_A)\frac{\delta s_A}{\hat{A}}L(t) \tag{75}$$

Note the scale effect (L)! A larger economy is a richer economy. More people means more idea creators.

8.4 Sector Details

8.4.1 Final Goods

$$Y = L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha} \tag{76}$$

Obviously, CRTS in “capital” (intermediate goods) and labor, *given* A.

Popular variant:

$$Y = L_Y^{1-\alpha} \int_0^A x_j^{\alpha} \tag{77}$$

Profit maximization:

$$\max_{L_Y, x_j} L_Y^{1-\alpha} \int_0^A x_j^{\alpha} - wL_Y - \int_0^A p_j x_j dj \tag{78}$$

F.O.C.

$$w = (1 - \alpha)Y/L_Y \tag{79}$$

$$p_j = \alpha L_Y^{1-\alpha} x^{\alpha-1} \tag{80}$$

8.4.2 Intermediate Goods

Patent protection -¿ only one firm manufactures each capital good.

Become monopolist by buying patent. Then maximize profits.

$$\max_{x_j} p_j x_j - r x_j \tag{81}$$

Downward sloping demands, so $p_j = p_j(x_j)$.

F.O.C.

$$p_j' x/p_j + 1 = r/p_j \tag{82}$$

The elasticity of demand is $\alpha - 1$, so we can solve for

$$p_j = r/\alpha \tag{83}$$

In this simple model, all capital goods sell for the same price.

Resulting profit:

$$\pi = \alpha(1-\alpha)Y/A \tag{84}$$

Note that profits are proportional to Y/A. Since $\widehat{Y/AL} = 0$ along a BGP, Since $\widehat{Y/A} = n$, so $\hat{\pi} = n$.

Define the aggregate capital stock:

$$K = \int_0^A x_j dj \tag{85}$$

and define

$$x = K/A \tag{86}$$

which is identical to x_j for all j. Since this value is constant across j we have

$$Y = AL_Y^{1-\alpha} x^\alpha = AL_Y^{1-\alpha} \left(\frac{K}{A}\right)^\alpha = K^\alpha (AL_Y)^{1-\alpha} \tag{87}$$

8.4.3 Research Sector

Must be able to earn equivalent of r by buying a patent, earning profits for a period (from the patent), and selling at the new price:

$$r = \pi/P_A + \widehat{P_A} \tag{88}$$

Along a balanced growth path, r is constant and $\widehat{P_A} = n$. (See 84 and comments, above.)

$$P_A^* = \pi/(r - n) \tag{89}$$

This gives us the price of a patent along a balanced growth path.

Query: who gets the profits?

Answer: due to free entry into the intermediate goods sector, the cost of a patent is bid up to the DPV of the profit stream. The inventors get all the profits.

There are no economic profits in the model.

8.5 Allocation of Labor

So far we have simply assume the fraction of labor in research is given, taking advantage of our anticipation that it will be constant along a BGP. Next we show this.

Labor in research and in the final goods sector earns its marginal product:

$$w_A = \bar{\delta} P_A \quad w_Y = (1 - \alpha) Y/L_Y \tag{90}$$

where $\bar{\delta}$ is treated as given by researchers, despite the externality in the number of researchers, because each researcher is individually negligeable.

Since labor is mobile, these two wages must be equal, so

$$\bar{\delta} P_A = (1-\alpha)Y/L_Y \tag{91}$$

Substitute our solution for P_A from (89),

$$\bar{\delta}\pi/(r-n) = (1-\alpha)Y/L_Y \tag{92}$$

Next, substitute our solution for π from (84):

$$\bar{\delta}\alpha(1-\alpha)Y/A(r-n) = (1-\alpha)Y/L_Y \tag{93}$$

Solve for L_Y

$$L_Y = A(r-n)/\alpha\bar{\delta} \tag{94}$$

Recalling that $\dot{A} = \bar{\delta} L_A$

$$L_Y/L_A = (r-n)/\alpha\hat{A} \tag{95}$$

This gets us there, since $L_Y/L_A = (1-s_A)/s_A = -1 + 1/s_A$:

$$s_A = \alpha/(\alpha + (r-n)/\hat{A}) \tag{96}$$

or, for a better match to your textbook,

$$s_A = \frac{1}{1 + (r-n)/(\alpha\hat{A})} \tag{97}$$

Note that at a higher interest rate, the DPV of a given profits stream is lower, which limits the amount of labor devoted to research.

What is the interest rate in this economy? It is *less* than the MPK ($\alpha Y/K$)

$$r = \alpha^2 Y/K = \alpha^2 (n + g + d)/s_K \tag{98}$$

This is quite different from the Solow model, since in the Solow model there is no compensation of researchers.

