Lecture Notes for International Finance

The Macroeconomics of Floating Exchange Rates

Note to My Students

These Lecture Notes are only for Econ 672, Spring 2021. You are permitted to make one copy of these notes for your own use. Please do not redistribute these notes in any form.

Alan G. Isaac

Contents

Preface

Exchange rate economics evolved quickly after the early 1970s. This book is a compact, selective survey that assumes only a very modest background in economics and in quantitative methods. I cover the literature that I believe will prove most useful to students of foreign exchange rates who wish to be practitioners. This means that I emphasize empirical models of exchange rate determination. There are three key questions addressed by these models. What are the fundamental determinants of the exchange rate? Can these be used to predict future exchange rates? Can policy stabilize the exchange rate and, if so, should it?

Since this book focuses on contemporary understandings of exchange rate determination, the models presented all take the *asset approach*. This means that the exchange rate is analyzed as an asset price. An older tradition of *flow analysis* of foreign exchange rates has not been an important part of contemporary macroeconomic research since the mid-1970s.

At the applied level, the algebraic details of exchange rate modeling complicated and opaque. Nevertheless, we may often develop a rigorous conceptual understanding with little or no algebra. Most of the material in this book requires minimal mathematical sophistication for mastery of the core concepts. Rigorous graphical developments often substitute for or supplement algebraic developments. My goal is to stress economic reasoning while providing supplementary algebraic presentations for those who find them useful. Algebra is often placed in separate sections or even appendices, allowing the reader to easily skip these on a first (or even second) reading. Similarly, discussions of empirical results emphasize the most accessible of the important studies.

My object is to produce a book that fairly presents exchange rate economics at several different levels. The result is a text that is more technically complete than most textbooks while being much more user friendly than most handbooks.

This book evolved from notes that I developed to teach International Finance at American University. Excepting the technical sections and the appendices, I have taught this material to undergraduates. I have taught most of the technical sections to Masters students. Some of the material in the appendices is more advanced, and I have generally reserved it for Ph.D. students specializing in International Finance. The bibliography is very selective and reflects the origins of the book: I have cited only the literature I feel is most useful to and most accessible to the student and the practitioner.

Alan G. Isaac
Department of Economics
American University
Washington, DC 20016
USA

April 22, 2021

Lecture 1

Introduction to International Macroeconomics

Modern economies are interdependent. Macroeconomic policy decisions often involve international considerations and may have international ramifications. Countries struggle to align their monetary and fiscal policies, defend their currencies, cope with international financial shocks, dodge excess international indebtedness and currency crisis, avoid high inflation and undesirable exchange rate fluctuations, and maintain international competitiveness and low unemployment even during global recessions. Making good macroeconomic policy requires an understanding of the international dimension, including the role of international financial markets and exchange rates.

1.1 The Open Economy

When we say that a country has an ***open economy***, we mean that it has important economic interactions with other economies. In contrast, we can imagine a ***closed economy*** or ***autarky***, which is self sufficient and does not have such interactions. A model of a closed economy does not have to reference the global economy. It is difficult to discuss an open economy without making explicit references to the global economy.

In this sense, all economies are open to some degree. (Even North Korea trades with other countries.) From the perspective of macroeconomic analaysis, we need to include open economy considerations in our models whenever these have important effects on our conclusions. (For example, effect on our forecasts or our policy recommendations.)

1.1.1 Modeling The Open Economy

All of the questions that arise in modeling a closed economy also arise in an open economy. In addition, a number of new questions arise. Is the exchange-rate regime a peg, a float, or something in between? Is financial capital internationally mobile, immobile, or something in between? Should we distinguish between goods that are traded internationally and those that are not? How much of an effect does domestic economic activity have on the world economy, and vice versa? Each of these questions multiplies the number of different configurations we can consider theoretically, and the diversity of actual economies and policy concerns lends relevance to many of these different configurations.

The response of economists to the bewildering array of possible economies has been the creation of a bewildering array of theoretical models. A central goal of this book is to reduce this complexity to manageable proportions. Many theoretical models are constructed from a few essential components, which function as building blocks. We will not survey the diverse models that economists have developed. Instead we will try to develop unifying themes that render understandable some important aspects of open economies.

This book explores some underlying common structures in a variety of specific models. We concentrate on simple, stylized models that highlight particular points under discussion. The nature of our questions will determine the components include in our models, which will be constructed from relatively simple theoretical building blocks. We use economic theory to understand the interaction of simple model components that can shed light on the behavior of actual economies.

Our macroeconomic models of the open economy will imply important effects of macroeconomic policy on macroeconomic outcomes. Most obviously, monetary policy is a key determinant of the nominal exchange rate. Since inflation and exchange rate depreciation both depend on monetary policy but may respond asymmetrically, we may also see important effects on the real exchange rate. Fiscal policy may also influence the real exchange rate.

Business cycle amplitude can also depend on macroeconomic policy. Real and financial adjustments may result from trade imblances, and policy makers may respond to macroeconomic shocks with changes in monetary and fiscal policy. Macroeconomic models provide the reasoning behind such policy responses. The success of our policy choices hinges on the adequacy of our models.

1.1.2 Economic Models

> The more accurate the map, the more it resembles the territory. The most accurate map possible would be the territory, and thus would be perfectly accurate and perfectly useless.
>
> — from the Notebooks of Mr. Ibis in *American Gods* by Neil Gaiman, p.546

At some stage in their studies, many students of economics become puzzled by the role of models in economic reasoning. Economic phenomena are inherently complex and interrelated, while economic models are relatively simple and unrealistic. It is important to realize that from the perspective of economic modeling, lack of realism is *good*. We judge a model by its utility for our purposes, not by its overall realism. Realism is essential to the model only when needed for the model to serve our purposes (understanding, forecasting, or policy guidance).

Just as you would not choose a scale model of a city to serve the purposes of a street map, you would not choose a scale model of an economy to guide your understanding of exchange rate determination. Good economic theory strives to isolate the considerations that are most important for our goals. A street map serves better than a topographic map for finding one's way in a city, but both are simplifications that have their uses. As in the production of a useful map, we produce a useful economic model by selecting components based on its intended use.

In keeping with this philosophy, we spend time on two quite different approaches to modeling the macroeconomy. The first approach has a long-run focus. We capture this in a very simple "Classical" model, wherein the domestic economy's output is held fixed at a level corresponding to the full and efficient employment of the economy's existing resources. (Economists often call this ***potential*** output or ***natural*** output.) The second approach has a short-run focus. We capture this in a very simple "Keynesian" model, wherein the price level of the domestic economy is predetermined. (This is a very stylized way of representing sticky prices in the domestic economy.) At times, as in our discussion of exchange rate overshooting, we will also discuss the transition between the short run and the long run.

1.1.3 Policy Goals

People often study economics more out of an interest in economic policy than in economic forecasting. However, good prediction may be a prerequisite for effective discretionary policy. One must anticipate the effects of proposed policies before choosing among them. For economists, good macroeconomic prediction remains elusive. For example, despite substantial efforts by exchange-rate economists, our ability to predict fluctuations in foreign exchange rates remains extremely limited. This can be viewed as a disappointment or as a challenge. In this book, we emphasize the challenge, and we try to shed light on how economists are grappling with this challenge.

Consider the following policy goals:

- high real income growth
- low variance of real income around it's trend
- low unemployment
- low inflation
- low variance of inflation
- fair income and wealth distributions
- current account "balance"

As we will see, there may be trade-offs between various policy goals. In such cases, you will need to decide which of these are most important.

1.2 Floating Foreign Exchange Rates

A ***foreign exchange rate*** is the rate at which two currencies trade for each other. In this book, except when otherwise specified, an exchange rate is the domestic currency price of a unit of foreign currency. (This is called a ***direct rate***.) For example, if the domestic currency is the U.S. dollar and the foreign currency is the British pound, the exchange rate is the number of dollars it takes to buy a pound.

With this convention, the exchange rate looks like any other price. Someone pondering whether exchange rates should float or be fixed may therefore be tempted to draw an analogy between the market for foreign exchange and the market for commodities. Strong arguments have been offered against price fixing in commodity markets; can these very arguments be marshalled against fixed exchange rates? In short, no.

Currencies fundamentally differ from commodities. Consider two neighboring cities currently using a single currency. Would you propose that the cities adopt two separate monies with a floating parity? Dealing in two separate monies will involve new costs, and in this case there is no obvious offesetting gain.[1]

The views of policy makers with respect to floating exchange rates have been extremely volatile. Milton Friedman reports how in 1969 top IMF officials "dismissed my proposal for floating exchange rates as utterly impractical," but only two years later (facing the collapse of the Bretton Woods system) described "floating rates as the only practicable system" (Friedman and Friedman, 1998, p.220). The optimal choice of exchange-rate regime remains an important area of research in international finance, and we will take it up in Lecture**??**. Currently the countries of the world use a variety of exchange rate regimes, but most major currencies float against each other. In the middle of the 20th century, the situation was quite different: most major currencies were part of a fixed exchange rate system.

Economists at that time had a number of expectations for a regime of floating exchange rates—expectations that tended to be rather sanguine. As it turns out, many of these expectations have not been met.[2]

Floating exchange rates would be fairly stable. Theoretical models of exchange rates suggested that money, interest rates, and real income were fundamental determinants of the exchange rate. Economists expected exchange rates to be largely determined by this small number of exchange-rate ***fundamentals***. Correspondingly, they expected exchange rates to be about as stable as those fundamentals. They expected fluctuations in exchange rates to be fairly predictable, since the fundamentals (particularly monetary policy) were expected to be fairly predictable. Divergent macroeconomic policies—especially divergent monetary policies—would cause exchange rate movements, of course, but economists perceived the ability of exchange-rate movements to automatically offset divergent policies as a virtue of the system. The following quotes are representative of this view:

> . . . instability of exchange rates is a symptom of instability in the underlying economic structure . . . a flexible exchange rate need not be an unstable exchange rate. If it is, it is primarily because there is underlying instability in the

[1] The theory of optimum currency areas considers the various costs and benefits that arise when different geographic regions adopts a common currency.

[2] The following discussion draws on Dornbusch and Frankel (1988). See also Friedman (1953a), Sohmen (1961), and Johnson (1969).

> economic conditions . . .
>
> —Friedman (1953b)

> The freedom of rates to move in response to market forces does not imply that they will move significantly or erratically; they will do so only if the underlying forces governing demand and supply are themselves erratic . . .
>
> —Johnson (1969)

The reality has been substantial, unexplained fluctuations in exchange rates. Exchange rates are volatile relative both to their past history and to the known fundamentals. This is known as the ***volatility puzzle***. Related to this, since most exchange rate variability is not explained by the usual lists of exchange rate fundamentals, empirical exchange rate models have performed poorly both in-sample and in out-of-sample forecasting.

Not only have nominal exchange rates been volatile, but so have real exchange rates. Economists draw a distinction between the nominal exchange rate, which is the domestic currency cost of foreign currency, and the real exchange rate, which is the cost in domestic goods of foreign goods. We define the ***real exchange rate*** (Q) as follows:

$$Q = \frac{SP^*}{P} \tag{1.1}$$

Here P is the domestic price level and P^* is the foreign price level. Taking the U.S. dollar (USD) as the domestic currency, can discover the units of the real exchange rate as follows:

$$\begin{aligned} \frac{SP^*}{P} &= \frac{(\#\text{USD}/\text{FCU})(\#\text{FCU}/\text{basket}^*)}{\#\text{USD}/\text{basket}} \\ &= \frac{\#\text{basket}}{\text{basket}^*} \end{aligned} \tag{1.2}$$

Here a * indicates the foreign country, and FCU is the foreign currency unit. So the real exchange rate is the number of baskets of the domestic good it takes to buy a basket of the foreign good. It is the cost of their good in terms of our goods. Volatility in the real exchange rate may cause volatility in the international demand for our goods, which can increase the volatility of our trade balance and even our unemployment rate.

Nowadays many economists believe that existing models cannot easily accommodate the short-run behavior of exchange rates. The list of fundamentals implied by accepted theory does not seem closely related to exchange rate movements. And there are other problems as well, which we will take up in future chapters. As one example, the appreciation of the early 1980s was large but gradual, while popular models predict that it should have taken place in a series of large jumps (in response to policy changes).[3]

Real exchange rates were expected to be very stable in the long run, but sticky prices would slow the adjustment toward long run equilibrium. While divergent macroeconomic policies might lead to volatility in the nominal exchange rate, as we have just

[3] However if the credibility of policy news accumulates slowly, then slower movements are predicted.

discussed, economists expected these nominal exchange rate movements to stabilize the real exchange rate—if not immediately, then after a short adjustment period.

Belief in the stability in the real exchange rate is called the ***purchasing-power-parity*** doctrine. The purchasing-power-parity doctrine holds that, at least in the long run, the real exchange rate approximates a constant value that is determined by real economic activity. As Lecture6 demonstrates, the reality has been that purchasing power parity fails badly in the short-run, and evidence that it prevails in the long-run remains problematic. Most movement in real exchange rates is unexpected and very persistent. Some economists even consider these persistent shocks to be "permanent", in the sense that they have no tendency to be automatically reversed. Long-run exchange rates have proved much harder to predict than the purchasing power parity doctrine seems to suggest.

Countries would be free to pursue divergent macroeconomic policies with less concern about international linkages. In particular, it was expected that flexible exchange rates would free countries to pursue independent monetary policies. And indeed, countries have seized this freedom in picking macroeconomic policy mixes. The generalized float has been associated with divergent monetary policies and divergent inflation rates. For example, the tight monetary policies initiated by the U.S. and U.K. in 1979 were independent of German or Japanese policy actions. Related to this policy independence, real interest rate differentials across countries have also increased. However, as continual G-7 meetings make clear, countries continue to be very interested in international policy coordination.[4] Further, monetary policy changes abroad can lead to large capital flows and large exchange rate movements, which may influence domestic monetary policy.

Countries would be insulated from external "shocks". Changes in foreign income, interest rates, or import demand were expected to have less influence on the domestic economy under flexible exchange rates. However it is certainly not obvious that international macroeconomic linkages have been reduced during by flexible exchange rates. In fact, the strong cross-country correlations between real GDPs appear to have increased during the floating rate period (Lumsdaine and Prasad, 2003). Possibly this correlation traces to large global shocks, such as the large oil price movements of the 1970s and 1980s. Another contributor may be real interest rate linkages across economies, which may in turn be responses to sticky prices and high capital mobility. And in fact some evidence suggests that floating exchange rates reduce the variability of inflation and GDP (Taylor, 1995).

Large trade imbalances would be rare, reducing protectionist pressures. A key concern in discussions of international monetary reform is now that large exchange rate fluctuations are an important source of protectionist pressure. In addition, the U.S. has run very large current account deficits since the early 1980s. Protectionism probably has nevertheless declined in the last two decades, it is difficult to asses the extent to which increases

[4]The G-7 meetings bring together the finance ministers from seven nations: Canada, France, Germany, Italy, Japan, United Kingdom, and United States.

in non-tariff protectionism has offset declines in tariff barriers (such as those negotiated under the GATT).

Central banks would hold fewer foreign exchange reserves. Central banks continue to accumulate foreign exchange reserves. Of course, it is also true that central banks have continued to actively intervene in foreign exchange markets. Some of these interventions have been huge, their size possibly mandated by high levels of international capital mobility.

For example, in 1998 the United States intervened to bolster the Japanese yen, which was tumbling in response to a financial panic that had spread around Asia. The yen subsequently rose about 30 percent, suprising giant investment funds and banks that had been betting on a further deterioration of the yen and were forced to abandon their positions with heavy losses. As another example, in late 2000 the European, American and Japanese central banks staged a coordinated currency intervention to prop up the declining euro. The European Central Bank had raised interest rates six times in a year without much affect on the euro's decline. Concerns had arisen that the decline in the euro would hurt the global economy by causing recessionary pressures in Europe: the relatively high oil prices in late 2000 hit Europe particularly hard due to the euro depreciation. (Oil is priced in dollars.) After a single-day intervention of perhaps USD 10B, the euro rose briefly from USD .85 to USD .9, but almost immediately lost about half that gain. By 26 Oct it hit a new low of 82.3 cents. In early November, the ECB tried intervening again, this time on its own. The market response was limited. In subsequent year, despite the absence of targeted interventions, the euro appreciated strongly against the dollar.

Speculation would be stabilizing rather than destabilizing. Foreign exchange speculators buy foreign exchange (or claims on foreign exchange) when they expect it to rise in value and sell foreign exchange when they expect it to fall in value. Economists believed speculators would smooth price fluctuations over time the way arbitragers smooth price fluctuations over space: increasing demand where price is relatively low, and increasing supply where price is relatively high. That is the basis of profitable speculation. Friedman (1953a) offered an extremely influential argument that since profitable speculation must be stabilizing, speculation must ultimately prove stabilizing: loss-making speculators eventually will be driven from the market.

As we have seen, exchange rates have been quite volatile, and the volatility puzzle continues to pose an empirical challenge. It may be true, as McKinnon (1976) argues, that this represents *insufficient* speculation (due, e.g., to capital constraints and institutional rules that stop traders from maintaining large open positions in foreign exchange). However modern theory, such as the theory of speculative bubbles, now tells us that even under rational expectations speculation can be destabilizing. In addition, evidence has accumulated that many market participants rely on technical analysis, which is essentially fancy extrapolation. De Long et al. (1987) incorporate this observation into a "noise trader" model that shows how destabilizing, profitable speculators may remain in the market. (They are rewarded for unintentionally bearing more risk.) This radically weakens the theoretical claim that speculation must be stabilizing.

The new exchange-rate risk would have little effect on international trade, since financial markets would provide adequate instruments for hedging this risk. For example, Johnson (1969) argued that "traders averse to uncertainty would be able to hedge their transactions through forward exchange markets, which would, if necessary, develop in response to demand." The volume of international trade has grown, and with it the demand for foreign exchange hedging has grown. *Hedging* is a method of reducing the risk of loss due to foreign exchange fluctuations, which we will discuss in chapter 2. Most currencies are convertible—they can be freely traded for other currencies—and low cost hedging through forward markets is widely available. In this sense the prediction has been born out. Yet Dornbusch and Frankel (1988) observe that most international trade is not hedged in the forward market.

Problems for Review

1. What is a foreign exchange rate? (Be precise.)
2. Suppose upon learning of your studies in economics a friend comments that economic models cannot be useful because they are unrealistic. How would you respond?
3. How prescient were postwar economists about the behavior of flexible exchange rates?
4. Why did economists believe that profitable speculation should be stabilizing?
5. Why is prediction a prerequisite to policy making?
6. In microeconomics you learn that it is inefficient for policy makers to fix the price of a commodity that is sold in competitive markets. Why does this argument not apply to foreign exchange markets, which are very competitive?

Lecture 2

International Financial Markets: Basic Concepts

In daily life, we find ourselves in constant contact with internationally traded goods. If you enjoy music, you may play a U.S. manufactured CD of music by a Polish composer through a Japanese amplifier and British speakers. You may be wearing clothing made in China or eating fruit from Chile. As you drive to work, you will see cars manufactured in half a dozen different countries on the streets.

Less visible in daily life is the international trade in financial assets, but its dollar volume is much greater. This trade takes place in the international financial markets. When international trade in financial assets is easy and reliable—due to low transactions costs in liquid markets—we say international financial markets are characterized by high ***capital mobility***.

Financial capital was highly mobile in the nineteenth century. The early twentieth century brought two world wars and the Great Depression. Many governments implemented controls on international capital flows, which fragmented the international financial markets and reduced capital mobility. Postwar efforts to increase the stability and integration of markets for goods and services included the creation of the General Agreement on Tariffs and Trade (the GATT, the precursor to the World Trade Organization, or WTO). Until recently, no equivalent efforts addressed international trade in securities. The low level of capital mobility is reflected in the economic models of the 1950s and 1960s: economists felt comfortable conducting international analyses under the assumption of capital immobility.

Financial innovations, such as the Eurocurrency markets, undermined the effectiveness of capital controls.[1] Technological innovations lowered the costs of international transactions. These factors, combined with the liberalizations of capital controls in the 1970s and 1980s, led to the development of highly integrated world financial markets. Economists have responded to this "globalization" of financial markets, and they now usually adopt perfect capital mobility as a reasonable approximation of conditions in the international financial markets.

[1]The term 'Eurocurrency' refers to deposits denominated in a currency that is not the currency of the financial center where the deposit is held, such as dollar deposits in London or dollar deposits in Japan. The second example makes it clear that the terms is misleading, as Europe need not be involved.

International capital flows surged after the oil shock of 1973–74, which spurred financial intermediation on a global scale. Surpluses in the oil-exporting countries and corresponding deficits among oil importers led to a recycling of "petrodollars" in the growing Euromarkets. Many developing countries gained new access to international capital markets, where they financed mounting external imbalances. Most of this intermediation occurred in the form of bank lending, and large banks in the industrial countries accepted huge exposures to developing country debt. The debt crisis of the 1980s led to a significant slowdown in capital flows to emerging markets.[2] The waning of the debt crisis led to new large-scale private capital inflows to emerging markets in the 1990s.[3] Private capital responded to the efforts of many Latin American countries to liberalize, privatize, open markets, and enhance macroeconomic stability. Countries in Central and Eastern Europe began a transition toward market economies, and rapid growth in a group of economies in East Asia had caught the attention of investors worldwide. Net long-term private flows to developing countries increased from $42 billion in 1990 to $256 billion in 1997. This time the largest share of these flows took the form of foreign direct investment (investment by multinational corporations in overseas operations under their own control). These flows totaled $120 billion in 1997 (Council of Economic Advisors, 1999, p.221). Bond and portfolio equity flows were 34 percent of the total in that year, while commercial bank loans represented only 16 percent, compared with about two-thirds in the 1970s Council of Economic Advisors (1999, p.222). Net flows have been large and growing, but gross cross-border inflows and outflows have grown even faster. The Mexican peso crisis of December 1994 led to a modest slowdown in capital flows to emerging markets in 1995, they surged again thereafter until the Asian crisis erupted in the summer of 1997.

2.1 Foreign Exchange Market

Foreign exchange is highly liquid assets denominated in a foreign currency. In principle these assets include foreign currency and foreign money orders. However most foreign exchange transactions are purchases and sales of bank deposits. A ***foreign exchange rate*** is the price of one nation's currency in terms of another's.

When goods, services, or securities are traded internationally, the currency denomination of the payment may be an issue. The most obvious role of the foreign exchange market is to resolve this issue. Suppose for example that a US exporter of calculators to Mexico wishes to receive payment in dollars while the importer possesses pesos with which to make payment. Transforming the pesos into dollars will generally take place in the foreign exchange market.

When we speak of the foreign exchange market, we are usually referring to the trading of foreign exchange by large commercial banks located in a few financial centers—especially London, New York, Tokyo, and Singapore. Foreign exchange transactions topped $250B/day by 1986. By 1995 the foreign exchange market had a *daily* transactions

[2]Loose monetary and fiscal policies in the borrowing countries, sharp declines in their terms of trade, and high international interest rates, triggered the debt crisis of the 1980s. Starting in Mexico in 1982, that crisis rapidly engulfed a large number of developing countries in Latin America and elsewhere.

[3]Debts were rescheduled, restructured, and finally reduced with the inception of the Brady Plan in 1989.

volume of over a trillion dollars in the major financial centers (BIS, 2002, Table B.1).[4] By 1998 volume had risen to more than USD 1.5 trillion per day (after making corrections to avoid double counting). This is about 60 times the global volume of exports of goods and services.[5] However 1998 marked a temporary peak of trading volume in the traditional foreign exchange markets: although the forward market continued to grow, trading volume fell sharply in the spot foreign exchange markets. By 2001 volume had fallen to about $1.2 trillion per day. However, as seen in Figure 2.1, by 2007 volume reached USD 3.2T per day. As seen in Table 2.1, the dollar was still involved in about 90% of transactions, with the USD/EUR volume being more than a quarter of the total.

About 31% of these transactions take place in London and 16% in New York, similar to the situation in the previous decade. In the 1980s Tokyo established itself as a major center, but it has lost ground to Singapore. In 1992 Tokyo had about 13% of the foreign exchange volume; the most recent survey pegs it at 9% and gives Singapore around 6%. About half the trading volume was the USD vs. the EUR and the JPY (BIS, 2002, Table B.7). With the adoption of the euro, trading shares were little changed, but there was some decline in volume associated with the elimination of intra-EMS trading. The share of trading in the euro against the dollar in 1999 roughly matched that of the German mark, French franc and Italian lira against the dollar in April 1998. Moreover, the euro/yen market appeared to be as small as the mark/yen market in 1998 (BIS, 2000, p.98).

Between 1995 and 1998, the share of electronic broking in spot foreign exchange market activity increased from about 10% to about 15%. The share doubled in the following two years, and in certain market segments, such as those involving the major currencies, electronic brokers reportedly covered between 50 and 80% of the market. The advance of electronic broking owes much to its lower costs, higher efficiency and, most importantly, greater transparency compared to traditional means of dealing. Spot foreign exchange markets have traditionally been opaque, given the difficulty of disseminating price information in the absence of centralised exchanges. Before the advent of electronic broking, dealers had typically to enter into a number of transactions to get information market prices. The role of such price discovery activities has been drastically reduced as traders turn to electronic brokers, which are able to instantly determine the "best" price available in the market. As a result, foreign exchange dealers now require fewer transactions than needed in traditional trading. Bid-ask spreads on the major exchange rates have fallen dramatically.

The foreign exchange market is constituted by a geographically dispersed web of brokers, traders, businesses, and individuals. There is always at least one active center, so the foreign exchange market is open twenty-four hours a day. The main participants in this market are retail customers, commercial banks, foreign exchange brokers, and central banks.

Retail customers generally transact with commercial banks. Commercial banks hold inventories of foreign exchange to satisfy the needs of their retail customers. When a retail customer purchases foreign exchange from a commercial bank, the bank's inventories of

[4] The Bank for International Settlements (BIS) is an international institution in Basle, Switzerland, that acts as a kind of central bankers' bank.

[5] In 1997 global exports were about $6.6 trillion, or about $25 billion per trading day (Council of Economic Advisors, 1999, p.224).

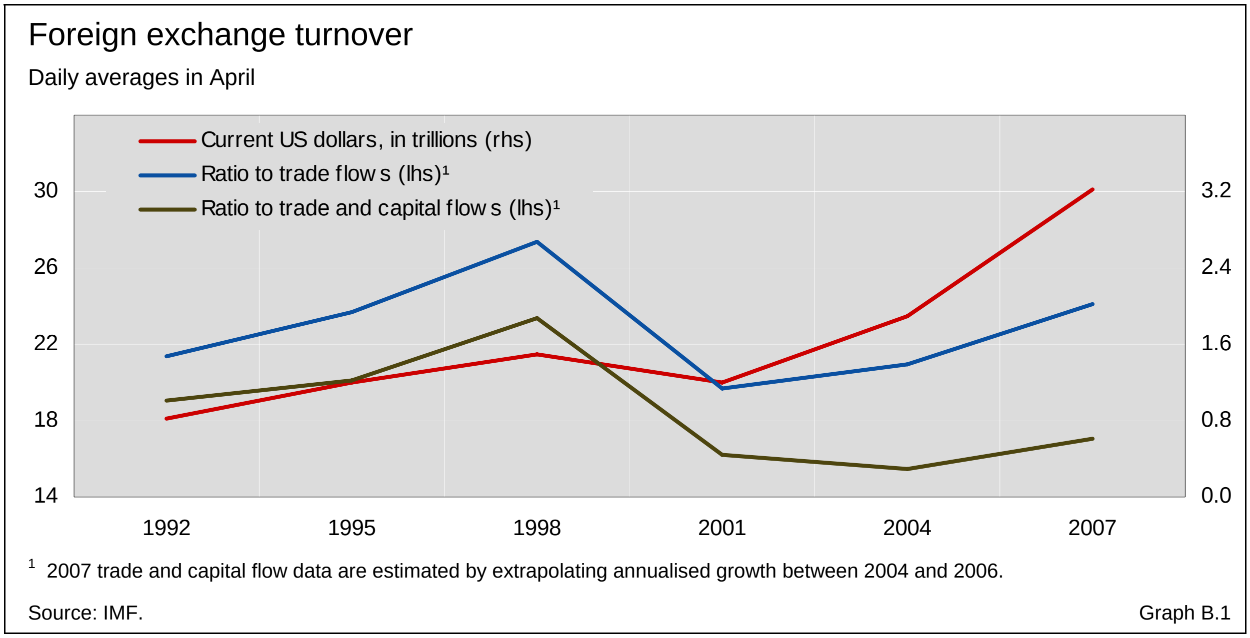


Figure 2.1
Source: BIS 2007

Table 2.1

Reported foreign exchange market turnover by currency pair[1]

Daily averages in April, in billions of US dollars and per cent

	2001		2004[2]		2007	
	Amount	% share	Amount	% share	Amount	% share
US dollar/euro	354	30	503	28	840	27
US dollar/yen	231	20	298	17	397	13
US dollar/sterling	125	11	248	14	361	12
US dollar/Australian dollar	47	4	98	5	175	6
US dollar/Swiss franc	57	5	78	4	143	5
US dollar/Canadian dollar	50	4	71	4	115	4
US dollar/Swedish krona[3]	…	…	…	…	56	2
US dollar/other	195	17	295	16	572	19
Euro/yen	30	3	51	3	70	2
Euro/sterling	24	2	43	2	64	2
Euro/Swiss franc	12	1	26	1	54	2
Euro/other	21	2	39	2	112	4
Other currency pairs	26	2	42	2	122	4
All currency pairs	1,173	100	1,794	100	3,081	100

[1] Adjusted for local and cross-border double-counting. [2] Data for 2004 have been revised. [3] The US dollar/Swedish krona pair could not be separately identified before 2007 and is included in "other". Table B.5

Source: BIS 2007

foreign exchange are depleted. When the customer sells foreign exchange, the bank's inventories increase. If the many retail sales and purchases were perfectly matched, there would be no net effect on the banks inventories of foreign exchange. But since sales and purchases are imperfectly offsetting, the bank's inventories move above or below their desired level. This is the basis of an active market in foreign exchange among commercial banks.

Commercial banks in the U.S. may trade foreign exchange directly with each other. More often, they rely on interbank intermediaries called foreign exchange ***brokers***. A broker is someone who "brings together" a buyer and a seller, without taking a position in foreign exchange. That is, the broker simply arranges the transaction for a fee. This fee is a ***spread*** between what a purchaser of foreign exchange pays (the ***ask price***) and what the seller of foreign exchange receives (the ***bid price***).

Use of a foreign exchange broker allows anonymous pricing, which is a reason central banks also rely on brokers for their foreign exchange transactions. Major brokerage houses are global and thereby able to service the interbank market around the clock.

2.2 Exchange Rates

We have been talking about the purchase and sale of foreign exchange. Of course, these transactions must take place at some price. We call that price the ***exchange rate***. That is, an exchange rate is the rate at which two different monies trade for each other. In this book, an exchange rate is the number of units of the domestic money required to purchase one unit of a foreign money. This type of exchange rate is called a ***direct quote***.[6] With this convention, an exchange rate is like any other price: the domestic currency cost of a purchase.

For example, if an American must spend USD 0.80 to buy a Canadian dollar, the exchange rate is CAD-USD 0.80. The first currency in the pair is called the ***base currency***. The second currency in the pair is called the ***quote currency*** (or counter currency). So in this example, the base currency is the Canadian dollar and the quote currency is the U.S. dollar. An exchange rate states how much of the quote currency you need to buy one unit of the base currency.

From the perspective of a Canadian facing the same relative price, it takes CAD 1.25 to buy a American dollar, so the exchange rate is USD-CAD=1.25. Note that we have the three letter ***ISO codes*** to describe the relationship between the Canadian dollar (CAD) and the U.S. dollar (USD).[7] In this book, we shall also use the codes for the Euro (EUR),

[6]The ***reverse quote***—the number of units of foreign money required to purchase one unit of the domestic money—is also often reported and referred to as an exchange rate. In fact US traders generally use the reverse quote convention, but we use the direct quote—a price like any other—because it is conceptually simpler.

[7]The codes were created by the International Organization for Standardization and are maintained as ISO Number 4217:1995, "Codes for the representation of currencies and funds." The ISO 4217 currency code is usually composed of the country's two-character ISO 3166 country code plus an extra character to denote the currency unit. For example, the code for United States Dollars is simply the US's two-character ISO 3166 code ("US") plus a one-character currency designator ("D"). For more discussion see http://www.xe.com/iso4217.htm. For the ISO 3166 codes see

the British pound (GBP), and the Japanese yen (JPY).

2.2.1 Spatial Arbitrage

As discussed above, the foreign exchange market is geographically dispersed but highly integrated. The primary reason for this is the extremely low transactions costs, both in the actual transfer of foreign exchange and in communication between different centers. Transfer costs are low because of the perfect homogeneity and minimal costs of geographical transport of the traded commodities (e.g., dollar bank deposits). Communication takes place continuously by telephone and computer network. As a result, exchange rates in different centers are closely aligned. This ***spatial arbitrage*** quickly eliminates differences in exchange rates between centers.

We will work with a stark definition of ***arbitrage***: buying an item where it is cheap and *simultaneously* selling it where it is dear. Naturally this raises demand and therefore price in the cheap location, whereas in the dear location supply rises and price falls. The price for the arbitraged item thereby moves toward a single value in all locations.

The limiting case of zero transactions costs captures the key intuition behind arbitrage. For example, suppose you received a quote of CAD-USD=0.81 in New York and CAD-USD=0.82 in London. You could simultaneously buy Canadian dollars in New York for USD 0.81 and sell them in London for USD 0.82, making a risk free profit. Such a transaction is arbitrage, a risk free profit with essentially no capital investment. Of course arbitrage opportunities are extremely attractive, so we expect quick, large responses to them. The demand for CAD rises in New York and the supply rises in London, quickly eliminating any differences in exchange rates between centers.

2.2.2 Triangular Arbitrage

Let us ignore transaction costs for a moment, and consider a situation where the law of one price prevails for each exchange rate. That is, there is a single price at which each currency can be bought or sold in terms of each other currency. For example, imagine that you had the extreme good fortune to be quoted the following exchange rates.

	↓ buys →	USD	CAD	JPY
USA	USD	1	1.25	100
Canada	CAD	0.8	1	100
Japan	JPY	0.01	0.01	1

Table 2.2: Imaginary Opportunity for Triangular Arbitrage

You can buy JPY 100 with USD 1; and you can then sell JPY 100 back for USD 1. Since we are ignoring transactions costs, you just end up back where you started. And this is true for every pair of currencies in table 2.2. But this does not mean that there are no arbitrage opportunities left. Suppose you bought CAD 1.25 for USD 1, but then bought

http://www.din.de/gremien/nas/nabd/iso3166ma/.

JPY 125 for your CAD 1.25. The same initial dollar purchase has netted you more yen. (We call the indirectly achieved exchange rate of USD-JPY=125 a ***synthetic*** cross rate.) Note that you could now sell your JPY 125 for USD 1.25, completing a ***triangular arbitrage*** that nets you a profit.

As before, we expect that the activity of arbitrageurs will lead to an adjustment of the exchange rates and an elimination of this profit opportunity. That is, we expect triangular arbitrage to align exchange rates so that there are no profits from sequentially buying and selling three currencies. As a result, it is not cheaper to acquire desired foreign currency indirectly (via a third currency) than directly.

In the absence of triangular arbitrage opportunities, should we consider the profitability of sequentially buying and selling larger numbers of currencies? The answer is no: the elimination of triangular arbitrage opportunities also eliminates the profits from longer sequences of buying and selling.

2.3 Forward Exchange

Up to now, our discussion of the foreign exchange market has focused on the ***spot*** rate: the price for immediate delivery of foreign exchange.[8] However it is also possible to use a *forward contract* to contract for future sale or purchase of foreign exchange. Like a spot contract, a forward contract specifies quantity of foreign exchange to be purchased or sold and a price at which the transaction is to take place. However it also specifies a future date on which the transaction is to take place. There are active *forward markets* in major currencies for one month, two months, three months, 6 months, 9 months, and a year out. Total volume on the forward exchange markets (including swaps, discussed below) exceeds that on the spot markets. Spot market purchases declined to 40 percent of foreign exchange transactions by 1998 and to less than a third by 2007.[9] It is not unusual for banks to quote rates up to ten years forward. The price specified in a forward contract is referred to as the *forward exchange rate*.[10]

Two primary functions of the forward market are hedging and speculation. Hedging is the purchase or sale of an asset in order to offset the risk involved in one's current financial position. For example, someone who expects a future payment in foreign exchange can offset the implied exchange risk (the risk of an unforeseen change in the spot rate) by selling that foreign exchange forward. Speculation is the purchase or sale of an asset in order to profit from the difference between the current value of the asset and its expected future value. For example, speculators can try to profit from any difference between the

[8] Although small spot transaction may take place with no delay, large spot transactions may allow up to two working days for delivery (depending on the currencies involved).

[9] Over-the-counter derivatives remain a small fraction of total transactions, but they have been the fastest-growing segment of the market.

[10] The forward exchange rate is sometimes called the *outright rate*. Rather than quote the outright rate, market makers may quote the much less volatile *swap rate*. The swap rate is the difference between the outright rate and the spot rate. For example, suppose the spot rate is CAD-USD=0.80 spot and you are quoted a swap rate of +0.01. (The CAD is trading at a forward premium.) This implies a forward rate of CAD-USD=0.81. The swap rate is less volatile than the forward rate because forward and spot rates tend to move together.

current forward rate and their expectations of the future spot rate. Only about 20% of foreign exchange trades directly involve nonfinancial customers (Council of Economic Advisors, 1999, p.224).

Banks often insist on a fairly large minimum size for forward exchange contracts, on the order of USD 100,000 in 2007. You can request an "indicative" quote from your bank, which is half way between the bid and ask prices. Ordinarily the quote will be the value of $10^4(F - S)$, which is called the ***forward points***.[11] For example, on January 28, 2010, when the EUR-USD spot rate was 1.3982 a two year forward points were −39, implying a two-year forward rate of 1.3943.

2.3.1 FX Swaps

You could combine a spot transaction with a forward transaction in the reverse direction. This locks in a rate of return based on the difference between the two rates. Foreign exchange swaps allow you to arrange this as a single transaction. For example, foreign exchange may be purchased spot and sold forward. The combined transaction is detailed in a single contract, the foreign exchange swap contract, which specifies the term of the swap and the swap rate. There are two settlement dates: the start date (or "near" date), when the currencies are first exchanged, and the end date (or "far" date), when they are exchanged back. The difference between the two exchange rates is called the swap rate (or swap *points*, if only the final digits are quoted). For reporting purposes, a swap is considered a single transaction. As seen in Table 2.3, foreign exchange swaps constitute the bulk of trading in the foreign exchange market.

For example, a CA firm may need USD 100,000 for three months. Say the initial exchange rate is CAD 1.2000 per USD with 31 swap points, so that the re-exchange rate is CAD 1.2031 per dollar. At the start date, the firm will pay CAD 120,000 for USD 100,000. At the end date the firm "returns" the USD 100,000 and receives CAD 120,310.

2.3.2 Hedging

Fluctuations in the spot exchange rate cause fluctuations in the value of assets denominated in a foreign currency. That is, holding such assets involves *currency risk*. The forward market can be used to *hedge* this risk. To see how, imagine that you are a U.S. exporter who expects to receive payment of JPY 1M in one month. Since the USD-JPY exchange rate may move during the month, you do not know what the dollar value of your receipts will be. You are exposed to currency risk. However, you could sell the JPY 1M forward today, which would eliminate the currency risk and lock in the dollar value of the receipts.[12]

[11] Traditionally the forward-points quote was in pips. A percentage in point, or pip, was the smallest exchange rate change commonly quoted for a currency pair. (With electronic trading, many currency prices include an extra decimal point, or 1/10 pip.)

[12] Of course, you do not need the forward market to hedge your future receipt. Instead you could borrow JPY 1M$/(1 + i^*)$ which you will pay back with your JPY 1M receipt when it arrives. In the meantime, you can put the money to work for you without exchange risk by selling it spot and lending it, so that you end up with

$$\text{JPY 1M}\, S\,(1 + i)/(1 + i^*)$$

Table 2.3

Global foreign exchange market turnover[1]
Daily averages in April, in billions of US dollars

	1992	1995	1998	2001	2004[2]	2007
Spot transactions	394	494	568	387	631	1,005
Outright forwards	58	97	128	131	209	362
Up to 7 days	…	50	65	51	92	154
Over 7 days	…	46	62	80	116	208
Foreign exchange swaps	324	546	734	656	954	1,714
Up to 7 days	…	382	528	451	700	1,329
Over 7 days	…	162	202	204	252	382
Estimated gaps in reporting	44	53	60	26	106	129
Total "traditional" turnover	820	1,190	1,490	1,200	1,900	3,210
Memo: Turnover at April 2007 exchange rates[3]	*880*	*1,150*	*1,650*	*1,420*	*1,970*	*3,210*

[1] Adjusted for local and cross-border double-counting. Due to incomplete maturity breakdown, components do not always sum to totals. [2] Data for 2004 have been revised. [3] Non-US dollar legs of foreign currency transactions were converted from current US dollar amounts into original currency amounts at average exchange rates for April of each survey year and then reconverted into US dollar amounts at average April 2007 exchange rates. Table B.1

Source: BIS 2007

Exchange rate variability creates an incentive to hedge, to reduce the variance of profits. The extent of hedging will depend on its cost, of course. If you can sell forward your anticipated foreign exchange receipts at a price close to what you expect will prevail in the spot market, then hedging will look attractive. If the forward sale yields much less revenue than expected from a future spot sale, you are less likely to hedge. You may also be less likely to engage in international trade. To what extent is concern about the negative effects of exchange rate variability on international trade offset by the availability of hedging through the forward market?

2.3.3 Speculation

The forward market can also be used for pure speculation. Suppose for example that you expected that in one month a spot rate of CAD-USD=0.81 while the current one month forward rate was CAD-USD=0.8. You might choose to bet on your expected future spot rate by buying Canadian dollars forward for U.S. dollars, in anticipation of selling them for a profit. Of course this is risky: if the future spot rate turns out to be CAD-USD=0.79 you will lose from your speculative activity. Nevertheless, the further the forward rate deviates from your expected future spot rate, the greater your incentive to place a bet

So choose the method of hedging that leaves you with the most dollars at the end of the transactions. When

$$F = S\frac{1+i}{1+i^*}$$

you will be indifferent between the two methods.

on your expectation. This suggests that speculative activity will keep forward rates from wandering too far from the expected future spot rates of major participants in the foreign exchange markets.

Empirically, however, forward rates appear to be more closely linked to current than to future spot rates. This raises some puzzling questions that are explored in chapter 10. One possibility is that volatile new information is constantly shifting the spot rate and the expected spot rate together, and the correlation between the spot rate and the forward rate reflects this. In this case, the forward rate might be the best possible predictor of the future spot rate, even though it is a poor predictor by absolute standards. In fact, Levich (1982) and Brasse (1983) find that the forward rate outperforms professional forecasters. Another possibility is that the forward rate is not closely linked to the expected future spot rate. Note that as a matter of proximate causality, commercial banks set the forward rate to insure that covered interest parity holds (Levich, 1981). This places emphasis on the current spot rate and interest rates as determinants of the forward rate, rather than on the expected future spot rate. (See Llewellyn (1980) for a discussion.)

Speculation in the spot market may exceed that in the forward market. Goodhart (1988) found that London bankers speculate on exchange rate movements by issuing debt in one currency in order to buy another currency spot.

2.3.4 Foreign Exchange Futures

The large minimum transaction size keeps some players out of the forward market. The futures market offers a close substitute, with a few differences.

- traded on organized exchanges (e.g., in the US, on the International Money Market at the Chicago Mercantile Exchange), which maintain zero net positions
- fewer currencies are traded
- contracts mature at standardized dates, usually the third Wednesday of March, June, September, and December
- contracts are for standardized values (e.g., a EUR contract will have a face value of EUR 125,000.
- contracts are "marked to market" on a daily basis: you must deposit an initial margin with your broker when buying a futures contract, and losses or gains to the value of your contract are debited or credited to your account on a daily basis.

The margin requirement means there is an interest cost associated with the purchase of a futures contract, but in the U.S. part of your margin deposit can be in the form of Treasury bills. Another difference is that forward contracts are generally fulfilled, while futures contracts are generally "reversed" before maturity, so that no currency delivery takes place.

2.4 Covered Interest Parity

The forward rate allows you to lock in an effective return from holding foreign currency. That is, you can spend S_t to buy a unit of foreign exchange today (time t) and then immediately sell it forward $T-t$ days in the future for $F_{t,T}$.[13] Your effective return is then

$$\text{fd}_{t,T} \stackrel{\text{def}}{=} \frac{F_{t,T} - S_t}{S_t} \tag{2.1}$$

We call fd the forward discount on the domestic currency or, equivalently, the forward premium on foreign exchange. For example, if the EUR-USD is 1.20 spot but is 1.23 forward 180 days, then the 180-day forward discount on the dollar is (1.23-1.20)/1.20=0.025. Using this computation, figure 2.2 illustrates variations in the forward discount for a given forward rate and various spot rates.

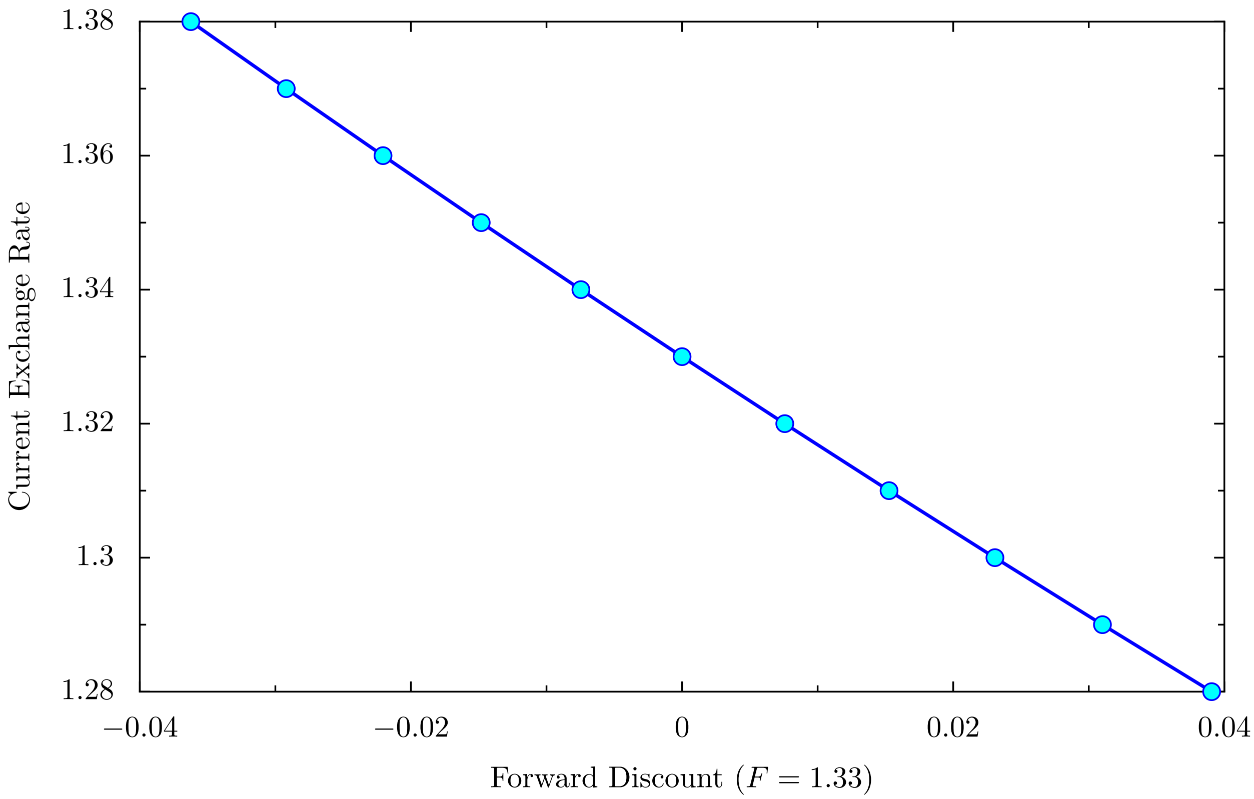


Figure 2.2: Computing the Forward Discount on the Domestic Currency

The forward discount on the domestic currency is positive if the forward rate is greater than the spot rate—that is, if the domestic currency price of foreign exchange is higher in the forward market than in the spot market. In this case, we say that the domestic currency trades at a forward discount. The forward discount on the domestic currency is negative

[13] In practice you would undertake these two transactions in a single contract to reduce the transactions cost. The forward discount gives the effective return over the period $[t, T]$: a percentage return that has not been annualized.

if the forward rate is less than the spot rate. In this case, we say that the domestic currency trades at a forward premium. Note that, ignoring the unlikely event of your forward contract going unfulfilled, the return defined by the forward discount is essentially risk free.

Of course, if effective return is your concern you can do better than this. Instead of holding foreign currency from t to T, you can invest in a foreign currency denominated interest bearing asset. Let $i^*_{t,T}$ be the risk free return available on foreign currency denominated assets. Then without incurring additional risk, you can raise your effective return to $i^*_{t,T} + \mathrm{fd}_{t,T}$.[14] This is the basis of the concept of covered interest parity.

Let $i_{t,T}$ denote the risk free effective return on domestic currency denominated assets. We have seen that $i^*_{t,T} + \mathrm{fd}_{t,T}$ is the risk free effective return available on foreign currency denominated assets. Since each of these is a risk free rate of return measured in the domestic currency, they must be equal.

$$i_{t,T} = i^*_{t,T} + \mathrm{fd}_{t,T} \tag{2.2}$$

Equation (2.2) is known as the covered interest parity condition. When no confusion can arise as to the periods involved, we express (2.2) more simply as (2.3).

$$\mathrm{fd} = i - i^* \tag{2.3}$$

We say portfolio capital is "mobile" to the extent that international financial markets are frictionlessly integrated. (Frictions include transactions costs or government controls.) In such circumstance, domestic and foreign residents are on equal footing in the purchase and sale of domestic and foreign assets. We then expect opportunities for risk-free profit making—arbitrage opportunities—to be quickly and completely exploited. Thus the most basic measure of international capital mobility is the disappearance of arbitrage opportunities in international financial markets. Of particular interest for our current purposes is that different ways of obtaining a riskless interest yield in any one currency should yield identical rates of return. That is, covered interest parity should obtain. Covered interest parity is thus a basic condition of perfect capital mobility, and deviations from covered interest parity are a primary indicator of imperfections or frictions in international capital markets.

Deviations from covered interest parity can be due to transactions costs, capital con-

[14] This development of the covered interest parity condition neglects the forward sale of the interest payment on the foreign currency denominated asset. That is, to eliminate all currency risk we need to sell forward $1 + i^*_{t,T}$ units of foreign exchange (not just one unit). If we take this into account, we get

$$i_{t,T} = i^*_{t,T} + \mathrm{fd}_{t,T}(1 + i^*_{t,T})$$

as our covered interest parity condition. Since the product $\mathrm{fd}_{t,T} i^*_{t,T}$ will often be very small, we will keep things simple by ignoring it.

Alternatively, if we interpret i as the log of one plus the domestic interest rate, i^* similarly, and fd as the log of one plus the forward discount, the original statement holds exactly.

Note that we have assumed the availability of risk free interest bearing assets. Country risk may be important for some assets, leading to deviations from covered interest parity. These deviations, known as the country premia, are small for the major industrial nations (Isard, 1988).

trols, political risk (e.g., fear of capital controls), or limitations on the supply of arbitrage funds. The last of these may help explain why CIP does not appear to hold well over long horizons (e.g., several years). Political-risk is sometimes discussed under the rubric of 'safe-haven effects,' where an increase in perceived political risk leads to an increased demand for assets from countries with low political risk.

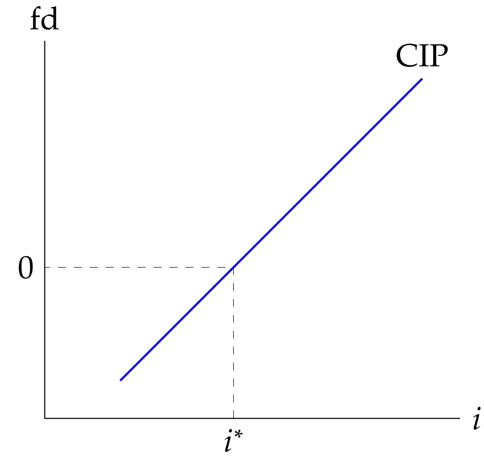


Figure 2.3: Covered Interest Parity

2.5 Capital Mobility

When financial assets are freely tradable across international borders at negligeable transactions costs, we speak of *perfect capital mobility*. For many countries, especially developed economies, perfect capital mobility seems to be a reasonable approximation of actual market conditions. Yet in a world with highly integrated financial markets, we might expect returns on similar domestic and foreign assets (Harberger, 1978). Returns on domestic and foreign assets often differ, even when the assets appear comparable in risk and liquidity characteristics.[15] In this section, we briefly explore three reasons for this divergence: risk premia, transactions costs, and capital controls.

2.5.1 Risk Premium

Even economies with perfect capital mobility may not have *perfect capital substitutability*. An individual treats two assets as perfect substitutes when the decision between them is based solely on their expected rate of return. An immediate implication is that when market participants consider two assets to be perfect substitutes, these will offer a common expected rate of return. Most assets, however, are not perfectly substitutable in an

[15]Furthermore, domestic savings and investment are highly correlated, which is something of a puzzle in highly integrated financial markets (Feldstein and Horioka, 1980; Frankel, 1991). However, Montiel (1994) notes that Feldstein-Horioka tests indicate a relatively high degree of capital mobility for most Latin American countries, despite the extensive legal barriers.

individual's portfolio. For example, an individual will usually care about the risk characteristics of assets, which generally differ among assets. (See chapter 10 for a more complete discussion.)

Compare the *ex post* real return from holding the domestic asset, with the uncovered real return from holding the foreign asset,

$$r_t \overset{\text{def}}{=} i_t - \pi_{t+1} \tag{2.4}$$

$$r_t^{df} \overset{\text{def}}{=} i_t^* + \Delta s_{t+1} - \pi_{t+1} \tag{2.5}$$

Define the ***excess return*** on the domestic asset as difference between these two *ex post* real rates of return.

$$\begin{aligned} \text{er}_{t+1} &\overset{\text{def}}{=} r_t - r_t^{df} \\ &\overset{\text{def}}{=} (i_t - \pi_{t+1}) - (i_t^* + \Delta s_{t+1} - \pi_{t+1}) \\ &= i_t - (i_t^* + \Delta s_{t+1}) \end{aligned} \tag{2.6}$$

This is the *ex post* difference in the uncovered returns.

A similar relationship must hold for the expected excess returns and the expected rate of depreciation of the spot rate. Consider an individual facing interest rates i and i^* who expects the spot rate depreciation to be Δs^e. If this individual understands (2.6) and has a minimal consistency in her expectations, then her expected excess return from holding the domestic asset with interest rate i instead of the foreign asset with interest rate i^* must be

$$\text{er}_{t+1}^e = i_t - (i_t^* + \Delta s_{t+1}^e) \tag{2.7}$$

A key determinant of an individual's asset allocation decisions will be the expected excess return from holding that asset. (We discuss this in detail in chapter 10.) We refer to the *equilibrium* expected excess return as the risk premium (rp) in the foreign exchange market. For convenience we will ignore any variation in the spot depreciation expected by different market participants, so we can just write the risk premium without ambiguity.

$$\begin{aligned} \text{rp}_t &\overset{\text{def}}{=} \text{er}_{t+1}^e \\ &= i_t - (i_t^* + \Delta s_{t+1}^e) \end{aligned} \tag{2.8}$$

In an ***efficient market***, the expectations of market participants are fully reflected in the equilibrium market price. That is, we call a market ***efficient*** when the market price is an equilibrium price that fully reflects the beliefs of market participants. When capital is highly mobile, we expect the foreign exchange markets to be very efficient in this sense. In the example we are considering, interest rates will fully reflect the spot rate depreciation expected by market participants (since this determines their expected excess returns and thereby their asset demands). Capital mobility will also assure covered interest parity, so we can also express the risk premium as

$$\text{rp}_t = \text{fd}_t - \Delta s_{t+1}^e \tag{2.9}$$

2.5.2 CIP Again

Give our decomposition of the forward discount on the domestic currency into a risk premium on domestic assets and an expected rate of depreciation of the domestic currency, we can offer an alternative representation of covered interest parity.

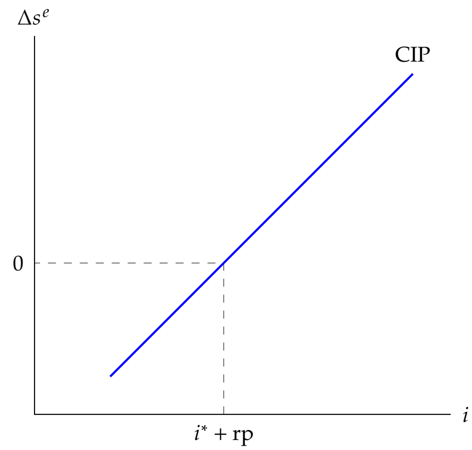
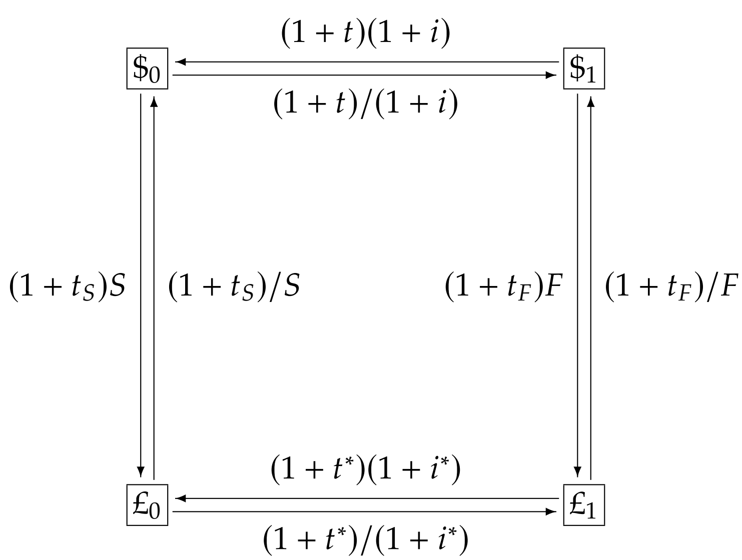


Figure 2.4: Covered Interest Parity

2.5.3 Transaction Costs



Source: Deardorff (1979).

Figure 2.5: Foreign Exchange and Securities Markets

Although we will often work with the simplification of referring to a single exchange rate, the real world is a bit more complicated. First of all, there is a small difference between purchase and sale prices in the foreign exchange market. Foreign exchange is sold at the *ask rate*, while it is purchased at the lower *bid rate*. The *spread* between the selling and buying rates is a source of profits for the banks and brokers in the foreign exchange market. In the late 1990s, the lower cost and greater transparency of electronic broking led to its domination of the spot foreign exchange market, driving the wholesale bid-ask spreads on most major currencies below three hundredths of a U.S. cent (BIS, 2000, p.99). Second, not everyone participates in the wholesale market. The modest foreign transactions of a small business or an individual will take place at much higher spreads. This is to be expected, for small transactions any fixed costs of foreign exchange transactions will have to be reflected in the spread (unless explicitly charged). The bid-ask spread varies by customer type. A tourist buying or selling foreign exchange at a bank may pay around 1%, creating a spread of 2%. The same tourist using a credit card may face a much lower spread, generally much closer to the wholesale spread.

In every case, the intuition behind arbitrage remains the same. For example, suppose you received a quote of CAD-USD=0.80-0.81 in New York and CAD-USD=0.82-0.83 in London. You could buy Canadian dollars in New York for USD 0.81 and simultaneously sell them in London for USD 0.82, making a risk free profit. Arbitrage opportunities exist as long as the bid price in one center is higher than the ask price in another. We therefore expect such occurrences to quickly disappear.

Competition and the Law of One Price

You may have noticed that in the presence of transaction costs the arbitrage argument does not assure that bid prices (or ask prices) are identical across centers. Arbitrage merely assures that bid and ask prices overlap. However we can expect identical bid and ask prices across centers due to the competition among traders in the foreign exchange markets. Any bank wishing to sell foreign exchange must offer it at a price as low as that of its competitors. Any bank wishing to buy foreign exchange must offer a price as high as its competitors. Thus all banks who wish to be involved in both buying and selling foreign exchange will have the same bid and ask prices.

Synthetic Cross Rates

Once we return to the consideration of transactions costs, synthetic cross rates may be preferable to direct rates despite triangular arbitrage. Essentially, a *synthetic* version of a contract relies on multiple transactions to achieve the goal of the contract. Clearly the transactions costs of multiple transactions will be an issue in the construction of synthetic contracts. Yet moving between two currencies can be cheapest using the U.S. dollar as an intermediary, despite the need to incur transactions costs twice. The reason is that the high trading volume in the U.S. dollar results in low spreads of the U.S. dollar against other currencies.

Although there are an increasing number of exceptions, most currencies simply are not traded against each other by professional traders. Even the most actively traded currencies tend to be traded against the dollar. Suppose we have twenty actively traded currencies; then each will have nineteen exchange rates. That is 380 prices to keep track of in the foreign exchange markets. But if all twenty currencies are always quoted against the dollar, there are only nineteen prices to keep track of. At the retail level, the background reliance on the dollar is invisible: the customer is simply quoted the cross-rate.

2.5.4 Capital Controls

Capital controls were common in the twentieth century. The use of capital controls surged during World War I and again during the Great Depression. After World War II, many countries continued to use capital controls in an effort to deal with balance-of-payments difficulties. The IMF explicitly permitted such controls.[16]

During the 1970s and 1980s, the developed countries removed most of their controls on the international trade in financial assets. The free flow of international capital was presumed to increase income and growth, and developing countries were encouraged to follow suit. In the 1990s, many developing countries nevertheless continued to rely on extensive capital controls, often as a way to insulate their macroeconomic policy from international considerations (Grilli and Milesi-Ferretti, 1995; Johnston and Tamirisa, 1998). Controls on capital outflows were the most prevalent. But countries also turned to controls on capital inflows, primarily to control direct foreign investment and real estate purchases (Johnston and Tamirisa, 1998). Latin American countries in particular, and especially Chile and Colombia, turned to controls on capital inflows in an attempt to slow large capital inflows (Edwards, 1998).

Capital flows are one way to move consumption between the present and the future, and there can be many good reasons to do this. Demographic shifts are one reason: an aging population may find it prudent to save for retirement by investing abroad. This may explain why the more rapidly aging Japanese population buys more assets from the U.S. than they sell there (Neely, 1999). Real investment opportunities are another reason. It can pay to borrow abroad in order to make high yielding domestic investments. For example, from 1960–1980 South Korea borrowed heavily in international markets, financing its domestic investments during a period of strong growth (Neely, 1999). Finally, it may pay to borrow in order to consume in the face of temporary negative real-income shocks, including declines in export prices, rises in import prices, recession, or natural disasters. For example, after a devastating earthquake in 1980, Italy borrowed abroad to help finance disaster relief and rebuilding (Neely, 1999). These examples highlight the benefits of international capital flows.

However in the 1990s, a series of financial crises diminished the reputation of capital flows. In 1992–3, the European Monetary System. In December 1994, after a decade of reforms had finally led to renewed confidence and large capital inflows, Mexico experienced a major currency crisis. In 1997–8, several East Asian economies experienced large capital outflows that forced abandonment of exchange rate pegs and precipitated banking crises and severe recessions. As policy makers searched for ways to prevent a recurrence of such crises, many focused on the benefits of limiting the mobility of financial capital.

[16]Consider the following section of the IMF's Articles of Agreement, signed in 1944. "Article VI. Section 3. Controls of capital transfers: members may exercise such controls as are necessary to regulate international capital movements . . . "

On September 1, 1998, Malaysia adopted capital controls. The Malaysian strategy was to discourage short-term capital flows while permitting long-term capital to flow freely. Early assessments of the results were largely favorable, although some observers worried that they were replacing rather than enabling reform of a fragile financial sector (Neely, 1999).

Speculative Attacks

In 1997, government policies in Thailand led to high nominal interest rates and a stock market decline. Currency traders began to question whether the central bank could sustain these policies and whether a devaluation of the baht was imminent. High capital mobility meant that these speculators could rapidly move huge sums in order to place their bets against the baht. George Soros, a famous currency speculator who had profited handsomely from the collapse of the British pound in 1992, began with others to bet heavily against the baht. In May, trading soared from the usual USD 1B/day to more than USD 10B/day. Other Asian central banks aided the Thai central bank in its defense of the baht, and initially Soros and other traders left the field with huge losses. Ultimately, however, the speculators proved correct in their anticipation that the baht parity could not be sustained.

2.6 Monetary Authorities

Monetary authorities may hold foreign-currency denominated assets and intervene in foreign exchange markets. Table 2.4 shows reserve asset holdings of U.S. monetary authorities. The Exchange Stabilization Fund (ESF) was established in 1934 at the United States Treasury Department, with the mission to stabilize dollar exchange rates. Exchange rate policy is set by the Treasury, in consultation with the Federal Reserve. When the Secretary of the Treasury authorizes foreign exchange market transactions, these are executed by the Federal Reserve Bank of New York (FRBNY). (The FRBNY also invests foreign currency assets for the ESF.) Foreign currency assets are split between The Federal Reserve has a portfolio of domestic and foreign securities, which is called the System Open Market Account (SOMA). As seen in Table 2.4, reserve asset holdings are split between the ESF and the SOMA.

Problems for Review

1. What is foreign exchange? (Be precise.)

2. What are the ISO codes for the US dollar, the British pound, the Japanese yen, the Euro, and the Canandian dollar?

3. Turn to the table of cross rates in a recent issue of the Financial Times to update the first row of table 2.2. Complete the rest of the table by calculating cross rates.

Table 2.4: U.S. Foreign-Currency Reserve Assets

	ESF	SOMA
Euro-denominated assets	14,649.8	14,673.1
Cash deposits at official institutions	7,643.3	7,666.5
Repurchase agreements b	2,233.3	2,233.3
Securities held outright	4,773.3	4,773.3
German government securities 2,391.5 2,391.5		
French government securities 2,381.8 2,381.8		
Japanese-yen-denominated assets	9,477.6	9,477.5
Cash deposits at official institutions	3,132.7	3,132.7
Securities held outright	6,344.9	6,344.9
Reciprocal currency swaps (EUR)		511

Carrying Value in Millions of U.S. Dollars, as of September 30, 2013
ESF=U.S. Treasury Exchange Stabilization Fund
SOMA=Federal Reserve System Open Market Account
Source: `http://www.ny.frb.org/newsevents/news/markets/2013/fxq313.pdf`, Table 2
Note: the ESF's SDR assets and liabilities are not listed.

4. What is a "synthetic" cross rate?

5. Get exchange rate data from the Financial Times at http://www.FT.com and update table 2.2. (Registration is free.)

6. Find a paper offering exchange rate quotes, and determine the EUR-USD spot exchange rate, 30 day forward rate, and 90 day forward rate. Do the same for the EUR-JPY and JPY-USD exchange rates. How closely does the triangular arbitrage condition hold?

7. You are a Canadian exporter expecting a payment of USD 1M in three months. You want to hedge your currency risk. After looking at prevailing interest rates and exchange rates, you decide to hedge forward. Ignoring transactions costs, must US exporters expecting payment in Canadian dollars wish to hedge forward or spot?

8. Suppose there are no profit opportunities in triangular arbitrage. Prove that arbitrage opportunities involving more than three currencies have also been eliminated.

2.A Exponents and Logarithms

In order to test whether the available data are supportive of the core predictions of the monetary approach, we often need to specify the functional form of relationships that we have been describing only in the most general theoretical terms. Many empirical tests of the monetary approach require us to be specific about the functional form for money demand. The most widely used functional form is log-linear. This section offers a very brief review of logarithms. (The next section develops the log-linear specification of the monetary approach to the determination of flexible exchange rates.)

The discussion in this section is indebted to Maor (1994).

The simplest way to motivate logarithms is by thinking again about the geometric progression $2^0, 2^1, 2^2, \ldots$. The monotonicity of the exponential function means that it is an injection (one-to-one), so that the inverse mapping is also a function. We call the inverse mapping the *logarithmic* function. Continuing with our base 2 example, the exponential function maps $2 \mapsto 4$ and $3 \mapsto 8$, so the logarithmic function maps $4 \mapsto 2$ and $8 \mapsto 3$.

Consider any member of this geometric progression, x, so that

$$x = b^m \tag{2.10}$$

This states a relationship between the number x, the base b, and the exponent m. We have a second way of stating this relationship: we say that m is the logarithm with base b of the number x.

$$\log_b x = m \tag{2.11}$$

So $x = b^m$ and $\log_b x = m$ are just two different ways of saying exactly the same thing.

Looking again at our geometric progression, we see immediately that there is a simple rule for multiplying any two terms of the progression:

$$b^m \times b^n = b^{m+n} \tag{2.12}$$

For example,

$$b^2 \times b^3 = (b \times b) \times (b \times b \times b) = b \times b \times b \times b \times b = b^5 \tag{2.13}$$

In this sense, we have transformed the problem of multiplication into the simpler problem of addition: the addition of the exponents.

Here is another way of making the same observation.

$$\log_b(x \times y) = \log_b x + \log_b y \tag{2.14}$$

For example, suppose $x = b^m$ and $y = b^n$ (so that $\log_b x = m$ and $\log_b y = n$), then

$$\log_b(x \times y) = \log_b(b^m \times b^n) = \log_b b^{m+n} = m + n \tag{2.15}$$

With a little work, we can similarly transform division into the simpler problem of subtraction. We would like to use the rule

$$\frac{b^m}{b^n} = b^{m-n} \tag{2.16}$$

Here is another way of making the same observation. We would like to be able to say that

$$\log_b(x/y) = \log_b x - \log_b y \tag{2.17}$$

To see these are the same, recall the relationship between logarithms and exponents, and suppose $x = b^m$ and $y = b^n$ (so that $\log_b x = m$ and $\log_b y = n$). With the rule $b^m/b^n = b^{m-n}$ we can then write

$$\log_b(x/y) = \log_b(b^m/b^n) = \log_b b^{m-n} = m - n \tag{2.18}$$

For example

$$\frac{b^3}{b^2} = \frac{b \times b \times b}{b \times b} = b \times \frac{b \times b}{b \times b} = b = b^1 \tag{2.19}$$

But what about b^2/b^3? Applying our rule we get $b^2/b^3 = b^{-1}$, so we will need to know what is meant by a negative exponent. We will extend our exponential notation to define $b^{-m} = 1/b^m$. So $b^{-1} = 1/b^1$, $b^{-2} = 1/b^2$, etc. Then

$$\frac{b^2}{b^3} = \frac{b \times b}{b \times b \times b} = \frac{1}{b} \times \frac{b \times b}{b \times b} = \frac{1}{b^1} = b^{-1} \tag{2.20}$$

We just need to think of our geometric progression as extending in both directions:

$$\ldots, b^{-3}, b^{-2}, b^{-1}, b^0, b^1, b^2, b^3, \ldots \tag{2.21}$$

Our rule that multiplication can be characterized in terms of the addition of exponents produces a related rule:

$$(b^m)^n = b^{mn} \tag{2.22}$$

For example

$$\begin{aligned}(b^3)^2 &= b^3 \times b^3 \\ &= (b \times b \times b) \times (b \times b \times b) \\ &= (b \times b \times b \times b \times b \times b) \\ &= b^6\end{aligned} \tag{2.23}$$

Here is another way of making the same observation.

$$\log_b x^n = n \log_b x \tag{2.24}$$

For example, suppose $x = b^m$ (so that $\log_b x = m$) then

$$\log_b x^n = n \log_b x = n \log_b b^m = n \times m \tag{2.25}$$

In summary, we have developed three useful rules in our discussion of logarithms.

$$\log_b(xy) = \log_b x + \log_b y \tag{2.26}$$
$$\log_b(x/y) = \log_b x - \log_b y \tag{2.27}$$
$$\log_b x^k = k \log_b x \tag{2.28}$$

Let's take an example rooted in computers: base 2 arithmetic. Consider the case where $b = 2$:

n	...	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	...
2^n	...	1/8	1/4	1/2	1	2	4	8	16	32	64	128	256	512	...

We can multiply 16×32 as follows. Noting that $\log_2(16 \times 32) = \log_2 16 + \log_2 32 = 4 + 5 = 9$, we conclude $16 \times 32 = 2^9 = 512$.

2.B Expectations, Consistency, and Efficiency

By now it should be clear that we cannot go very far in our thinking about financial markets without thinking about expectations. Economists consider this a major problem, since expectations cannot be observed. Rather than simply treat expectations as an unobserved exogenous influence on financial markets, economists impose certain structure upon expectations. Two common structures are the expectations be self-consistent and informationally efficient.

2.B.1 Law of Iterated Expectations

Linearity

For any two variables x and y, and any two constants a and b,

$$\mathcal{E}_t(ax + by) = a\mathcal{E}_t x + b\mathcal{E}_t y \tag{2.29}$$

Forecast Errors

The *forecast error* is the difference between the expected outcome and the realized outcome. Expectational consistency requires that the expected forecast error be zero.

$$\mathcal{E}_t\{s_{t+1} - \mathcal{E}_t s_{t+1}\} = 0 \tag{2.30}$$

When combined with linearity, this implies

$$\mathcal{E}_t s_{t+1} = \mathcal{E}_t\{\mathcal{E}_t s_{t+1}\} \tag{2.31}$$

which is quite natural, since today's expectation is known today. But it also implies the stronger relationship

$$\mathcal{E}_t s_{t+k} = \mathcal{E}_t\{\mathcal{E}_{t+k-1} s_{t+k}\} \tag{2.32}$$

We say that expectations satisfy the law of iterated expectations. It is easy to imagine exceptions to the law of iterated expectations. For example, today I might know that the spot rate will be pegged at a specific value at time $t + k$, but I might also be planning a hypnosis session tomorrow where I will be induced to believe it will take on a different value. Requiring expected forecast errors to be zero disallows such anticipated loss of knowledge over time.

An Application

When the forward exchange market is efficient, the forward rate fully reflects the beliefs of the market participants. This relates the interest differential to the expected value of the future spot rate and the risk premium on the domestic currency.

$$i_t - i_t^* = \Delta s_{t+1}^e + \mathrm{rp}_{t+1} \tag{2.33}$$

For the moment, let us suppose the risk premium is zero. We can write the expected rate of depreciation as the expected change in the log of the spot rate

$$\Delta s_{t+1}^e = s_{t+1}^e - s_t \tag{2.34}$$

so we have

$$s_t = s_{t+1}^e - (i_t - i_t^*) \tag{2.35}$$

In the next period we will have

$$s_{t+1} = \mathcal{E}_{t+1} s_{t+2} - (i_{t+1} - i_{t+1}^*) \tag{2.36}$$

Applying the law of iterated expectations, we know that our expectation today of our next period's expectation is just our expectation today of the two period ahead spot rate.

$$\mathcal{E}_t \mathcal{E}_{t+1} s_{t+2} = \mathcal{E}_t s_{t+2} \tag{2.37}$$

Secondly, individuals should expect that (2.35) will hold in the future.

$$s_{t+1} = \mathcal{E}_{t+1} s_{t+2} - (i_{t+1} - i_{t+1}^*) \tag{2.38}$$

Together these give us

$$\mathcal{E}_t s_{t+1} = \mathcal{E}_t s_{t+2} - \mathcal{E}_t (i_{t+1} - i_{t+1}^*) \tag{2.39}$$

$$\mathcal{E}_t s_{t+2} = \mathcal{E}_t s_{t+3} - \mathcal{E}_t (i_{t+2} - i_{t+2}^*) \tag{2.40}$$

$$\vdots \tag{2.41}$$

Applying these relationships repeatedly to (2.35), we get

$$\begin{aligned} s &= \mathcal{E}_t s_{t+1} - (i_t - i_t^*) \\ &= \mathcal{E}_t s_{t+2} - \sum_{k=0} 1(i_{t+} - i_{t+k}^*) \\ &= \lim_{T\to\infty} \{\mathcal{E}_t s_{t+T+1} - \sum_{k=0}^{T} (i_{t+} - i_{t+k}^*)\} \end{aligned} \tag{2.42}$$

Informational Efficiency

Economists have meant a number of different things by the efficient markets hypothesis. Most often, the hypothesis is that markets are not only efficient, but are also informationally efficient. That is, not only do market prices fully reflect the beliefs of market participants, but the beliefs of market participants fully reflect the available information. When this is the case, economists like to say that expectations are "rational". What they mean is that market participants do not make systematic (i.e., predictable) forecasting errors.

Let the information available at time t be $\mathcal{I}_t$. Let $\mathcal{E}\{\cdot|\mathcal{I}_t\}$ denote the mathematical expectation conditional on this information. Then expectations are "rational" if

$$\Delta s_{t+1}^e = \mathcal{E}\{\Delta s_{t+1}|\mathcal{I}_t\} \tag{2.43}$$

When there is no danger of ambiguity about the information set, we will just write

$$\Delta s_{t+1}^e = \mathcal{E}_t \Delta s_{t+1} \tag{2.44}$$

Following Fama (1970), we can distinguish various degrees of market efficiency. A market is informationally efficient relative to $\mathcal{I}_t$ if "abnormal returns" cannot be obtained using only the information in $\mathcal{I}_t$. A market that is efficient relative to the past history of prices is called *weakly efficient*. A market that is efficient relative to all publically available information is called *semi-strong form efficient*. A market that is efficient relative to all publically and privately available information is called *strong form efficient*.

Consider the forecast error

$$\varepsilon_{t+1} = \Delta s - \mathcal{E}\{\Delta s_{t+1}|\mathcal{I}_t\} \tag{2.45}$$

Note that the expected value of the forecast error, given information $\mathcal{I}_t$, is zero.

Lecture 3

The Monetary Approach to Flexible Exchange Rates

This chapter is our first attempt to understand exchange rate determination. A good model will help us understand the past and anticipate the future. In Lecture2 we saw that many economic decisions are affected by expectations of future exchange rates. Yet exchange rates are notoriously hard to predict. One way economists hope to improve our exchange rate predictions is by discovering the fundamental determinants of exchange rate movements. The monetary approach to flexible exchange rates focuses on domestic and foreign money supply and money demand. Monetary policy is given the central role in exchange rate determination. The determinants of domestic and foreign money demand also prove to be fundamental determinants of the exchange rate.

3.1 Key Ingredients of the Monetary Approach

The monetary approach has two key ingredients: exogeneity of the real exchange rate, and a simple Classical model of price level determination.[1] Exogeneity of the real exchange rate means that inflation at home or abroad will not affect how much foreign goods cost in terms of domestic goods. The Classical model of price determination says roughly that the price level is proportional to the money supply, so that monetary policy is the key determinant of inflation rates.

Eventually, we will explore both of these constituents in some detail. Suffice it to say that as short-run descriptions of real economies, both appear quite unrealistic. However as long-run descriptions, they show somewhat more promise. So the monetary approach to flexible exchange rates is best seen as a description of long-run outcomes. As a description of short-run outcomes, it serves as a reference model that highlights some core concerns in our attempt to understand exchange rate determination.

[1]Some authors treat uncovered interest parity as central to the monetary approach. We will not need this restriction. We treat the risk premium as exogenous but possibly nonzero.

3.1.1 Exogenous Real Exchange Rates

Let P be the domestic consumer price index and P^* be the foreign consumer price index. For now, we will keep things simple by thinking of each price index as the monetary cost of a fixed consumption basket. Equation (3.1) defines the real exchange rate, Q.

$$Q \stackrel{\text{def}}{=} \frac{SP^*}{P} \tag{3.1}$$

We call Q the *real exchange rate* because it tells you the rate at which domestic goods must be given up to obtain foreign goods. The monetary approach to flexible exchange rates assumes that Q is exogenous. This exogeneity assumption fits naturally with the Classical model of price determination, which generally treats real variables as exogenous.

Given the real exchange rate, the nominal exchange rate and the relative price level have a determinate relationship given by (3.2).

$$S = Q\frac{P}{P^*} \tag{3.2}$$

Here S is the exchange rate, P is the domestic price index, P^* is the foreign price index, and Q is the exogenous real exchange rate. For any given Q, equation (3.2) requires that exchange rate movements offset price level movements so that the rate at which goods actually exchange for each other remains unchanged.

Purchasing Power Parity

Most presentations of the monetary approach to flexible exchange rates assume that the real exchange rate is not only exogenous but that it is invariant. This is called the *purchasing power parity* assumption.[2] Constancy of the real exchange rate implies that the exchange rate is proportional to the relative price level. For example, suppose domestic inflation leads to a doubling of the domestic prices, while foreign inflation is zero. Then a doubling of the exchange rate will leave the real exchange rate unchanged. As a theory of exchange rate determination, this is only a beginning: it does not explain the determination of relative price levels.[3] That is our next project.

3.1.2 The Classical Model of Price Determination

In the Classical model of price level determination, the supply of money determines the (perfectly flexible) price level. In the simple Classical model, monetary policy has no influence on real economic activity. The real interest rate and real income are determined in the goods and labor markets independently of monetary policy. This *dichotomy* (i.e.,

---

[2]Given a common base year and identical price index construction, the relative price level, P/P^*, is often called the *purchasing power parity exchange rate* (or sometimes simply the *purchasing power parity*). See for a detailed discussion of purchasing power parity. You may simplify for now by setting $Q = 1$ and thinking of PPP as an application of the law of one price.

[3]In fact some economists consider it less than a beginning, since they feel that the relevant price indices (P and P^*) are not observable Hodrick (1978); MacDonald (1993).

independence of the real sector from the monetary sector) is a helpful simplification when we model monetary phenomena such as the nominal interest rate, the inflation rate, or the exchange rate. It allows us to treat real income, the real interest rate, and even the real exchange rate as exogenous when we are modeling the determination of the price level.

Four key assumptions of the simple Classical model are relevant to the determination of the price level.

- real money demand (L) is a stable function of real income (Y) and the nominal interest rate (i).
- the money market is in continuous equilibrium.
- in addition to real income (Y) and the real interest rate r, the nominal money supply (H) is exogenous.
- the price level (P), *not* the interest rate (i), moves to clear the money market.

The first two of these assumptions are common to almost all macroeconomic models. The last two assumptions are specific to models in the Classical tradition.

The first assumption is implied by standard theoretical treatments of the demand for money.[4] It embodies the theoretical dependence of liquid transactions balances on the desired level of transactions and the opportunity cost of liquidity. The nominal interest rate represents the opportunity cost of holding money rather than less liquid assets, and real income proxies the real value of monetized transactions in the economy. A rise in the interest rate therefore decreases money demand, while a rise in real income increases money demand. Our real money demand function can therefore be written as $L[\overset{-}{i},\overset{+}{Y}]$.

The second assumption is also standard to almost all macroeconomic models, and since adjustments can take place very rapidly in asset markets, it is very reasonable. It says that the real money supply is always equal to real money demand:

$$\frac{H}{P} = L[i, Y] \tag{3.3}$$

Equation (3.3) is a characterization of money market equilibrium, where H is the money supply, P is the price level, i is the nominal interest rate, Y is real income, and $L[\cdot,\cdot]$ is the real money demand function. So (3.3) says that real money supply equals real money demand, where real money demand is a stable function of i and Y.

The third assumption lists the standard exogeneity assumptions of the simple Classical model. For the moment we will additionally treat the nominal interest rate as exogenous—an assumption we will soon drop. The final assumption, perfect price flexibility, is very strong. It implies that our model of money market equilibrium can be interpreted as a model of price level determination.

We are now ready to model our Classical theory of price determination. The theory states that nominal money supplies and real money demand determine the price level.

[4]In empirical work, however, this formulation is used for *long-run* real money demand. Short-run money demand functions are generally characterized in terms of a partial adjustment to the desired long-run level (Goldfeld and Sichel, 1990).

Specifically, the price level is determined as the ratio of nominal money supply to real money demand, where real money demand is determined by real income and the nominal interest rate. Algebraically, we simply solve (3.3) for the price level, which gives us (3.4) a simple Classical model of price determination.

$$P = \frac{H}{L[i, Y]} \tag{3.4}$$

Basic Predictions of the Classical Model

Equation (3.3) suggests some immediate predictions of our simple Classical model of price level determination. Given the interest rate, an exogenous increase in the money supply raises the price level so as to leave the real money supply unchanged.[5] This is the core Classical story about price determination: changes in the money supply simply change the price level proportionately without causing any real changes in the economy. Changes in money demand also affect the price level, of course. An increase in real income raises real money demand, and the price level falls to restore equilibrium. Similarly, an increase in the interest rate lowers real money demand, and the price level rises to restore equilibrium.

3.2 The Crude Monetary Approach Model

The monetary approach applies our simple Classical Model to the determination of the price level in both at home and abroad. The foreign country equivalent of (3.4) is (3.5).

$$P^* = \frac{H^*}{L^*(i^*, Y^*)} \tag{3.5}$$

Here an asterisk indicates a foreign value. In each county, the price level is determined by the ratio of the nominal money supply to real money demand. It follows that the relative price level is determined by the ratio of the relative nominal money supply to relative real money demand.

$$\frac{P}{P^*} = \frac{H/H^*}{L[i, Y]/L^*[i^*, Y^*]} \tag{3.6}$$

This is true even if these two economies are completely closed. Our consideration of specifically open-economy considerations begins with the introduction of the exchange rate.

In the monetary approach, the exchange rate is determined directly by the relative price level via purchasing power parity (PPP). We use (3.2) and (3.6) to write the *crude*

[5] Recall that the Classical Model treats H and Y as exogenous. Interest rate determination is considered in section 3.3.2.

monetary approach model to exchange rate determination as (3.7).

$$S = Q \frac{H/H^*}{L[i,Y]/L^*[i^*,Y^*]} \tag{3.7}$$

We call the model "crude" because it remains incomplete: we have not yet modeled the determination of interest rates. Nevertheless, the crude monetary approach does express the exchange rate in terms of variables for which data are readily available—interest rates, incomes, and money supplies—and such data have been used to test it. (See section 3.2.2.) These variables are often referred to as exchange rate *fundamentals*, and an implication of the monetary approach is that these exchange rate fundamentals should help us explain and predict the behavior of the exchange rate.

3.2.1 Core Predictions of the Monetary Approach to Flexible Exchange Rates

The core prediction of the monetary approach to exchange rate determination is that relative money supplies affect the exchange rate. Looking at (3.7), we can see that an increase in the relative money supply leads to a depreciation of the exchange rate.

$$\uparrow H/H^* \longrightarrow \uparrow S$$

Since purchasing power parity is a core ingredient of the monetary approach to flexible exchange rate determination, this result is unsurprising. The exchange rate changes in response to a change in the relative money supply because the relative price level changes. A higher relative money supply implies a higher relative price level, and by PPP the exchange rate rises with the relative price level. A domestic money supply increase produces a depreciation of the domestic currency.

The prediction is even more specific than this. If the relative money supply doubles, so does the relative price level; if the relative price level doubles, so does the spot rate. So according to the monetary approach to flexible exchange rates, the movement in the spot rate is proportional to the movement in the relative money supply. Proportionality means for each percentage change in the relative money supply, there is a one percent change in the spot rate. That is, the elasticity of the spot rate with respect to the relative money supply is unity.

Symmetrically, an increase in relative money demand leads to an appreciation of the exchange rate.

$$\uparrow L/L^* \longrightarrow \downarrow S$$

This reflects the effects of relative money demand on the relative price level, along with purchasing-power parity. We know that a rise in Y or a fall in i will raise domestic money demand, which increases relative money demand. So the monetary approach predicts that a rise in domestic income or a fall in the domestic interest rate will appreciate the domestic currency. Symmetrically, a rise in Y^* or a fall in i^* will raise foreign money demand, which decreases relative money demand. So the monetary approach predicts that a rise in foreign income or a fall in the foreign interest rate will depreciate the domestic

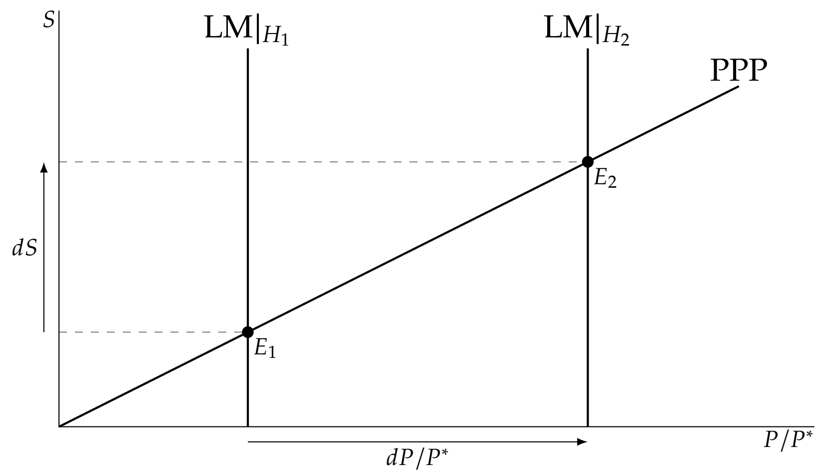


Figure 3.1: Effects of a Money Supply Increase
An exogenous increase in the domestic money supply ($H_2 > H_1$) causes proportional increases in the domestic price level and exchange rate.

currency.

Of course money demand can change for other reasons, such as deregulation and financial innovation. (As examples of innovation, think of the spread of credit cards or ATM machines.) Many observers believe that such factors destabilized money demand in the 1980s and 1990s. The monetary approach predicts that financial innovation that reduces the demand for money will cause the domestic currency to depreciate.

The exchange rate is the domestic currency price of foreign money, so it is natural that an increase in the supply of foreign money will appreciate the domestic currency (i.e., lower the price of foreign exchange) while an increase in the demand for foreign money will depreciate the domestic currency (i.e., increase the price of foreign exchange). This is just what happens in the monetary approach.

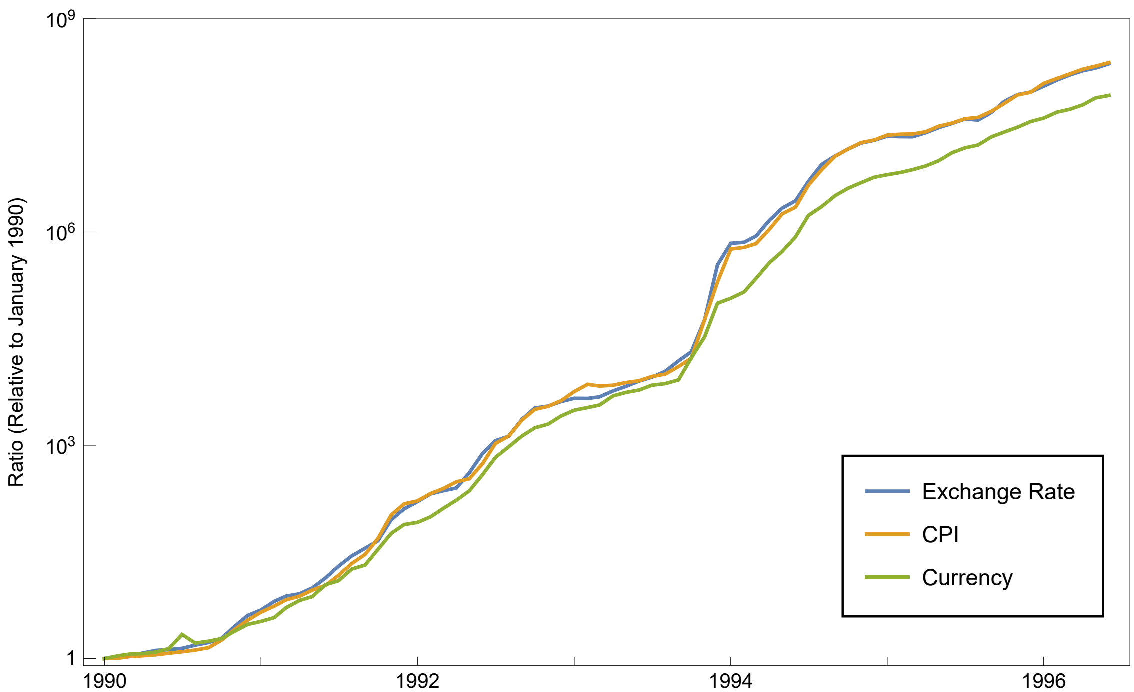
1% Increase In:	H	H^*	Y	Y^*	i	i^*
Resulting ΔS:	1%	-1%	−	+	+	−

Table 3.1: Core Predictions: Crude Monetary Approach

Inflation in Zaire

We have seen that a core prediction of the monetary approach to flexible exchange rates is that, ceteris paribus, money supply increases cause inflation and corresponding depreciation. Even though other factors are always changing, this theory would certainly predict that very large, sustained increases in the money supply will produce inflation and associated currency depreciation. This makes the experience of high inflation countries relevant to a first assessment of the monetary approach. In particular, hyperinflations seem to offer natural testing grounds for the monetary approach model. After all, one of its core components is the Classical model of price determination. which emphasizes long-run relationships between the money supply and other nominal variables (such as prices or exchange rates). In a hyperinflation, changes in such nominal variables dwarf all the real changes in the economy. We might therefore hope that, during a hyperinflation, the kinds of relationships emphasized by the Classical model would be more readily exposed to view.

Consider the example of Zaire. In the mid-1990s, Zaire experienced an unusually long and severe hyperinflation. During the late 1980s the country had experienced high inflation, around 70% per year. At the end of 1990, things took a turn for the worse. Haughton (1998) reports that in five years consumer prices rose a total of 6.3 billion percent. Beaugrand (1997) reports inflation of nearly 10,000% in 1994. In the midst of all this was a failed currency reform in 1993. On October 22, 1993, at the rate of one for every 3 million (old) zaire, and was initially fixed at 3nz/$ (Beaugrand, 1997).[6] By 1997, a 1-million-zaire note was worth about $3 in Kinshasa. Figure 3.2 illustrates the simultaneous inflation and depreciation, as predicted by the monetary approach to flexible exchange rates.



Data Source: Beaugrand (1997).

Figure 3.2: Money, Prices, and Exchange Rates

Inflation in Zimbabwe

Between early 2006 and early 2007, prices in Zimbabwe rose by more than 1,000%.

[6] In an interesting aside (Haughton, 1998) reports: "The new notes were not accepted by residents of Kasai province, who continue to use only the old notes; inflation in the province is said to be negligible."

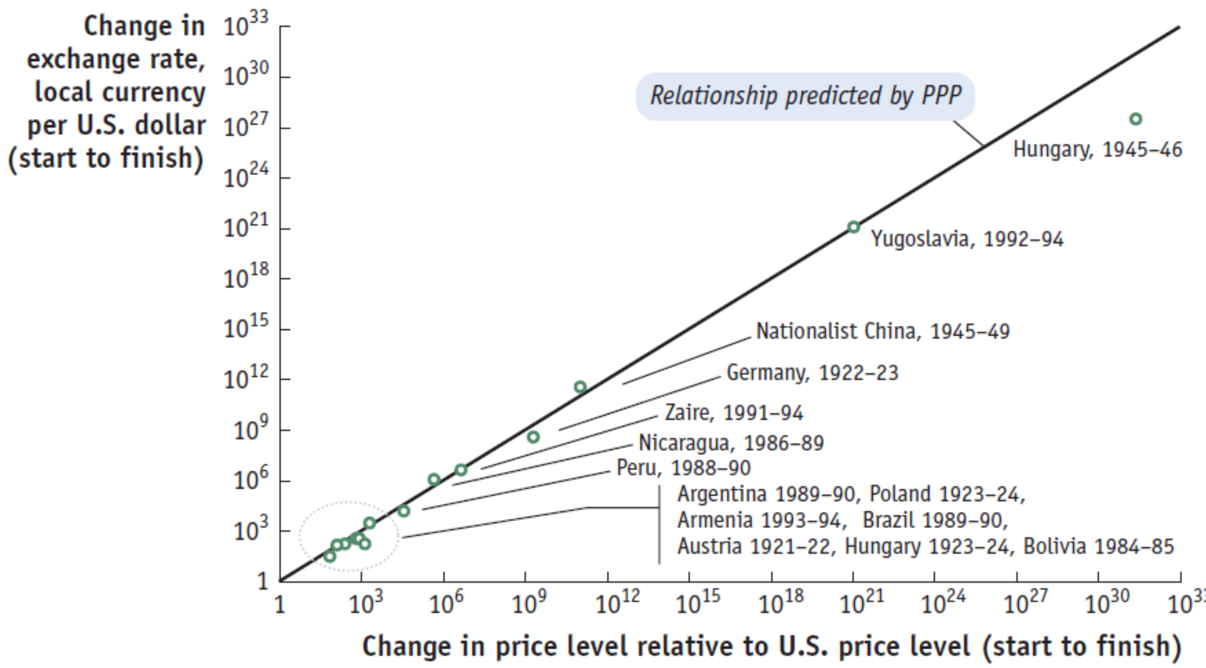


Figure 3.3: Some Hyperinflations
Chart Source: Feenstra and Taylor, 2014

The Mugabe government nevertheless refused to devalue the Zimbabwean dollar, which on the black market was worth only 5% of its official value. Gideon Gono, governor of the Reserve Bank, announced plans to reduce broad money supply growth from "over 1,000 percent to between 415 and 500 percent by December 2007 and subsequently to under 65 percent by December 2008." While this showed a good awareness of the source of inflation, the next month Gono offered a change in policy that was parodic: he announced that price increases would be illegal from March through June. This policy was abandoned when shortages quickly emerged. In July 2008, Gono estimated the inflation rate at 2.2 million percent and was immediately challenged by analysts who calculated it at more that 12 million percent. By one report, a loaf of bread rose in price from a billion Zimbabwean dollars to 100 billion in about a day. The new 100 billion dollar note issued in mid-July was not enough to buy a loaf of bread by the end of July. Dollarization, although illegal, became increasingly common.

Other Hyperinflations

3.2.2 Early Tests of the Monetary Approach Model

Naturally economists wish to determine the extent to which the available data are supportive of the core predictions of their models. Before we can develop specific tests of the monetary approach to flexible exchange rates, we need to find data that appropriately represent the variables in our model. We must also *parameterize* relationships that we have been describing only in the most general terms, perhaps by specifying a functional form for the money demand function. So before proceeding to the empirical test of the monetary approach, we need to discuss relative real money demand in a bit more detail.

We have been writing our money demand functions as $L[i, Y]$ and $L^*[i^*, Y^*]$. This represents in a general fashion our assumption that money demand is a stable function of the nominal interest rate and real income. Monetary theory tells us that real money demand responds positively to real income, which is a proxy for the level of transactions in the economy, and negatively to the interest rate, which is a proxy for the opportunity cost of holding money. The monetary approach to flexible exchange rates tells us that these money demand responses are also relevant to exchange rate determination: the spot rate responds negatively to real income, due to the implied rise in money demand, and negatively to the interest rate, due to the implied fall in money demand. When we conduct an econometric investigation of the monetary approach to flexible exchange rates, we let the data suggest numerical values for the sizes of these responses. For example, we estimate the elasticity of the spot rate with respect to real income. We would like to learn whether our theoretical predictions match the relationships present in the data. We are therefore interested in the sign and the size of our estimates.

Table 3.2 lists some of the early econometric tests of the monetary approach to flexible exchange rates. The first study in the list is by far the most famous: Jacob Frenkel's study of the post-WWI German hyperinflation. As we have discussed, since nominal changes dwarf real changes during a hyperinflation, we might hope that the kinds relationship between money and exchange rates stressed by the monetary approach will be more readily detected in the data. In the case of the German hyperinflation, Frenkel felt this justified ignoring changes in domestic income and all foreign variables, since the changes in these variables were very small compared to the huge changes in the spot rate and German money supply that took place during the hyperinflation. He estimated the response of the spot rate to only two variables: the German money supply, and the opportunity cost of money.

		Estimated Response of the Spot Rate to:					
Study:	Sample:	H	H^*	Y	Y^*	i	i^*
DEM/GBP							
Frenkel (1976)	1921.02–1923.08	0.975	–	–	–	0.591	←
Bilson (1978b)	1972.01–1976.04	1.0013	-1.0081	-1.0184	0.9990	0.0228	←
Bilson (1978a)	1970.04–1977.05	1.0026	-0.9846	-0.9009	1.0183	1.3853	←
USD/DEM							
Hodrick (1978)	1973.04–1975.09	1.52	-1.39	-2.23	0.073	2.53	1.93
GBP/USD							
P&W (1979–80)	1972–1977	0.63	←	-0.77	←	0.14	←

Note: The reported "responses" are money supply elasticities, income elasticities, and opportunity-cost semi-elasticities of the spot rate. Opportunity costs of holding money are proxied by interest rates or forward discounts. A dash indicates an omitted variable. A left arrow indicates that the domestic and foreign money demand parameters are constrained to equality.

Table 3.2: Early Tests of the Monetary Approach

Frenkel's results are summarized in table 3.2. The dashes indicate responses that he did not estimate, and the left arrow indicates that he constrained to equality the response to the domestic and foreign interest rate.[7] His estimates, using monthly German data, are correctly signed, significantly different from zero, and of plausible magnitude. In addition, money supply elasticity of the spot rate differs insignificantly from unity. Many economists viewed Frenkel's results as providing dramatic support for the monetary approach.

Bilson (1978a; 1978b) offers additional early evidence on the crude monetary approach model. Bilson (1978b) estimated a model of the DEM/GBP exchange rate using monthly data for the period 1972.01–1976.04. The results are reported in table 3.2.[8] Overall the results look fairly good. However the coefficient on his opportunity cost measure is not statistically different from zero, and Bilson required considerable econometric manipulation to arrive at the reported results. The study therefore lends modest support to the crude monetary approach. Bilson (1978a) extends this sample to 1970.04–1977.05. (Note that this includes some years preceding the general float.) After resorting to a complicated estimation procedure, Bilson finds the results reported in table 3.2.[9] His estimates have the correct sign and reasonable magnitude. All the coefficients are significantly different from zero. Again the coefficients on money supplies are close to unity, the value implied by the monetary approach.

Keep in mind that flexible prices, PPP, and stable money demand lie at the core of the monetary approach. It is therefore rather surprising, in a non-hyperinflationary setting, that the monetary approach performed so well with monthly data. Price flexibility and PPP are most likely to hold in the longer run. Indeed, as we shall see, PPP performs very poorly in the short run, and an important reason appears to be price stickiness. Furthermore, the assumption of a stable money-demand function has become increasingly suspect during the post-Bretton Woods era (Boughton, 1988).

Hodrick (1978) and P&W (1979–80) also offer early empirical tests of (3.7) using monthly data from developed countries. Hodrick (1978, eq.7) examine the USD/DEM exchange rate from April 1973 to September 1975. The first row of table 3.1 reports his estimated reponses of the spot rate to money, interest rates, and income.[10]

Recall that theory predicts that the spot rate moves proportionally to the relative money supply, or equivalently that it is unit elastic with respect to the relative money supply. In table 3.2, we therefore would hope to find numbers close to 1 in the H column and close to -1 in the H^* column. While we do not find a perfect match, the estimated responses were close enough for Hodrick to conclude that the data did not reject this prediction of the monetary approach. His results are somewhat favorable to the theory, in the sense that his estimated responses generally have the predicted signs, are significantly different than zero, and are of plausible magnitude. Moreover, he reports that exchange rate fundamentals are

[7] We discuss his opportunity cost measure in more detail below.

[8] The reported results are for his equation (45), which reports long-run coefficients calculated from an estimation incorporating a partial adjustment money demand equation. (See problem 8.) He also incorporates prior expectations about the coefficients in his estimation procedure.

[9] This is his equation (12). Not reported in the table is his coefficient on a time trend, which suggests a half a percent appreciation of the DEM per month. This adds up: over the period, it makes an important contribution to the model's predicted values.

[10] See Hodrick's article for a discussion of the data and of his variable definitions.

explaining about two-thirds of the variation in the spot rate. Nevertheless, there are a couple of noticeable problems. German income appears to have a weak influence on the exchange rate. (The response is smaller than expected; in fact, Hodrick finds that it is insignificantly different from zero.) Worse, the spot rate response to the German interest rate is the wrong sign (and differs significantly from zero).[11] The support for the flexprice monetary approach model offered by Hodrick's research is modest at best.

Putnam and Woodbury's results are somewhat more encouraging. Their estimates are also reported in Table 3.2.[12] They use data from 1972–77 to examine the GBP/USD exchange rate. Table 3.2 reports their results for monthly data; they also report results for quarterly data, which look similar. Fewer coefficients are reported because they reported results when relative money, relative income, and the relative interest rate are the regressors, thus constraining the domestic and foreign responses to be the same. (They report that relaxing these constraints produces comparable results.) All the coefficients have the correct sign and are significantly different from zero. The coefficient on relative income is disappointingly far from its predicted value of zero, but the regression otherwise provides good support for the monetary approach to flexible exchange rates.

Using the Forward Discount

When we begin to think about empirical applications of the monetary approach, we have to decide what measured quantities will represent each of the variables in our exchange rate equation (3.7). For example, there are many different foreign and domestic interest rates: which are appropriate for our purposes? Rather than choose an interest rate, some researchers have turned to the forward discount on domestic currency.

Use of the forward discount instead of domestic and foreign interest rates requires a special assumption, which is fairly standard in the empirical literature testing the monetary approach to flexible exchange rates. This is the assumption of a common interest rate response in foreign and domestic money demand. Relative real money demand is given a special form:

$$L[i, Y]/L^*[i^*, Y^*] = \mathcal{L}[i - i^*, Y, Y^*] \tag{3.8}$$

With this assumption, we can write

$$S = Q\frac{H/H^*}{\mathcal{L}[i - i^*, Y, Y^*]} \tag{3.9}$$

In this version of the crude monetary approach, the foreign and domestic interest rates enter only as an interest rate differential ($i - i^*$). Recall that developed the covered interest parity condition.

$$\text{fd} = i - i^* \tag{3.10}$$

With high capital mobility, covered interest parity should hold closely for assets that

[11] Hodrick suggests that German capital controls adopted in February 1973 might offer an explanation, but in the context of the monetary approach this is not evident.

[12] The estimated "responses" are money supply elasticities, income elasticities, and interest rate elasticities of the spot rate. They have corrected for first order autocorrelation with a Hildreth-Lu procedure. See their article for a discussion of the data and for variable definitions.

are good substitutes. So one natural way to measure the interest rate differential in the monetary approach model is with the forward discount. This yields (3.11).

$$S = Q\frac{H/H^*}{\mathcal{L}[\mathrm{fd}, Y, Y^*]} \tag{3.11}$$

Frenkel (1976) used this approach in his study of the post-WWI German hyperinflation.[13] Bilson 1978a; 1978b also uses the forward discount to proxy the relative opportunity costs of holding domestic and foreign money.

In-Sample Fit and Out-of-Sample Forecasting

Later studies proved less supportive of the crude monetary approach to flexible exchange rates. It turns out that the in-sample fit of monetary approach models depends on the choice of exchange rate and the sample period. This is rather discouraging news. In addition, only a small fraction of the actual variation in the exchange rate is explained by the monetary approach model. However in-sample fit is only one criterion for model performance, and arguably it is not the most interesting criterion. Since the mid-1980s, economists have been more interested in evidence of the predictive power of their models. In particular, exchange rate research has concentrated on the out-of-sample forecasting ability of existing models. A model that is capable of improving our exchange rate forecasts can be very useful even if its in-sample fit is disappointing.

The classic study of the out-of-sample forecasting ability of modern exchange rate models is Meese and Rogoff (1983). Meese and Rogoff pit a one-step-ahead random-walk model against the core inflation and other models. Using monthly data (1973.03-1981.06) for several countries, they found that the one-step-ahead random walk model outperforms the structural models in forecasting the exchange rate. However Schinasi and Swamy (1987) argue that the one step ahead forecasts favor the random walk model, which uses the lagged exchange rate. They find much greater forecast error in the multi-step random walk forecasts. And indeed, adding a lagged dependent variable to the structural models greatly improves their forecasting performance.[14] Cerra and Saxena (2010) explore these results in a large panel of countries and produce more favorable results. In particular, they find a strong relationship between exchange rates and relative money supplies and that versions of the monetary model can outperform a random walk in out-of-sample forecasting.

3.2.3 Volatility Puzzle

Recall that the crude monetary approach to flexible exchange rates, as represented by (3.7), explains the exchange rate in terms of a small number of exchange rate fundamentals: interest rates, incomes, and money supplies. Although we expect short-run deviations from this relationship, we might hope that such deviations are not large relative to the volatility

[13]Frenkel's justification of using the forward discount was somewhat different than that offered here.

[14]Other studies finding some advantages from including the lagged spot rate include Woo (1985), Somanath (1986), Boughton (1987), Driskill et al. (1992), and MacDonald and Taylor (1994).

of the exchange rate. However the exchange rate is surprisingly volatile relative to these fundamentals. In order to highlight this *volatility puzzle* for the monetary approach, Flood and Rose (1999) aggregate these fundamentals in order to offer a rough comparison of the variability of the exchange rate and the variability of the fundamentals.[15] Two problems emerge. First, for the 1980s and 1990s Flood and Rose (1999) find no link across countries between the volatility of aggregate fundamentals and exchange-rate volatility. Second, they can find no evidence that countries tend to experience periods of volatile exchange rates at the same time as they experience volatility in the aggregate fundamentals. For example, the collapse of the Bretton Woods system of pegged exchange rates produced a tremendous increase in exchange-rate volatility without any noticeable increase in the volatility of the aggregate fundamentals. To explain this, it seems we need to discover a new exchange rate fundamental that behaves entirely differently under pegged and floating exchange-rate regimes.

3.3 Exchange Rates and Monetary Policy

A basic contention of the monetary approach to flexible exchange rates (MAFER) is that monetary policy is an important determinant of the behavior of the exchange rate. A change in monetary policy may be a change in the *level* of the money supply or by a change in its *growth rate*. This suggests two different monetary policy thought experiments that expose basic predictions of the monetary approach.

3.3.1 Money Supply Shocks

An unanticipated exogenous disturbance of the economy is called a *shock*. As a first thought experiment, consider the following money supply shock: a one-time, permanent, unanticipated increase in the level of the domestic money supply (H). Since this is a one-time shock, there is no change in the expected growth rate of the money supply. For simplicity, let the money supply growth rate be zero both before and after the shock.

The MAFER is a Classical style model, so in response to this shock, the domestic price level should increase proportionally. This keeps the real money supply is unchanged. The MAFER assumes purchasing power parity, so the exchange rate must increase in proportion to the price level. Consequently, the model predicts a depreciation of the exchange rate that is proportional to the increase in the money supply. This is easily seen

[15] In addition to identical interest rate semi-elasticities, much empirical work assumes identical income elasticities as well. Constraining these to unity yields the Flood and Rose (1999) aggregate measure of exchange rate fundamentals. Relative money demand becomes

$$L[i, Y]/L[i^*, Y^*] = \mathcal{L}[i - i^*, Y/Y^*] = e^{-(i-i^*)}\, Y/Y^*$$

and the aggregate fundamentals are therefore

$$\frac{M/M^*}{Y/Y^*} e^{i-i^*}$$

in equation (3.7): if H doubles on the right-hand side, S must double on the left-hand side to maintain the equality.

The behavior of the money supply, the real money supply, the price level, and the spot rate are illustrated in figure 3.4.[16] The rise in the nominal money supply, H, takes place at time t_0. The change in the spot rate and the price level proportional to the change in the nominal money supply, so the real money supply remains unchanged.

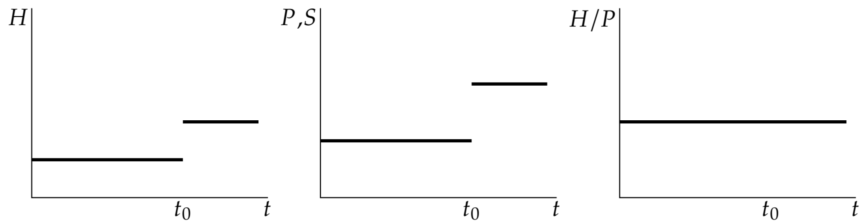


Figure 3.4: Dynamics of a Money Supply Shock
Increasing the money supply (H) at time t_0 produces a proportional increase in the price level (P) and in the exchange rate (S), reflecting the neutrality of money. The real money supply is therefore unchanged.

Although this first thought experiment is based on a one-time event, it also illustrates what would happen in a Classical economy where the money supply followed a random walk. (That is the situation where each period the money supply is equally likely to rise or to fall.) Consider a point forecast of the next month's money supply, in such circumstances. To be as right as possible on average, predict that the money supply will remain at its present level. Next month the same rule applies, even if the money supply turned out higher or lower than you had expected. Always predict that it will stay at its new level. In this sense, under a random walk, changes in the level of the money supply are permanent. Consequently, changes in the money supply tell us nothing about the future money supply growth rate, which is always expected to be zero.[17]

3.3.2 Money Growth Shocks

The next experiment we want to consider is a one-time, permanent, unanticipated change in the *growth rate* of the money supply. As a preliminary, we need to think a bit more carefully about the role of interest rates in the monetary approach model.

Interest Rates and Inflation

The Classical model of interest rate determination begins with the following definition of the real interest rate.

$$r \stackrel{\text{def}}{=} i - \pi^e \tag{3.12}$$

[16] Absolute heights are meaningless in the graphs: only relative changes matter. To keep the presentation as simply as possible, we have plotted P and S in the same graph. (This may be seen as a harmless normalization, where $Q = 1$ and $P^* = 1$.)

[17] Almost nothing changes if the expected growth rate is some other constant, say 0.5%/month; just add this expected growth to the forecasts.

The real interest rate is the expected real rate of return from holding an interest bearing asset. In an inflationary environment, part of the nominal rate of return on an asset simply compensates for rises in the price level. For example, if an individual holds his wealth in assets paying 10% per year when inflation is 10% per year, then the nominal interest rate of 10% per year simply maintains the purchasing power of wealth but does not augment it. The Classical model treats the real interest rate as exogenous, which turns (3.12) into a theory of interest rate determination.

$$i = r + \pi^e \tag{3.13}$$

Equation (3.13) is often called the *Fisher equation*. It decomposes the nominal interest rate into the real interest rate r and expected inflation π^e. Since the real interest rate is exogenous in the Classical model, variations in expected inflation imply variations in the nominal interest rate. We can use the Fisher equation (3.13) to substitute for the nominal interest rate in our representation of money market equilibrium (3.3). The result is (3.14).

$$\frac{H}{P} = L[r + \pi^e, Y] \tag{3.14}$$

The representation of money market equilibrium in equation (3.14) predicts a decline in real balances when expected inflation is higher. Expected inflation should be high when actual inflation is persistently high, this is a prediction that we should be able to roughly test simply by looking at real money balances in high inflation economies. Consider table 3.3, which is based on data from several hyperinflations. The dates are the first and last months for which the inflation rate exceeded 50%/month. The inflation rates in the third column are the average monthly inflation rates for the period. The final column shows the smallest level of real balances reached during the hyperinflation as a ratio of the level of real balances held at the beginning of the hyperinflation. Despite tremendous growth in the money supply during these hyperinflations, the real money supply is declining. High inflation raises the expected cost of holding money. The table shows that the decline in real balances during hyperinflations is dramatic, which supports the prediction of equation (3.14).

Country	Dates	$\%\Delta P$/month	$(H/P)_{\min}/(H/P)_0$
Austria	1921.10–1922.08	47	0.350
Germany	1922.08–1923.11	322	0.030
Greece	1943.11–1944.11	365	0.007
Hungary	1923.03–1924.02	46	0.390
Hungary	1945.08–1946.07	19,800	0.003
Poland	1923.01–1924.01	81	0.340
Russia	1921.12–1924.01	57	0.270

Source: McCallum (1989, p.135), based on data in Cagan (1956).

Table 3.3: Inflation and Real Money Demand

Additional Predictions of the Monetary Approach

While monetary policy is sometimes characterized in terms of changes in terms of changes in the level of the money supply, it is more often characterized in terms of the *rate of growth* of the money supply. Our next thought experiment determines the predictions of the monetary approach for changes in money supply growth rates.

The simplest case to consider is a one-time, permanent increase in the rate of growth of the money supply. Suppose such a change takes place today. Given our previous work on the monetary approach to flexible exchange rates, we might expect that the rate of inflation immediately adjusts to the new growth rate of the money supply, and that by purchasing power parity this becomes the new rate of depreciation as well. This is basically correct, but there must be an additional one time adjustment of the price level. This is because a higher inflation rate implies a higher nominal interest rate, which lowers real money demand.

Recall the Fisher equation for the domestic interest rate.

$$i = r + \pi^e$$

Similarly, the foreign interest rate can be written as

$$i^* = r^* + \pi^{*e}$$

Substituting for the nominal interest rates in (3.7) allows us to write our exchange rate determination equation as (3.15).

$$S = Q\frac{H/H^*}{L[r + \pi^e, Y]/L^*[r^* + \pi^{*e}, Y^*]} \tag{3.15}$$

Equation (3.15) is known as the *core-inflation* formulation of the monetary approach to exchange rate determination because it focuses attention on the determinants of expected inflation. This equation is particularly useful for developing the more detailed predictions of the monetary approach to flexible exchange rates.

Too see this, apply (3.15) to our monetary policy experiment. Looking at (3.15), it is clear that unless we say something specific about the behavior of expectations, we will have little to say about the behavior of the exchange rate. Let expectations adjust very quickly to the policy change. For any level of inflation expectations, we know from our previous monetary policy experiment that the price level must rise proportionately to the money supply increases. It is therefore natural to set expected as well as actual inflation equal to the new money supply growth rate. The higher expected inflation rate lowers domestic money demand. Prices must rise not only because the money supply is increasing but also because real money demand has fallen. Thus we have a brief *magnification effect*: the inflation rate measured over a period near the time of the policy change will actually exceed the new money supply growth rate. Since exchange rate movements are determined by purchasing power parity, the rate of depreciation must display a similar magnification effect (Frenkel, 1976).

Figure 3.5 illustrates these outcomes. The change in the money supply growth rate

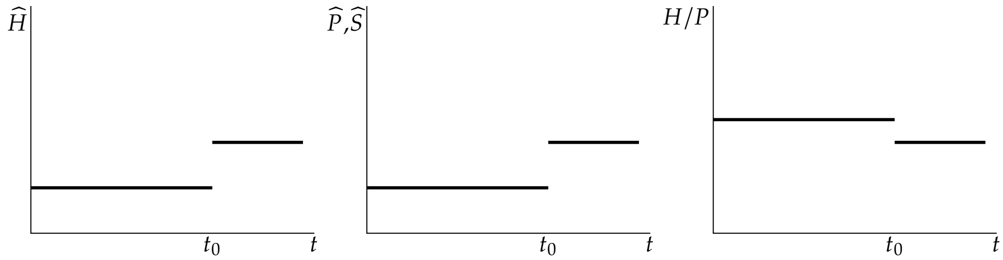


Figure 3.5: Increase $\widehat{H}$ at t_0

An increase in money growth cause an increase in inflation and currency depreciation. Real balances fall as a consequence of the Fisher Effect.

takes place at time t_0. We plot the time path of the growth rate of the money supply ($\hat{H}$), the rate of inflation ($\hat{P}$), and the rate of depreciation ($\hat{S}$). We also graph the level of real balances over time. Note that although the level of the money supply does not jump, the spot rate and price level do jump. This jump is the source of the "magnification effect". In contrast with our previous experiment, the real money supply is changed by this experiment. This change derives from the higher interest rate implied by higher expected inflation, which in turn is a result of the change in monetary policy.

3.4 Anticipated Monetary Policy

In section 3.3 we considered two changes in monetary policy: a change in the *level* of the money supply, and a change in its *growth rate*. In both cases we considered an *unanticipated* change in policy. We now consider what happens when these changes in policy are *anticipated*.

3.4.1 Transitory Shocks

As an entry point to the consideration of expecations, this subsection explores a simple model of exchange rate responses to a transitory shock. Here, a *transitory shock* is an exogenous change that is expected to last for a single period. The key new consideration is the role of expectations.

Recall that our monetary approach model consists of two key ingredients: purchasing power parity, and the Classical model of price determination.

$$S = QP/P^* \tag{3.16}$$

$$P/P^* = \frac{H/H^*}{\mathcal{L}} \tag{3.17}$$

We have been working with a very simple model of money demand, where $\mathcal{L}$ depends on the interest differential and relative income.

$$\mathcal{L} = \mathcal{L}[i - i^*, Y/Y^*] \tag{3.18}$$

In turn, we recognized that covered interest parity holds under perfect capital mobility, so that

$$i - i^* = \text{fd} = \Delta s^e + \text{rp} \tag{3.19}$$

So our model of the spot rate involves relative money demand, which depends on expected depreciation. This suggests that any discussion of transitory shocks will require careful attention to the role of expectations. In this section, our approach will be to treat individuals as understanding that they are dealing with a transitory shock, which allows them to form correct expectations about the future. Since individuals know that the exchange-rate will return to its long-run equilibrium level next period, we have

$$\Delta s^e = (S^e_{+1} - S)/S = (\bar{S} - S)/S \tag{3.20}$$

This must affect our description of short run equilibrium in the money market, since we must allow for the effects of interest rates on money demand, and in the short run interest rates must move with the spot rate. The relative price level is now determined as

$$P/P^* = \frac{H/H^*}{\mathcal{L}\left[\frac{\bar{S}-S}{S} + \text{rp}, Y/Y^*\right]} \tag{3.21}$$

and therefore must be negatively related to the current spot rate. The monetary approach model to temporary shocks thereby becomes the following.

$$S = Q\frac{H/H^*}{\mathcal{L}\left[\frac{\bar{S}-S}{S} + \text{rp}, Y/Y^*\right]} \tag{3.22}$$

Money Supply Shock

Suppose that the economy is initially in a long-run equilibrium, with spot rate $\bar{S}$, when it receives an unanticipated transitory shock to level of money supply. Since individuals know that the exchange-rate will return to its long-run equilibrium level next period, we have

$$\Delta s^e = (S_{+1} - S)/S = (\bar{S} - S)/S \tag{3.23}$$

As with a permanent shock, the domestic currency must depreciate in response to the increase in H. But individuals anticipate that there will be a subsequent appreciation, which lowers the interest differential and increases the relative demand for money. This moderates the short-run movement of the exchange rate, which is therefore less than in the case of a permanent shock.

3.4.2 An Anticipated Change in the Money Supply

Next, consider a one-time, permanent, *anticipated* increase in the level of the money supply. The behavior of the money supply is identical to the first monetary policy experiment we considered in section 3.3. However, the increase in the money supply at time t_0 is now known at the earlier time t_a. That is, at time t_a we correctly anticipate an increase in the

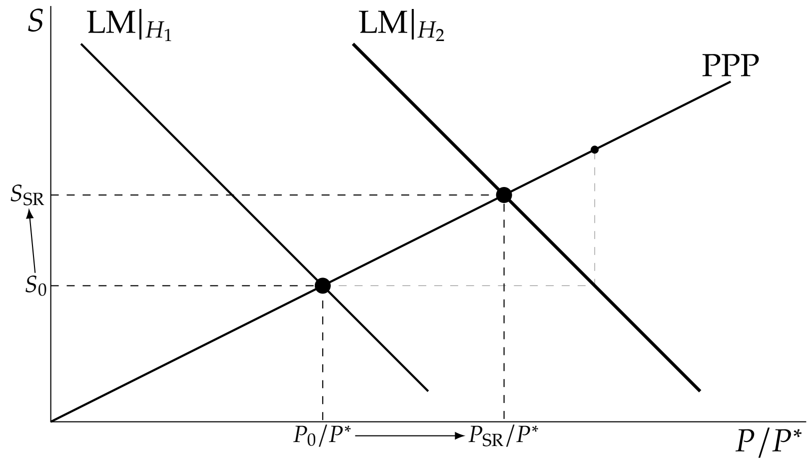


Figure 3.6: Short-Run Effects of Transitory Monetary Policy
A transitory expansion of the money supply ($H_2 > H_1$) temporarily raises the domestic price level and depreciates the domestic currency, but less than proportionally. When the expansion reverses, the initial situation (P_0, S_0) is restored.

money supply, which will take place at time t_0.

Since the behavior of the money supply is identical to our earlier experiment with an *unanticipated* change in the money supply, we might think that prices, exchange rates, and real balances should also behave in the same fashion. And indeed, if expected inflation were unaffected by the anticipated increase in the money supply—for example, if expectations were "static" in the sense of being exogenously fixed—the outcomes would in fact be identical. But when the increase in the money supply is anticipated, it is not plausible to treat expectations as static.

When we expect the money supply to increase, we predict an increase in the price level. This must show up immediately in the nominal interest rate, which is the real interest rate plus expected inflation. The rise in the nominal interest rate decreases money demand, and this in turn increases the equilibrium price level. By the purchasing power parity relationship, the increase in the domestic price level shows up as a proportional depreciation of the exchange rate. So an anticipated *future* money supply increase leads to inflation and depreciation *today*. This is true no matter how far in the future is the anticipated change, although the farther away it is the smaller today's response will be.

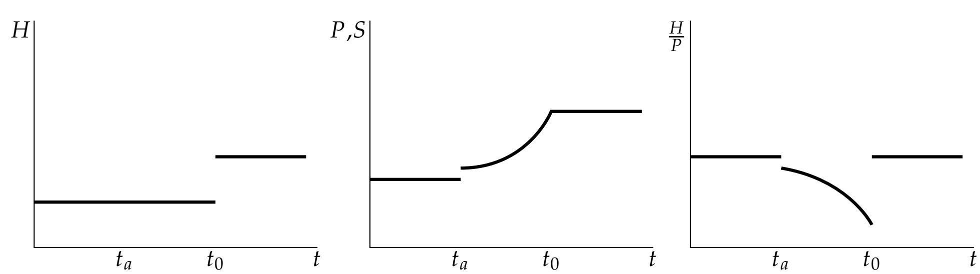


Figure 3.7: Anticipated Increase in H

Figure 3.7 characterizes the outcomes when we anticipate at time t_a the increase in the level of the money supply that does not take place until time t_0. We can easily characterize economic outcomes before t_a and after t_0. Before t_a there is no expected inflation, corresponding to the constant money supply. In line with this, actual inflation is zero, as is exchange rate depreciation. After t_0 expected inflation will be correct if it again matches the zero money supply growth rate. So after t_0 we should have expected inflation, actual inflation, and exchange rate depreciation each at 0% per year. The difficulty lies in determining what happens between t_a, the time the policy is anticipated, and t_0, the time the policy is implemented.

Expectations play a critical role between t_a and t_0. To aid us in working through the economic outcomes during this period, we will work with expectations that are very accurate: the actual and anticipated economic outcomes are the same. One crucial key to understanding the outcomes lies in the recognition that there cannot be any anticipated jumps in the price level.

Here is why. Suppose you expected an upward jump in the price level next Friday. This jump will lower the value of any money balances you are holding, so you would like to get rid of your money balances the day before. But so would everyone else, and knowing this you expect money demand to fall on Thursday, with the implication that you must expect the jump in the price level to take place on Thursday instead of Friday. Of course you face the same difficulty in expecting the price level to jump on Thursday. Or Wednesday. Or any future day. The conclusion is that if there is to be a jump in the price level, it must be unanticipated. So any jump in the price level must take place at t_a, as soon as the policy change is anticipated.

One implication of this analysis is that, in the case of anticipated changes in the money supply, the contemporaneous link between expected inflation and expected changes in the money supply is broken. Consider price determination in the Classical model, solving (3.14) for (3.24). A rise in expected inflation causes an *immediate* jump in the price level and the exchange rate.

$$P = \frac{H}{L[r + \pi^e, Y]} \tag{3.24}$$

Now we have enough information to predict the economic outcomes between t_a and t_0. At t_a, the price level rises. Since the money supply has not yet changed, this determines an initial fall in real balances. The lower real balances are compatible with money market equilibrium because of the increase in expected inflation. Remember, expected inflation tracks actual inflation. But since the money supply has not yet increased, the rise in prices implies a fall in real balances. Inflation expectations must rise over time so as to maintain money market equilibrium at the falling level of real balances. Until the money supply actually increases, the price level and the exchange rate keep rising. Correspondingly, real balances keep falling.

In the middle graph you can see the changes in the price level: the actual inflation. In the last graph, you can see the changes in real balances, which are due to changes in expected inflation. Since actual and expected inflation are tied together by our assumption that individuals form very accurated expectations, we need the inflation implied by the middle graph to be consistent with the expected inflation implied by the last graph.

If individuals are very good at predicting the money supply increase, prices should have risen just enough by the time of this increase that no further changes are necessary to restore equilibrium in the money market. That is, the total change in the price level over the period t_a to t_0 will be just proportional to the increase in the money supply that takes place at time t_0. At this new price level, the increased money supply is just adequate to raise real balances back to their old level, which is the equilibrium level of real balances when expected inflation is zero.

3.4.3 An Anticipated Change in Money Growth

In section 3.3.2, we characterized changes in monetary policy in terms of the *rate of growth* of the money supply. Our final monetary policy thought experiment determines the predictions of the monetary approach for *anticipated* changes in money supply growth rates.

Suppose at time t_a we correctly anticipate a one-time, permanent, increase in the rate of growth of the money supply, which will take place at time t_0. Suppose annual money growth will rise from zero to ten per cent. Once again, the price level and the inflation rate must respond immediately to our expectations of a future policy change. Correspondingly, via the purchasing power parity condition, the exchange rate also responds. Recall that increases in expected inflation immediately increase the exchange rate (by reducing relative money demand and thereby increasing the relative price level).

In figure 3.8, we can once again easily characterize economic outcomes before t_a and after t_0. Before t_a there is constant expected inflation, corresponding to the constant money supply growth rate. Let us say this growth rate is 5% per year. In line with this, actual inflation is 5% per year, as is exchange rate depreciation. After t_0 expected inflation will be correct if it matches the new money supply growth rate of 10% per year. So after t_0 we should find expected inflation, actual inflation, and exchange rate depreciation each to be 10% per year. The difficulty once again lies in determining what happens between t_a, the time the policy is anticipated, and t_0, the time the policy is implemented.

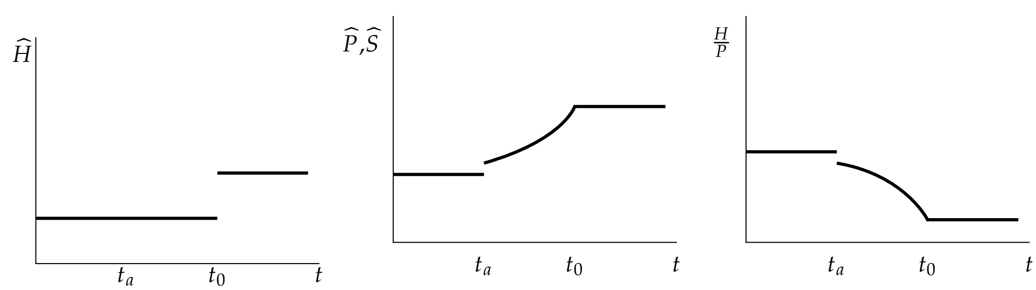


Figure 3.8: Anticipated Increase in $\widehat{H}$

As before, economic reasoning rules out *anticipated* jumps in the price level. This allows us to predict the economic outcomes between t_a and t_0. At t_a, the price level rises. Since the money supply has not yet changed, this determines an initial fall in real balances. The lower real balances are compatible with money market equilibrium because of the increase in expected inflation. Expected and actual inflation rise together over time so as to maintain money market equilibrium. The price level and the exchange rate are rising faster than the money supply. Correspondingly, real balances keep falling. If individuals are very good at predicting the monetary policy change, prices should have risen just enough by the time of the policy change so that no further changes are necessary to restore equilibrium in the money market. That is, the total change in the price level over the period t_a to t_0 will reduce the real money supply to the level that is compatible with an expectation of 10% annual inflation. At time t_0 the money supply starts growing by 10% per year, and the economy is in equilibrium with 10% per year in (actual and expected) inflation and exchange rate depreciation.

3.5 Conclusion

The monetary approach to flexible exchange rates has two key constituents: purchasing power parity and a simple Classical model of price determination. The result is a very simple model of exchange rate determination that, like the Classical model of price determination, should have interest primarily as a long run description of exchange rate determination.

Early empirical tests yielded encouraging support for the monetary approach. Later tests proved much less satisfactory. However much of the empirical work on the monetary approach has been conducted with small samples of monthly data for countries with low average inflation rates. The failure of the monetary approach in such tests does not bear on its usefulness as a description of the fundamental long-run influences on the exchange rate. These can be better tested over long periods of time or in situations, such as the German hyperinflation, where the sheer magnitude of the monetary changes ensure that their influence will be felt even in the short run.

Predictions of the monetary approach include the following. An increase in the domestic money supply leads to a proportional depreciation of the spot exchange rate. An increase in domestic income generates an exchange rate appreciation. An increase in the domestic interest rate causes an exchange rate depreciation, and for this reason there is a “magnification effect” of changes in the money supply growth rate. It is natural to wonder how we might test these predictions, which require a characterization of expected inflation and its links to monetary policy. The next chapter addresses this.

Expansionary monetary policy might be represented either as a change in the *level* or as a change in the *growth rate* of the money supply. In each case, the policy change may be a complete surprise or it may be anticipated. This leads to four possible scenarios. A primary lesson is that any effort to model exchange rates must pay careful attention to the role of expectations.

Problems for Review

1. Let S be the USD/GBP exchange rate, P^* be the pound cost of a consumption basket in the U.K., and P be the dollar cost of a consumption basket in the U.S. Using the units of S, P^*, and P, find the units of the real exchange rate, $Q = SP^*/P$.

2. Suppose you are a U.S. resident with fifteen thousand dollars (USD 15k) for living expenses. You are contemplating a year abroad. Let the dollar-franc exchange rate be USD/FFR 0.2. Is this enough information to determine your relative material standard of living in the U.S. and France? Why or why not?

3. Given our discussion of the crude monetary approach model, what variables did Frenkel (1976) "dump" into the constant terms of his key exchange rate regression? How about Bilson (1978a)?

4. Consider a French resident who invests FFR 10,000 for one year at an annual interest rate of 10%. What is the nominal value of the investment at the end of the year? Suppose the French CPI rises from 100 to 110 that year. What is the real value of the investment at the beginning of the year? What is the real value of the investment at the end of the year? What is the inflation rate? What is the real rate of return on the investment over the year?

5. Produce graphs similar to those in figure 3.4 to illustrate the effects of a one-time, permanent increase in Y.

6. Produce graphs similar to those in figure 3.4 to illustrate the effects of a one-time, permanent increase in i. (You may assume this increase is due to a one-time, permanent increase in the exogenous real interest rate.)

7. Produce graphs similar to those in figure 3.5 to illustrate the effects of a one-time, permanent increase in $\hat{Y}$. (Assume a unitary income elasticity of money demand.)

8. Bilson (1978b) gets reasonable results for the crude monetary approach model only by adopting a partial adjustment formulation for money demand. In this formulation, domestic money market equilibrium obtains when

$$h - p = \beta_0 + \beta_1 y + \beta_2 i + \beta_3 (h_{t-1} - p_{t-1})$$

   Assume the same formulation for the foreign country, and derive the new spot rate equation. (Specifically, show that the current spot rate depends on its own past value.)

9. Suppose news at time t_a leads to the expectation that money supply growth will fall to zero at time t_0. Assuming expectations are correct, graph the behavior of the money supply, real balances, the price level, and the spot rate. [Comment: although the price level falls at t_a, it may end up higher than it started by t_0.]

3.A Empirical Monetary Approach Models

In this section, we develop the popular log-linear representation of the monetary approach to the determination of flexible exchange rates.

3.A.1 Characterizing Money Demand

Recall that our money demand function, $L[i, Y]$, represents in a general fashion the dependence of money demand on the interest rate and income. When we move to empirical considerations, we will need to be more specific about the form of these influences. A popular and convenient functional form for money demand is

$$L[i, Y] = Y^{\phi} e^{-\lambda i} \tag{3.25}$$

where e is the "natural base" (see the appendix to There are two parameters: ϕ is the income elasticity of money demand, and $-\lambda$ is the interest rate semi-elasticity of money demand. These determine the response of real money demand to real income and to the interest rate. The most important thing to note is that ϕ and λ are positive, so that real money demand is increasing in real income and decreasing in the nominal interest rate. Imposing money market equilibrium yields

$$\frac{H}{P} = Y^{\phi} e^{-\lambda i} \tag{3.26}$$

Taking logarithms of both sides of (3.26), we get (3.27).

$$h - p = \phi y - \lambda i \tag{3.27}$$

Here $p = \ln P$ is the logarithm of the price level, $h = \ln H$ is the logarithm of the money supply, and $y = \ln Y$ is the logarithm of income.

Price level determination as represented by (3.4) can now be restated as (3.28).

$$p = h - \phi y + \lambda i \tag{3.28}$$

We will adopt the functional form of equation (3.25) for both the domestic and foreign country. So the foreign price level is determined as

$$p^* = h^* - \phi^* y^* + \lambda^* i^* \tag{3.29}$$

There are two new parameters: ϕ^* is the income elasticity of foreign money demand, and λ^* is the interest rate semi-elasticity of money demand.

Relative Money Demand

Relative money demand becomes

$$L/L^* = Y^{\phi} e^{-\lambda i} / Y^{*\phi^*} e^{-\lambda^* i^*} \tag{3.30}$$

With this specification of relative money demand, if we take the logarithm of (3.7), we get the following restatement of the crude monetary approach.

$$s_t = q + h_t - h_t^* - (\phi y_t - \phi^* y_t^*) + (\lambda i_t - \lambda^* i_t^*) \tag{3.31}$$

Here $p^* = \ln P^*$ is the logarithm of the foreign price level, $h^* = \ln H^*$ is the logarithm of the foreign money supply, $y^* = \ln Y^*$ is the logarithm of foreign income, and i^* is the foreign interest rate.

3.A.2 Testing the Monetary Approach

Of course we do not expect (3.31) to hold exactly. When we test the monetary approach, we allow for temporary deviations from purchasing power parity and from our representation of relative money demand to add an "error term" to (3.31). This lack of perfect fit forces us to consider statistically based tests of the monetary approach, such as the early regression tests of Hodrick (1978) and P&W (1979–80) discussed in section 3.2.2. Using monthly data from April 1973 to September 1975, Hodrick (1978, eq. 7) found the following for the USD/DEM exchange rate.

$$s = 7.85 + 1.52h - 1.39h^* + 2.53i + 1.93i^* - 2.23y + 0.073y^* \tag{3.32}$$

In section 3.2.2, we also noted that the monetary approach is usually developed under the further assumption that the foreign and domestic countries have identical interest rate semi-elasticities. In our log-linear specification, this implies that $\lambda = \lambda^*$, yielding (3.33).[18]

$$s_t = q + h_t - h_t^* - (\phi y_t - \phi^* y_t^*) + \lambda(i_t - i_t^*) \tag{3.33}$$

In this version of the crude monetary approach, the foreign and domestic interest rates enter only as an interest rate differential $(i_t - i_t^*)$, allowing us to substitute the forward discount from interest parity condition $\text{fd} = i - i^*$. This yields (3.34).

$$s_t = q + h_t - h_t^* - (\phi y_t - \phi^* y_t^*) + \lambda \text{fd}_t \tag{3.34}$$

The numbers reported in table 3.2 are the estimated coefficients from equation 3.34, as we see more explicitly in table 3.4.

Implicitly, Frenkel dumped domestic income and all foreign variables into the constant

[18] We also noted that some work assumes identical income elasticities ($\phi = \phi^*$) as well as identical interest rate semi-elasticities. Here is a quick way to develop the crude monetary approach under these assumptions. Note that by taking the logarithm of (3.2), purchasing power parity can be characterized by

$$s_t = q + p_t - p_t^*$$

When domestic and foreign money demands have identical parameters, we have

$$\begin{aligned} h_t - p_t &= \phi y_t - \lambda i_t \\ h_t^* - p_t^* &= \phi y_t^* - \lambda i_t^* \end{aligned}$$

Solving these money market equilibrium conditions for prices and substituting into the PPP equation yields

$$s_t = q + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(i_t - i_t^*)$$

		Estimated Coefficient on				
Study:	Sample:	fd	h	h^*	y	y^*
Frenkel (1976)	1921.02–1923.08	0.591	0.975	–	–	–
Bilson (1978b)	1972.01–1976.04	0.0228	1.0013	-1.0081	-1.0184	0.9990
Bilson (1978a)	1970.04–1977.05	1.3853	1.0026	-0.9846	-0.9009	1.0183

Table 3.4: Germany/U.K.: Early Estimates of Equation (3.34)

term. His equation (4″) is

$$s = -5.135 + 0.975h + 0.591\text{fd}$$

As discussed in section 3.2.2, all the estimated coefficients are statistically significant, and the coefficient on the money supply differs insignificantly from unity. This was viewed as dramatic support for the monetary approach. The table reports equation (45) of Bilson (1978b), which estimates a model of the DEM/GBP exchange rate using monthly data for the period 1972.01–1976.04. Despite considerable econometric manipulation to arrive at the reported results, Bilson found the coefficient on fd not to be statistically different from zero. Bilson (1978a) extends this sample to 1970.04–1977.05. The results reported in table 3.4 are from his equation (12), which includes a time trend intended to capture trends in relative money demand. His equation (12) is

$$s = -1.3280 + 1.0026h - 0.9846h^* + 1.3853\text{fd} - 0.9009y_t + 1.0183y_t^* - 0.0049t$$

Although the estimated coefficient of -0.0049 on t appears small, a half a percent appreciation of the DEM per month adds up. As discussed previously, all the coefficients are statistically significant, and the coefficients on money supplies are close to unity.

3.B Partial Adjustment of Money Demand

In our discussions of the monetary approach, we have treated the current value of real money demand as a simple function of current income and interest rates (see). However, applied work on money demand generally uses this functional form only for "long-run" money demand. For example, we might write

$$(h-p)_t^{lr} = \phi^{lr} y_t - \lambda^{lr} i_t$$

Since the work of Goldfeld (1973), short run, money demand is generally assumed to adjust only partially to its long-run level h_t^{lr}.

$$\begin{aligned}(h_t - p_t) - (h_{t-1} - p_{t-1}) &= (1-\alpha)[(h_t^{lr} - p_t) - (h_{t-1} - p_{t-1})] \\ \text{or} \qquad h_t - p_t &= (1-\alpha)(h_t^{lr} - p_t) + \alpha(h_{t-1} - p_{t-1})\end{aligned}$$

Substituting for long run real money demand yields

$$h_t - p_t = \phi y_t - \lambda i_t + \alpha(h_{t-1} - p_{t-1})$$

where $\phi = (1-\alpha)\phi^{lr}$ is the short-run income elasticity of money demand and $\lambda = (1-\alpha)\lambda^{lr}$ is the short-run interest rate semi-elasticity of money demand. As Bilson (1978b) and Woo (1985) emphasize, this change of functional form has implications for the monetary approach. Money market equilibrium is now characterized by

$$(h_t - h_t^*) - (p_t - p_t^*) = \phi(y_t - y_t^*) - \lambda(i_t - i_t^*) + \alpha[(h_{t-1} - h_{t-1}^*) - (p_{t-1} - p_{t-1}^*)]$$

When combined with purchasing power parity, we then have a new exchange rate equation

$$s_t = q_t + (h_t - h_t^*) - \phi(y_t - y_t^*) + \lambda(i_t - i_t^*) - \alpha(h_{t-1} - h_{t-1}^*) + \alpha s_{t-1} - \alpha q_{t-1}$$

Of course the inflation rate depends on many variables in the short run. In the monetary approach model, however, attention centers on the rate of growth of the money supply, and money supply growth rates (Δh and Δh^*) are treated as exogenously set by the central bank. The expected inflation differential is therefore treated as deriving from perceived monetary policy. Recalling that for simplicity we are holding income constant, this implies that when money growth rates are expected to be constant $\pi^e - \pi^{*e} = \Delta h^e - \Delta h^{*e}$. Under such conditions, we can write our exchange rate determination equation as

$$S = \frac{H/H^*}{\mathcal{L}[\Delta h^e - \Delta h^{*e}, Y/Y^*]} \tag{3.35}$$

However, when changes in monetary policy are anticipated, we *cannot* use (3.35).

We might try to avoid choosing an interest rate by turning to the Fisher equation, which allows us to rewrite this as

$$s_t = \lambda(r_t - r_t^*) + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\pi_t^e - \pi_t^{*e})$$

If we treat the real interest rate differential as fixed, we no longer have to worry about picking the right interest rate. But now we must worry about finding proxies for unobserved expectations.

Capital Mobility

Suppose you are considering the purchase of an asset denominated in foreign currency. For example, you might be a U.S. citizen contemplating the purchase of a Canadian bond. If you expect the U.S. dollar to lose value (depreciate) against the Canadian dollar while you hold the bond, this will raise your expected dollar returns from holding the Canadian bond.

Let us calculate the real returns, from the domestic perspective, to holding the domestic and foreign interest bearing asset. The real return from holding the domestic interest bearing asset is $i - \pi$. The real return from holding the domestic interest bearing asset is

$i^* + \Delta s - \pi$. If the two assets are expected to have the same real return, then

$$i = i^* + \Delta s^e \tag{3.36}$$

This is "uncovered interest parity" (UIP). When UIP holds, the interest differential equals the expected rate of depreciation.

$$i - i^* = \Delta s^e \tag{3.37}$$

The difference in the nominal rates of return is offset by an expected change in the price of foreign exchange.

Uncovered interest parity is not implied by covered interest parity. Ensuring UIP requires more than just frictionless financial markets. However, we can imagine special conditions under which UIP should hold, so that the assets of two different currency denominations should be expected to yield the same real returns. Suppose for example that investors are indifferent as to the currency denomination of their assets as long as they have the same expected return. That is, although investors care about their average returns they do not care about the volatility of these return. In particular, they do not care about "currency risk". This situation is often referred to as *perfect substitutability* or *risk neutrality*, and it implies that the expected real rate of return on the domestic asset equals the expected real rate of return from holding the foreign asset: $i - \pi^e = i^* + \Delta s^e - \pi^e$. Perfect substitutability implies uncovered interest parity (UIP):

$$i - i^* = \Delta s^e \tag{3.38}$$

Note the UIP is an equilibrium condition, not a theory of interest rate determination. It is a relationship that must hold when foreign and domestic assets are perfect substitutes. This is a very strong condition on international capital markets. An even stronger condition arises when we add a condition known as ex ante relative purchasing power parity, which just says that changes in the real exchange rate cannot be anticipated. The idea that real exchange rate changes cannot be anticipated is captured by treating the expected change in the real exchange rate as zero: $\Delta q^e = 0$. From the definition of the real exchange rate, this implies

$$\Delta s^e + \pi^{*e} - \pi^e = 0 \tag{3.39}$$

We will refer to this condition as *expected PPP*. It says that there is no expected change in the real exchange rate. This may be because the real exchange rate has been constant and is expected to stay that way. More realistically, expected PPP results when any past changes in the real exchange rate are expected to be permanent.

Perfect substitutability and expected PPP together imply a very strong condition on world capital markets. Combining expected (3.38) with (3.39), we see that the expected real returns earned by domestic resident on domestic assets will equal the real returns expected by foreign residents on foreign assets.

$$i - \pi^e = i^* - \pi^{*e}$$

This situation is known as real interest parity. Most empirical work on the monetary approach assumes expected PPP and UIP. However, some early tests of the monetary approach took place under weaker assumptions than real interest parity, and we will begin by looking at those.

Real Interest Parity

Risk Neutrality

We now add a fourth standard component of the monetary approach to exchange rate determination: the uncovered interest rate parity condition.

$$i_t - i_t^* = \Delta s_t^e$$

Substituting this into the crude monetary approach solution yields the standard flex-price monetary approach solution for the behavior of the exchange rate:

$$s_t = h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\Delta s_t^e) \tag{3.40}$$

Immediately, we see that exchange rate expectations are a crucial determinate of the current exchange rate. Unfortunately, these expectations are not directly observable. Empirical tests of the monetary approach must therefore grapple with the problem of finding a useful proxy for these expectations. We consider three approaches to modeling expectations in our exchange rate models: the core inflation approach (Frankel 1979), regressive expectations, and rational expectations.

Expected PPP

Note that if purchasing power parity is expected to hold next period, then consistency requires

$$s_t^e = p_t^e - p_t^{*e}$$

This can be combined with the purchasing power parity condition for the current period to yield a link between expected depreciation and the expected inflation differential.

$$\Delta s_t^e = \pi_t^e - \pi_t^{*e}$$

We can therefore rewrite the standard monetary approach solution as the core inflation approach.

$$s_t = h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\pi_t^e - \pi_t^{*e}) \tag{3.41}$$

Comparing (3.41) with (3.40) makes it clear that this formulation just assumes away the contribution of the real interest differential. Thus if we include a constant term in our empirical work, say because we wish to allow for constant terms in the money demand equations, (3.41) and (3.40) are not readily distinguishable.

Comment: some work has been done under the assumption of interest inelastic money demand (e.g., Florentis et al. 1994). This yields the same models as above, with $\lambda = 0$.[19]

[19] The result is identical to that implied by the standard monetary approach when fundamentals follow a

Comment: We have written

$$\begin{aligned} i_t - i_t^* &= \Delta s_t^e \\ &= \pi_t^e - \pi_t^{*e} \end{aligned}$$

Implying

$$i_t - \pi_t^e = i_t^* - \pi_t^{*e}$$

That is, open interest rate parity plus expected PPP implies real interest rate parity.

As discussed in a previous section, this model is incomplete in that it does not include a characterization of interest rate determination. Introduction of the Fisher equation changes this situation.

$$i_t = r_t + \pi_t^e$$

in order to characterize the interest rate differential as

$$i_t - i_t^* = r_t - r_t^* + \pi_t^e - \pi_t^{*e} \tag{3.42}$$

Combining (3.B) and (3.42) yields (3.43), the core-inflation characterization of the monetary approach model.

$$s_t = q + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(r_t - r_t^* + \pi_t^e - \pi_t^{*e}) \tag{3.43}$$

Equation (3.43) highlights some key insights of the monetary approach to flexible exchange rates. Most important is the observation that, ceteris paribus, doubling the domestic money supply doubles the exchange rate.[20]

So far we have simply reviewed the derivation of the core-inflation version of the monetary approach model. We now observe that if PPP holds every period then informed individuals should expect it to hold. That is, the expected depreciation of the exchange rate should equal the expected-inflation differential.

$$\Delta s^e = \pi^e - \pi^{*e}$$

This condition is known as expected purchasing power parity (EPPP). Under EPPP, the monetary approach model of flexible exchange rates can be summarized as

$$s = h - h^* - \phi(y - y^*) + \lambda(s^e - s) \tag{MAFE}$$

The key difference in this approach is that the real interest differential is constrained to zero. But if we wish to determine absolute price levels in the two countries as well as their relative price level, we need to explicitly add the Fisher effect and exogenous real interest rates. We also need the assumption that (3.2) is expected to hold every period.

random walk. (See the next chapter.)

[20]Note that among the ceteris held paribus is expected inflation. Note too that doubling the money supply *adds* ln 2 to the log of the money supply.

$$\frac{L[i, Y]}{L^*[i^*, Y^*]} = \mathcal{L}[i - i^*, Y/Y^*]$$

This allows us to rewrite (3.7), the determination of the exchange rate, as (3.33).

$$S = Q\frac{H/H^*}{\mathcal{L}[i - i^*, Y/Y^*]} \tag{3.44}$$

The implied characterization of the foreign price level is given in (3.45), which is the foreign country equivalent of (3.28).

$$p^* = h^* + \phi^* y^* - \lambda^* i^* \tag{3.45}$$

Lecture 4

The Monetary Approach under Rational Expectations

LECTURE3 showed that expectations of future inflation are an important determinant of the current exchange rate. This creates a difficulty for research on exchange rate determination, since economists cannot observe expectations and know very little about expectations formation. This book considers several different ways that economists have struggled to grapple with this difficulty. One approach is the rational expectations hypothesis: the expectations of individuals are assumed to match the predictions of our model. It is more accurate to call these expectations ***model consistent***, but the convention of calling them ***rational*** is well established among economists. This chapter offers a further exploration of the link between exchange rates and expectations, when expectations formation is "rational" in this restricted sense.

4.1 Expectations and Exchange Rates: The Monetary Approach

LECTURE3 explored the crucial role of expectations in exchange-rate determination. The economic consequences of anticipated monetary policy changes intimately involved the process of expectations formation. It proved helpful to assume expectations closely matched actual outcomes. In a nutshell, that assumption is the rational expectations hypothesis. The rational expectations hypothesis states that the expectations relevant to economic outcomes are appropriately proxied by the forecasts derived from an economist's model.

To keep the algebra as simple as possible, this chapter works with the log-linear version of the monetary approach model. Recall that there are two basic components of the monetary approach to the determination of flexible exchange rates: purchasing power parity (PPP), and the classical model of price determination. As always, PPP says that the spot rate is proportional to the relative price level, where the factor of proportionality is the exogenous real exchange rate. A log-linear representation of purchasing power parity as follows.[1]

$$s_t = q_t + p_t - p_t^* \tag{4.1}$$

Next, consider a crude Classical model of the relative price level. As always in this model, the relative price level is determined by relative nominal money supplies and relative real money demands. Represent a log-linear representation of purchasing power parity as follows.Next, consider the log-linear representation of the relative price level, as determined by the Classical model.[2]

$$p_t - p_t^* = h_t - h_t^* - [\phi(y_t - y_t^*) - \lambda(i_t - i_t^*)] \tag{4.2}$$

Combining this Classical model of relative price determination with purchasing power parity yiels a crude monetary approach to floating exchange rates.

$$s_t = q_t + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(i_t - i_t^*) \tag{4.3}$$

LECTURE3 looked at some of the empirical applications of this simple model. One potential problem for empirical tests of the crude monetary approach model is that it may mistake other influences on the exchange rate for a response to the interest rate. For example, the empirical version of the model allows for random shocks to money demand. Let u_t be the money demand shock at time t, so that

$$s_t = q_t + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(i_t - i_t^*) - u_t \tag{4.4}$$

A money demand shock affects the current price level and thus the current exchange rate, but it also influences expected future inflation and thus the interest differential. For example, suppose u_t is entirely temporary: it does not persist past period t. Then a positive shock will increase money demand and lower the price level today, but next period the shock will be absent and the price level will rise again. So a positive money demand shock will have both a direct and indirect effect on the spot rate: by raising money demand it appreciates the exchange rate, but by raising expected future inflation (and the interest differential) it depreciates the exchange rate. Since the observable interest differential will be positively correlated with the unobserved money demand shocks, the estimate of λ will be biased downward.[3]

There are other difficulties as well. For example, there is the question of which interest rate should be used empirically, out of the array of possibilities. In this section we algebraically derive the predictions of the monetary approach to flexible rates under the rational expectations hypothesis. This allows us to overcome some of the problems that have concerned us, although it raises some new questions as well.

---

[1]Here, s is the natural logarithm of the spot exchange rate. Q is the natural logarithm of the real exchange rate, p is the natural logarithm of the domestic price level, and p^* is the natural logarithm of the foreign price level. For the moment, do not impose a constant real exchange rate, but do continue to treat it as exogenous.

[2]Here h and h^* are the domestic and foreign money supplies, and y and y^* are the domestic and foreign money supplies, all in natural logs. Also, i and i^* are the domestic and foreign interest rate, but in natural levels rather than as logarithms. For algebraic simplicity, this formulation equates the foreign and domestic money demand parameters.

[3]This was noted by Driskill and Sheffrin (1981) and, in a similar context, Sargent (1977).

Recall from Lecture2 that the covered interest parity condition equates the interest differential $(i - i^*)$ to the forward discount on the domestic currency (fd).

$$i_t - i_t^* = \text{fd}_t \tag{4.5}$$

Substituting (4.5) into (4.4) yields the forward-discount variant of the monetary approach associated with Frenkel (1976).

$$s_t = q_t + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda \underbrace{\text{fd}}_{i-i^*} - u_t \tag{4.6}$$

Recall as well the decomposition of the forward discount into two pieces: the expected rate of depreciation (Δs^e) and the risk premium (rp). When including time subscripts, let Δs_{t+1}^e represent the expectation at time t of the percentage rate of depreciation of the spot rate over the period t to $t+1$.

$$\text{fd}_t = \Delta s_{t+1}^e + \text{rp}_t \tag{4.7}$$

Equations (4.5) and (4.7) imply that the interest differential also bears a simple relationship to expected depreciation and the risk premium.

$$i_t - i_t^* = \Delta s_{t+1}^e + \text{rp}_t \tag{4.8}$$

Substituting (4.8) into (4.4) yields the monetary-approach equation that is the focus of this chapter.

$$s_t = q_t + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda\underbrace{(\Delta s_{t+1}^e + \text{rp}_t)}_{i-i^*} - u_t \tag{4.9}$$

4.1.1 Fundamentals

Combine all the exogenous determinants of the exchange rate into a single variable, $\tilde{m}$.

$$\tilde{m}_t \stackrel{\text{def}}{=} q_t + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda \text{rp}_t - u_t \tag{4.10}$$

This notation allows us to rewrite (4.9) in a simpler form.

$$s_t = \tilde{m}_t + \lambda \Delta s_{t+1}^e \tag{4.11}$$

Except for expected depreciation, the portmanteau variable $\tilde{m}$ holds all the exchange-rate determinates. In the context of the monetary approach to floating exchange rates, income and money are often called exchange-rate ***fundamentals***. However as a convenient shorthand, this chapter applies that term to the broader collection in $\tilde{m}$. Equation (4.11) expresses the spot exchange rate in terms of $\tilde{m}$ and expected depreciation. The spot rate is determined both by the fundamentals and by expectations. Expectations are a crucial determinant of the spot rate: an increase in expected depreciation causes the spot rate to depreciate. Unless $\lambda = 0$, the fundamentals alone are not enough to determine the exchange rate.

Equation (4.11) implies that if $\Delta s^e = 0$, then exchange rate fundamentals directly determine the exchange rate. An expectation $\Delta s^e = 0$ is not an expectation that the exchange rate will remain unchanged, as under a credibly fixed exchange rate. Rather, it roughly means that upward and downward movements are equally likely. When there is no reason to believe that depreciation or appreciation is more likely, there is no reason to bet on a movement in one direction rather than another. Typically, changes in the exchange rate are very hard to predict, especially in the short run. It is as if the exchange rate follows a ***random walk***: each period it is just as likely to rise as to fall.

Since expectations are not observable, the crucial role of expectations for exchange-rate determination presents an evident difficulty for empirical work. Economists have made many proposals to deal with that, but this book focuses on two. One proposal is to use survey methods to ask market participants about their expectations. Although this proposal seems to have recently gained some ground, traditionally economists have resisted it. Instead, they propose to model expectations formation. This finesses the observability problem by attempting to endogenize expectations.

As a preliminary to such an attempt, decompose expected depreciation into the current spot rate and the expected future spot rate. Substituting into (4.11) produces the following.

$$s_t = \tilde{m}_t + \lambda \underbrace{(s^e_{t+1} - s_t)}_{\Delta s^e_{t+1}} \tag{4.12}$$

Solving for the spot rate then yields (4.13), the fundamental relationship between the spot exchange rate and its future expectation.

$$s_t = \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}s^e_{t+1} \tag{4.13}$$

Once again this solution drives home a key message: the current level of the spot rate depends on its expected future value.

4.2 Solving under Rational Expectations

LECTURE3 developed some predictions of the monetary approach by tying expectations about the economy to its actual evolution. Much of the empirical work on the monetary approach proceeds in this fashion, and economists typically refer to this close relationship between expectations and actual outcomes as ***rational*** expectations. This section presents an algebraic solution of the monetary approach to exchange rate determination under rational expectations.

In the context of economic modeling, the ***rational expectations hypothesis*** (RATEX) states that the expectations included in a model are appropriately proxied by the forecasts of the model. At the conceptual level, an attractive feature of the rational expectations hypothesis is that it produces a consistent relationship between expectations formation and the structure of the economy. The economic actors implicit in the model act on expectations that are consistent with the structure of the model. Rational expectations are

model consistent.

Under rational expectations, the expected future exchange rate becomes one of the variables explained by the model. Let $\mathcal{E}_t$ denote a mathematical expectation conditional on all information available at time t, including past values of the fundamentals and the structure of the model. Then for the MAFER model of this chapter, represent the rational expectations hypothesis as follows.

$$s^e_{t+1} = \mathcal{E}_t s_{t+1} \tag{4.14}$$

This is an apparently simple way of endogenizing the expectations of the future, but it has very strong implications. This chapter will show that it implies that the entire expected future of the economy is relevant to the current spot rate. Specifically, as highlighted by Bilson (1978b, p.78), RATEX implies that the current spot rate is a weighted sum of all expected future exchange rate fundamentals.

$$s_t = \frac{1}{1+\lambda} \sum_{i=0}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_t \tilde{m}_{t+i} \tag{4.15}$$

In (4.15), the weights decline into the future, since $\lambda/(1+\lambda) < 1$. This resembles a discounted-present-value calculation, so economists often refer to (4.15) as the *present-value solution* for the spot exchange rate. Since the present-value solution involves expectations of future fundamentals, it is not yet clear how it can assist empirical work. Before addressing that question, the next section works through the algebra that produces (4.15).

4.2.1 The Rational Expectations Algebra

This section provides a detailed algebraic derivation of (4.15), the present-value solution for the spot exchange rate. Some readers may prefer to skip ahead to section 4.3, which explores how to remove the unobserved expections from the solution.

Combine the basic solution (4.13) with the rational expectations hypothesis (4.14) to produce (4.16).

$$s_t = \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}\mathcal{E}_t s_{t+1} \tag{4.16}$$

One approach to eliminating the expectations in (4.16) is *recursive substitution*. Begin the observation that (4.16) applies at each point in time. Therefore it implies (4.17).

$$s_{t+1} = \frac{1}{1+\lambda}\tilde{m}_{t+1} + \frac{\lambda}{1+\lambda}\mathcal{E}_{t+1} s_{t+2} \tag{4.17}$$

This is just the same relationship, one period forward in time. Taking expectations (at time t) of both sides of (4.17) yields equation (4.19).

$$\mathcal{E}_t s_{t+1} = \frac{1}{1+\lambda}\mathcal{E}_t\tilde{m}_{t+1} + \frac{\lambda}{1+\lambda}\mathcal{E}_t\mathcal{E}_{t+1} s_{t+2} \tag{4.18}$$

Substitute (4.18) into (4.16) to produce (4.19).

$$s_t = \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}\frac{1}{1+\lambda}\mathcal{E}_t\tilde{m}_{t+1} + \left(\frac{\lambda}{1+\lambda}\right)^2 \mathcal{E}_t\mathcal{E}_{t+1}s_{t+2} \tag{4.19}$$

Next, repeat the substitution process. This time, substitute for $\mathcal{E}_{t+1}s_{t+2}$. After that, substitute for $\mathcal{E}_{t+2}s_{t+3}$. And so on. Proceding in this way for n successive substitutions produces (4.20).

$$\begin{aligned} s_t = {} & \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}\mathcal{E}_t\{\frac{1}{1+\lambda}\tilde{m}_{t+1}\} \\ & + \cdots \\ & + \left(\frac{\lambda}{1+\lambda}\right)^n \mathcal{E}_t\mathcal{E}_{t+1}\cdots\mathcal{E}_{t+n-1}\{\frac{1}{1+\lambda}\tilde{m}_{t+n}\} \\ & + \left(\frac{\lambda}{1+\lambda}\right)^{n+1} \mathcal{E}_t\mathcal{E}_{t+1}\cdots\mathcal{E}_{t+n}s_{t+n+1} \end{aligned} \tag{4.20}$$

The Law of Iterated Expectations says that $\mathcal{E}_t\{\mathcal{E}_{t+i}s_{t+i+1}\} = \mathcal{E}_t s_{t+i+1}$. This just means that your current best guess of your future best guess about the future spot rate is just your current best guess about that future spot rate. In otherwords, your current guess uses all the information you have available, and therefore differs from your future best guess only due to new information you will receive in the future and cannot use now. This allows us to simplify (4.20).

$$s_t = \frac{1}{1+\lambda}\sum_{i=0}^{n}\left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_t\tilde{m}_{t+i} + \left(\frac{\lambda}{1+\lambda}\right)^{n+1} \mathcal{E}_t s_{t+n+1} \tag{4.21}$$

Noting that $\lambda/(1+\lambda) < 1$, assume $\lim_{n\to\infty}\left(\lambda/(1+\lambda)\right)^{n+1}\mathcal{E}_t s_{t+n+1} = 0$.[4] Then the recursive-substitution solution becomes (4.22), which as promised is identical to (4.15).

$$s_t = \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_t\tilde{m}_{t+i} \tag{4.22}$$

The Role of News

Consider the implication of (4.22) for the difference between the realized spot rate and the anticipated spot rate. Taking expectations at time $t-1$ yields

$$\mathcal{E}_{t-1}s_t = \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_{t-1}\tilde{m}_{t+i} \tag{4.23}$$

[4]The situation is a bit more complicated than this. See the discussion of "bubbles" in section 4.2.1. For now, simply assume that the weighted sum of expected future fundamentals will converge.

Subtracting (4.23) from (4.22) yields (4.24).

$$s_t - \mathcal{E}_{t-1}s_t = \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i (\mathcal{E}_t\tilde{m}_{t+i} - \mathcal{E}_{t-1}\tilde{m}_{t+i}) \tag{4.24}$$

That is, the difference between the spot rate differs from its anticipated value to the extent that expectations about the fundamentals have been revised.

Bubbles

Recall that recursive substitution produced (4.21), repeated here for convenience.

$$s_t = \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_t\tilde{m}_{t+i} + \lim_{n\to\infty}\left(\frac{\lambda}{1+\lambda}\right)^{n+1} \mathcal{E}_t s_{t+n+1}$$

The observation that $\lambda/(1+\lambda) < 1$ motivated the assumption that

$$\lim_{n\to\infty}\left(\frac{\lambda}{1+\lambda}\right)^{n+1} \mathcal{E}_t s_{t+n+1} = 0 \tag{4.25}$$

This assumption produces the present-value solution. However, exchange-rate *bubbles* may generate a non-zero limit for this term. This section offers a simple illustration of this possibility.

Let s^f be the present-value solution. That is

$$s_t^f \stackrel{\text{def}}{=} \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_t\tilde{m}_{t+i} \tag{4.26}$$

Consider a ***bubble solution*** of the form $s_t^f + b_t$, where b_t grows over time independently of the model fundamentals. Can this be a solution to the rational expectations model? That is, can a bubble solution satisfy (4.16)? Suppose it does; then

$$s_t^f + b_t = \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}\mathcal{E}_t\left(s_{t+1}^f + b_{t+1}\right) \tag{4.27}$$

$$b_t = \frac{\lambda}{1+\lambda}\mathcal{E}_t b_{t+1} \tag{4.28}$$

Any process that obeys (4.28) produces a solution that is compatible with rational expectations. For example, define a bubble b_t as the following explosive first-order autoregressive process.

$$b_t = \frac{1+\lambda}{\lambda}b_{t-1} + \eta_t \tag{4.29}$$

If η_t is white noise (i.e., mean zero and constant variance), then

$$\mathcal{E}_t b_{t+1} = \frac{1+\lambda}{\lambda} b_t \tag{4.30}$$

This produces an explosive bubble is can be part of the solution to the MAFER model under rational expectations.

4.3 An Observable Solution

We have seen that the exchange rate solution is a weighted sum of expected future fundamentals. Let us turn to the question of how to use this solution in empirical work. How can we handle the expected future fundamentals in the exchange rate solution? Until we can relate these expected future values to something we can observe and measure, our spot rate solution under rational expectations cannot be turned into a useful empirical model. For example, we cannot offer any simple relationship between the spot rate and the money supply (as we attempted to do in Lecture3) until we know how the expected future behavior of the money supply is related to its current and past behavior. Many economists approach this problem by characterizing the way the fundamentals evolve over time. Such a characterization is called a *data generating process* (DGP) for the fundamentals. This information can then be used in forming expectations about the future.

A common assumption in early empirical tests of the monetary approach is that the exchange rate fundamentals follow a random walk:

$$\tilde{m}_t = \tilde{m}_{t-1} + u_t \tag{4.31}$$

Here u_t is the unanticipated change in the fundamentals, which averages zero (and is not serially correlated). This just means that the fundamentals are just as likely to rise as to fall each period. In this case our "best guess" of the future fundamentals is their current value. That is

$$\mathcal{E}_t \tilde{m}_{t+i} = \tilde{m}_t \tag{4.32}$$

If the fundamentals are just as likely to rise as to fall, so is the spot exchange rate. That is, our best guess of the future spot rate is the current spot rate. Recalling (4.11), we can conclude that

$$s_t = \tilde{m}_t \tag{4.33}$$

(This can also be seen algebraically by substituting (4.32) into (4.15).) Recalling the definition of the exchange rate fundamentals, we can then gather data and test (4.33) empirically.

$$s_t = q + h - h^* - \phi(y - y^*) + \lambda \mathrm{rp} \tag{4.34}$$

Note how similar this is to the crude monetary approach: only the interest rate effect has disappeared.

Obviously the fundamentals cannot always be characterized as following a random walk. For example, if one country consistently has high inflation and another country consistently has low inflation, the random walk characterization of the exchange rate fundamentals for these two countries will be a poor one. As a result, for many exchange rates we need to characterize the exchange rate fundamentals by a more complicated data generating process. For example, we may assume that the fundamentals follow a simple autoregressive process.[5] Even this considerably complicates the problem of representing and summing up the expected future fundamentals. One useful alternative method for dealing with more general DGPs is the method of undetermined coefficients, which is briefly treated below.

4.4 The Data Generating Process

Before starting on the algebra, let us get our bearings. Remember, we have a model of the spot exchange rate under rational expectations

$$s = \frac{1}{1+\lambda}\tilde{m} + \frac{\lambda}{1+\lambda}\mathcal{E}_t s \tag{4.13}$$

that provides the basic structure of spot rate determination. We found that we could solve this model for the spot rate as

$$s_t = \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i \mathcal{E}_t \tilde{m}_{t+i} \tag{4.35}$$

where $\tilde{m}$ represents the exchange rate fundamentals (e.g., relative money supplies and the determinants of relative real money demand). Now the problem with this kind of formulation is that it still involves unobserved expectations. While it is informative at the theoretical level, it is not very useful empirically. We would like to have an *observable* solution for the exchange rate, a solution in terms of variables we can observe and measure. To move from a solution like (4.35) to an observable solution, we represent the fundamentals

[5] A more typical way of closing a rational expectations model is much more general: assume that the n exogenous variables follow a k^{th}-order vector autoregressive process (VAR) after differencing. This is a bit more complicated, so details will be relegated to appendix 4.D. Briefly, let X_t be the n-vector of exogenous variables at time t. For example, we might treat relative money supplies and relative incomes as the only components of our fundamentals. Let $X_t^\top = (h-h^*, y-y^*)$, so we can rewrite our fundamentals as $\tilde{m}_t = a^\top X_t$ where $a^\top = (1, -\phi)$. The VAR process simply relates X_t to its past values.

$$\Delta X_t = \sum_{j=1}^{k} B_j \Delta X_{t-j} + v_t \tag{4.67}$$

The B_j's are the matrices of coefficients on the lagged exogenous variables and v_t is a vector of errors. If we create a vector Z_t containing the current and lagged exogenous variables, then as shown in appendix 4.D, our solution for the exchange rate is a linear function of the current and lagged exogenous variables.

$$s_t = a^\top X_t + a^\top GC\Delta Z_t \tag{4.76}$$

where GC is a matrix defined in appendix 4.D. This can be estimated simultaneously with (4.67).

by a *data generating process* (DGP).

For example, we initially treated $\tilde{m}$ as following a random walk, which yielded a simple observable solution for the exchange rate. But while this may be a good approximation for some pairs of countries, for others it will be terrible. (For example, one country might have persistent high inflation when the other does not.) So consider a more general DGP: the following simple "autoregressive" representation of the fundamentals.

$$\tilde{m}_t = \mu_0 + \mu_1 \tilde{m}_{t-1} + u_t \tag{4.36}$$

We will show that given the DGP (4.36), the exchange rate solution is

$$s_t = \frac{\lambda \mu_0}{1 + \lambda - \lambda \mu_1} + \frac{1}{1 + \lambda - \lambda \mu_1} \tilde{m}_t \tag{4.37}$$

4.4.1 Anticipating Future Fundamentals

Suppose the DGP (4.36) governs the evolution of the fundamentals over time. The following equations just restate this relationship at various times.

$$\begin{aligned}
\tilde{m}_{t+1} &= \mu_0 + \mu_1 \tilde{m}_t + u_{t+1} \\
\tilde{m}_{t+2} &= \mu_0 + \mu_1 \tilde{m}_{t+1} + u_{t+2} \\
\tilde{m}_{t+3} &= \mu_0 + \mu_1 \tilde{m}_{t+2} + u_{t+3}
\end{aligned}$$

Consider expectations formed based on this DGP.

$$\begin{aligned}
\mathcal{E}_t \tilde{m}_{t+1} &= \mu_0 + \mu_1 \mathcal{E}_t \tilde{m}_t + \mathcal{E}_t u_{t+1} \\
&= \mu_0 + \mu_1 \tilde{m}_t \\
\mathcal{E}_t \tilde{m}_{t+2} &= \mu_0 + \mu_1 \mathcal{E}_t \tilde{m}_{t+1} + \mathcal{E}_t u_{t+2} \\
&= \mu_0 + \mu_1 (\mu_0 + \mu_1 \tilde{m}_t) \\
&= \mu_0 + \mu_1 \mu_0 + \mu_1^2 \tilde{m}_t \\
\mathcal{E}_t \tilde{m}_{t+3} &= \mu_0 + \mu_1 \mathcal{E}_t \tilde{m}_{t+2} + \mathcal{E}_t u_{t+3} \\
&= \mu_0 + \mu_1 (\mu_0 + \mu_1 \mu_0 + \mu_1^2 \tilde{m}_t) \\
&= \mu_0 + \mu_1 \mu_0 + \mu_1^2 \mu_0 + \mu_1^3 \tilde{m}_t \\
&\vdots
\end{aligned}$$

A pattern quickly emerges, as summarized by (4.38). This pattern underpins an important insight: all expectations of future fundamentals can be stated in terms of the current fundamentals.

$$\mathcal{E}_t \tilde{m}_{t+i} = \mu_0 \sum_{j=0}^{i-1} \mu_1^j + \mu_1^i \tilde{m}_t \qquad i \geq 1 \tag{4.38}$$

4.4.2 Finding An Observable Reduced Form: The Algebra

In principle, one may substitute the solutions (4.38) for expected future fundamentals into equation (4.35) and compute the nested summations. The algebra is not complicated but can be rather messy; see section 4.B. This section introduces an alternative: the method of undetermined coefficients. (Naturally, it will yield the same solution.)

The Method of Undetermined Coefficients

The method of undetermined coefficients begins with an educated guess the general form of our solution. This guess is not arbitrary; it relies on two pieces of information. First, recall that (4.35) expresses the current spot rate in terms of expected future fundamentals. Second, recall that all expected future fundamentals may be expressed in terms of current fundamentals. These two pieces of information suggest that the spot rate can be expressed in terms of the current fundamentals. This is captured by the following expression, which is an educated guess as to the general form of the observable solution.

$$s_t = \phi_0 + \phi_1 \tilde{m}_t \tag{4.39}$$

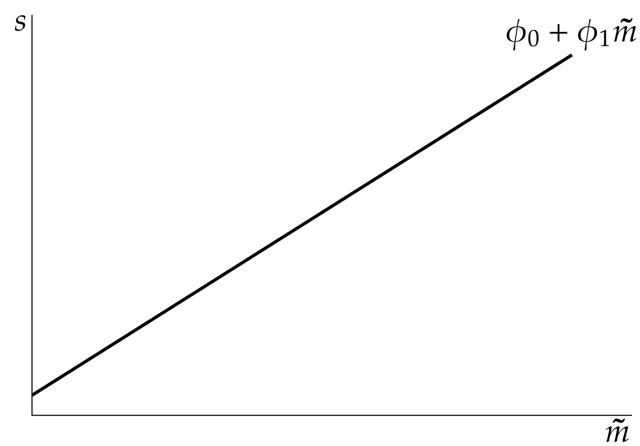


Figure 4.1: Exchange Rate and Fundamentals

This is an educated guess at the general functional form, but involves two undetermined coefficients, ϕ_0 and ϕ_1. How are these coefficients related to the underlying structural parameters? To explore this question, restate the current guess (4.39) one period forward, thereby producing the following.

$$s_{t+1} = \phi_0 + \phi_1 \tilde{m}_{t+1} \tag{4.40}$$

Taking the expecation of both sides produces the implied expression for the expected future spot rate.

$$\mathcal{E}_t s_{t+1} = \phi_0 + \phi_1 \mathcal{E}_t \tilde{m}_{t+1} \tag{4.41}$$

Now recall (4.13), the fundamental relationship between the spot exchange rate and its future expectation. A correct solution form must be compatible with this. To check for consistency, substitute the tentive expressions for s and s^e (with their undetermined coefficients) into equation (4.13).

$$s_t = \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}\mathcal{E}_t s_{t+1} \tag{4.42}$$

$$\begin{aligned}\phi_0 + \phi_1\tilde{m}_t &\equiv \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}[\phi_0 + \phi_1(\mu_0 + \mu_1\tilde{m}_t)] \\ &= \frac{\lambda}{1+\lambda}(\phi_0 + \phi_1\mu_0) + \frac{1+\lambda\phi_1\mu_1}{1+\lambda}\tilde{m}_t\end{aligned} \tag{4.43}$$

Next comes the magic. Equation (4.43) must hold for *any* level of the fundamentals $\tilde{m}_t$. That is what pins down the ϕs: the slope and intercept must be the same for the expression in $\tilde{m}_t$ found on each side of the equality (4.43).

$$\begin{aligned}\phi_0 &\equiv \frac{\lambda}{1+\lambda}(\phi_0 + \phi_1\mu_0) \\ \phi_1 &\equiv \frac{1+\lambda\phi_1\mu_1}{1+\lambda}\end{aligned} \tag{4.44}$$

The two equations in (4.44) involve two unknowns: ϕ_0 and ϕ_1. The second equation involves only ϕ_1; and it is easily solved.

$$\phi_1 = \frac{1}{1+\lambda-\lambda\mu_1} \tag{4.45}$$

With a solution for ϕ_1 in hand, solve the first equation for ϕ_2.

$$\phi_0 = \lambda\mu_0\phi_1 \tag{4.46}$$

$$= \frac{\lambda\mu_0}{1+\lambda-\lambda\mu_1} \tag{4.47}$$

Now replace the undetermined coefficients in the initial guess (4.39), producing the following observable reduced form for the spot exchange rate. (In empirical work, economists often estimate (4.48) and (4.36) simultaneously.)

$$s_t = \frac{\lambda\mu_0}{1+\lambda-\lambda\mu_1} + \frac{1}{1+\lambda-\lambda\mu_1}\tilde{m}_t \tag{4.48}$$

Equation (4.48) has an important message for exchange rate research. There is no simple relationship between exchange rates and money supplies. Note that the relationship between the spot rate and the money supply depends on all the parameters of the data generating process. For example, if $\mu_1 < 1$ so that money supply increases tend to be reversed over time, then the exchange rate will rise less than in proportion to a money supply increase.

In summary, the method of undetermined coefficients proceeds as follows:

1. Begin with a model and a data generating process (DGP) for the exogenous variables.

2. Based on the DGP, make an educated guess about the functional form of the solution, which will generally involve the exogenous variables, shocks to the system, and possibly lagged endogenous variables. That is, guess an "observable reduced form" for the system, where your guess involves "undetermined coefficients" that you want to express in terms of structural coefficients.

3. Find the expectations implied by your proposed solution form.

4. Plug these expectations into the model.

5. Use the implied identities in the coefficients to solve for the undetermined coefficients in terms of the structural coefficients (here, the μs and λ).

4.5 An Empirical Application

The monetary approach to flexible exchange rates has been tested under the assumption of rational expectations. After some early results lent some support to the model, a great deal of testing took place. Three salient supportive studies are Hoffman and Schlagenhauf (1983), Macdonald (1983), and Woo (1985). This section briefly discusses the first of these studies. (See section for a discussion of Woo (1985).)

Hoffman and Schlagenhauf (1983) produced an early test of the rational expectations version of the monetary approach to exchange rate determination.[6] Using monthly data (1974.1–1979.12), they model the spot rate of the dollar against the franc, pound, and mark. Their results are pretty good. Typically, they do not estimate the relative money supply coefficient—predicted by the monetary approach to be unity—in their rational expectations model. However, they do test this restriction jointly with several others, and fail to reject the restrictions. Since under the rational expectations hypothesis the structural model and DGP combine to produce a number of non-linear parameter restrictions, as seen in (4.76), economists viewed their failure to reject these restrictions statistically as very interesting support for the rational expectations version of the monetary approach. Their estimates for the income elasticity of money demand and the interest rate semi-elasticity of money demand are reported in table 4.1.

As suggested by the monetary approach, these estimates are all positive; they are also of plausible magnitude. Finally, Hoffman and Schlagenhauf found all of them significant at the 5% level. All in all, this early study appears to provide impressive support for the rational expectations version of the monetary approach to flexible exchange rates.

[6]They restricted the data generating process for the exogenous variables to be AR(1) for each variable individually (in differences). That is, their model of exchange-rate fundamentals is that relative money supplies and relative incomes individually follow an ARIMA(1,1,0) process. This imposes structure on the C matrix discussed in section 4.D. In the notation of that section, they estimated a differenced form of the exchange rate equation:

$$\Delta s_t = a'\Delta X_t + a'GC\Delta^2 Z_t$$

Sensitivity to↓ \ Exchange Rate:	USD/DEM	USD/FRF	USD/GBP
Interest rates	1.784	1.108	1.782
Real income	1.507	0.969	0.508

Source: Hoffman and Schlagenhauf (1983)

Table 4.1: Monetary Approach Estimates under Rational Expectations

4.6 Concluding Comments

A key lesson of this chapter is that there is no simple relationship between changes in the money supply and changes in the exchange rate. The effect of a change in the current money supply on the current exchange rate depends crucially on its effect on the expected future money supply. This implies that beliefs about the monetary policy reaction function are an important determinant of the contemporaneous link between exchange rates and money supplies. LECTURE10 further pursues this insight.

Recall that in the Classical model, nominal interest rates are determined by expected inflation. If individuals accurately link expected inflation to their anticipations of monetary policy, we can make more detailed predictions about the behavior of prices over time. This observation carries over immediately to the monetary approach to floating exchange rates. MAFER's assumption of purchasing power parity ensures that any predictions about the behavior of prices are also predictions about the behavior of the exchange rate.

Problems for Review

1. Use the method of undetermined coefficients to solve for an observable reduced form of the monetary approach to flexible exchange rates under rational expectations. Use the following DGP: $m(t) = 0.8m(t-1) + u(t)$ where $u(t)$ is white noise (i.e., is zero on average). Show all the steps in your solution procedure.

2. Use the method of undetermined coefficients to solve for an observable reduced form of the monetary approach to flexible exchange rates under rational expectations. Use the following DGP: $m_t = 0.8m_{t-1}+u_t$ where $u_t = \rho u_{t-1}+\varepsilon_t$ and ε_t is white noise. Make sure you *explicitly* solve for the undetermined coefficients in terms of the coefficients of the structural model. How does allowing u_t to follow an autoregressive process change the solution?

4.A Another Representation of the Algebra

Section 4.2.1 produced a rational expectations solution of the MAFER through repeated substitution. Use of the forward shift operator offers a simplified representation of the algebra. To reduce notational clutter in the present section, let $\mu = \lambda/(1+\lambda)$. Substituting into (4.16) produces an equivalent characterization of the spot exchange rate in terms of fundamentals and expectations.

$$s_t = (1 - \mu)\tilde{m}_t + \mu \mathcal{E}_t s_{t+1} \tag{4.49}$$

Since the expectation at time t of the current spot rate is just the current spot rate, and similarly for the current money supply, we can rewrite this as

$$\mathcal{E}_t s_t = (1 - \mu)\mathcal{E}_t \tilde{m}_t + \mu \mathcal{E}_t s_{t+1} \tag{4.50}$$

Now consider the ***forward-shift operator***, F, defined as $x_{t+n} = F^n x_t$. Use this operator to rewrite (4.50) as follows.

$$(1 - \mu F)\, \mathcal{E}_t s_t = (1 - \mu)\mathcal{E}_t \tilde{m}_t \tag{4.51}$$

As a naive algebraic manipulation, it is tempting to multiply both sides by the inverse of $(1 - \mu F)$. This would produce the following solution for the spot rate.[7]

$$\mathcal{E}_t s_t = (1 - \mu F)^{-1} (1 - \mu)\mathcal{E}_t \tilde{m}_t \tag{4.52}$$

Succumbing to this temptation proves permissible. Define the inverse to be[8]

$$\begin{aligned}(1 - \mu F)^{-1} &= 1 + \mu F + \mu^2 F^2 + \cdots \\ &= \sum_{i=0}^{\infty} \mu^i F^i\end{aligned} \tag{4.53}$$

Substitute this into (4.52) to produce a reduced form for the spot exchange rate.

$$\mathcal{E}_t s_t = (1 - \mu) \sum_{i=0}^{\infty} \mu^i \mathcal{E}_t \tilde{m}_{t+i} \tag{4.54}$$

Of course $\mathcal{E}_t s_t = s_t$, so (4.54) provides a rational-expectations solution for the spot rate. Naturally, it is equivalent to the solution (4.22), found by repeated substitution.

[7] More accurately, we would have

$$\mathcal{E}_t s_t = (1 - \mu F)^{-1} (1 - \mu)\mathcal{E}_t \tilde{m}_t + \eta \mu^{-t}$$

The term $\eta\mu^{-t}$ is permitted in the solution, since

$$(1 - \mu F)\, \eta \mu^{-t} = 0$$

However, this section ignores such “bubbles” in the solution. See section 4.2.1 for a discussion.

[8] Note that $(1 - \mu F)(1 + \mu F + \mu^2 F^2 + \cdots) = 1$.

4.B Direct Summation

This section shows how to produce the observable reduced form by direct summation.

$$
\begin{aligned}
s_t &= (1-\mu)\sum_{i=0}^{\infty}\mu^i \mathcal{E}_t \tilde{m}_{t+i} \\
&= (1-\mu)\sum_{i=1}^{\infty}\mu^i \mathcal{E}_t \tilde{m}_{t+i} + (1-\mu)\tilde{m}_t \\
&= (1-\mu)\sum_{i=1}^{\infty}\mu^i \left(\mu_0 \sum_{j=0}^{i-1}\mu_1^j + \mu_1^i \tilde{m}_t\right) + (1-\mu)\tilde{m}_t \\
&= (1-\mu)\mu_0 \sum_{i=1}^{\infty}\mu^i \sum_{j=0}^{i-1}\mu_1^j + (1-\mu)\sum_{i=0}^{\infty}\mu^i\mu_1^i \tilde{m}_t
\end{aligned}
\tag{4.55}
$$

So we have

$$
s_t = \phi_0 + \phi_1 \tilde{m}_t \tag{4.56}
$$

where ϕ_0 and ϕ_1 are the constant coefficients that are the infinite sums of structural form parameters in (4.55). It turns out that we can give much simpler representations of these coefficients.

Noting that

$$
\sum_{i=0}^{\infty}\mu^i\mu_1^i = \frac{1}{1-\mu\mu_1} \tag{4.57}
$$

we have

$$
\phi_1 = \frac{1-\mu}{1-\mu\mu_1} \tag{4.58}
$$

Finding ϕ_0 is a little more work. Let us begin with the inner summation.

$$
\sum_{j=0}^{i-1}\mu_1^j = \frac{1}{1-\mu_1}(1-\mu_1^i) \tag{4.59}
$$

Then we can write

$$
\phi_0 = \frac{1-\mu}{1-\mu_1}\mu_0 \sum_{i=1}^{\infty}\mu^i(1-\mu_1^i) \tag{4.60}
$$

Now note that

$$\begin{aligned}\sum_{i=1}^{\infty} \mu^i (1 - \mu_1^i) &= \sum_{i=1}^{\infty} \mu^i - \sum_{i=1}^{\infty} \mu^i \mu_1^i \\ &= \frac{\mu}{1-\mu} - \frac{\mu\mu_1}{1-\mu\mu_1} \\ &= \frac{\mu(1-\mu_1)}{(1-\mu)(1-\mu\mu_1)}\end{aligned} \tag{4.61}$$

So we get

$$\begin{aligned}\phi_0 &= \frac{1-\mu}{1-\mu_1}\mu_0 \frac{\mu(1-\mu_1)}{(1-\mu)(1-\mu\mu_1)} \\ &= \frac{\mu\mu_0}{1-\mu\mu_1}\end{aligned} \tag{4.62}$$

Substituting the above solutions for ϕ_0 and ϕ_1 yields an observable reduced form for the spot rate.

$$s_t = \frac{\mu\mu_0}{1-\mu\mu_1} + \frac{1-\mu}{1-\mu\mu_1}\tilde{m}_t \tag{4.63}$$

4.C Partial Adjustment of Money Demand

This chapter treats the current value of real money demand as a simple function of current income and interest rates. However, as noted in Lecture3, and as Bilson (1978b) and Woo (1985) emphasize, applied work on money demand generally uses this functional form only for *long-run* money demand. LECTURE3 shows that allowing for a partial adjustment characterization of money demand yields the following spot-rate equation.

$$s_t = q_t + (h_t - h^* t) - \phi(y_t - y_t^*) + \lambda(s_t^e - s_t - \text{rp}) - \alpha(h_{t-1} - h_{t-1}^*) + \alpha s_{t-1} - \alpha q_{t-1}$$

When combined with absolute purchasing power parity and uncovered interest parity, this produces a new exchange rate equation.

$$-\lambda s_t^e + (1+\lambda)s_t - \alpha s_{t-1} = (h_t - h_t^*) - \phi(y_t - y_t^*) - \alpha(h_{t-1} - h_{t-1}^*)$$

Woo follows Bilson (1978b) in incorporating lagged real balances in the money demand equation. Proceed with the solution under the assumption of rational expectations. Let $X_t^\top = (h_t - h_t^*, y_t - y_t^*)$, $a^\top = (-1, \phi)$, and $b^\top = (\alpha, 0)$. Then letting $\tilde{m}_t = a^\top X_t + b^\top X_{t-1}$ we can write

$$\begin{aligned}\lambda \mathcal{E}_t s_{t+1} - (1+\lambda)s_t + \alpha s_{t-1} &= -(h_t - h_t^*) + \phi(y_t - y_t^*) + \alpha(h_{t-1} - h_{t-1}^*) \\ &= a^\top X_t + b^\top X_{t-1} \\ &= \tilde{m}_t\end{aligned}$$

Now transform this by taking expectations at time t.

$$[\lambda F - (1+\lambda) + \alpha F^{-1}]\mathcal{E}_t s_t = \mathcal{E}_t \tilde{m}_t \tag{4.64}$$

With a slight abuse of notation, write the characteristic equation as the following quadratic equation.

$$\lambda F^2 - (1+\lambda)F + \alpha = 0$$

Solve this quadratic equation for its two solutions.

$$F_1, F_2 = \frac{1 + \lambda \pm \sqrt{(1+\lambda)^2 - 4\lambda\alpha}}{2\lambda}$$

Use these two solutions to rewrite (4.64) as follows.

$$(F - F_1)(F - F_2)\lambda F^{-1}\mathcal{E}_t s_t = \mathcal{E}_t \tilde{m}_t$$

This has the following general solution.

$$\lambda F^{-1}\mathcal{E}_t s_t = (F - F_1)^{-1}(F - F_2)^{-1}\mathcal{E}_t \tilde{m}_t + c_1 F_1^t + c_2 F_2^t \tag{4.65}$$

From the general solution, it is clear that the spot rate will tend to move explosively away from the fundamentals if either root is greater than unity in absolute value. Assuming that the parameters have the expected signs and magnitudes, $\lambda > 0$ and $\alpha \in (0, 1)$, there are two positive real roots.[9] In addition, the smaller root is less than unity while the larger root is greater than unity. Probably the easiest way to see this is to note that $dF_1/d\alpha > 0$ and $dF_2/d\alpha < 0$, and then consider the values of F_1 and F_2 at the extreme values of α.

Characteristic Roots: Relative Magnitudes

	$0 < \beta < 1$		$\beta \geq 1$	
	F_1	F_2	F_1	F_2
$\alpha = 0$	0	$1 + 1/\beta$	0	$1 + 1/\beta$
$\alpha = 1$	1	$1/\beta > 1$	$1/\beta \leq 1$	1

This is a situation of saddle-path instability. Economists deal with the instability deriving from F_2 by setting $c_2 = 0$. This is known as a ***transversality condition*** for the solution: it ensures that the exchange rate will approach its fundamentals in the long run. Putting it another way, it rules out explosive exchange rate bubbles. The transversality condition is serving another important role in this solution: without it, there is not enough information to determine a unique exchange rate. That is because there is only a single predetermined variable (s_{t-1}) for the second order difference equation (4.64). Highlight the role of this initial condition by multiplying (4.65) by $(F - F_1)$.

$$(F - F_1)\lambda F^{-1}\mathcal{E}_t s_t = (F - F_2)^{-1}\mathcal{E}_t \tilde{m}_t$$

[9]With $\lambda, \alpha > 0$ the requirement that $(1+\lambda)^2 - 4\lambda\alpha > 0$ is satisfied as long as $\lambda + 1/\lambda > 4\alpha - 2$. Given the nature of the partial adjustment mechanism (i.e., $\alpha < 1$), this is necessarily satisfied.

Noting $F_1 F_2 = \alpha/\lambda$, write this as follows.

$$(1 - \frac{1}{F_1}F)\alpha F^{-1}\mathcal{E}_t s_t = (1 - \frac{1}{F_2}F)^{-1}\mathcal{E}_t \tilde{m}_t$$

Equivalently, using summation notation,

$$\begin{aligned}
\mathcal{E}_t s_{t-1} - \frac{1}{F_1}\mathcal{E}_t s_t &= \frac{1}{\alpha}\sum_{i=0}^{\infty}\left(\frac{1}{F_2}\right)^i \mathcal{E}_t \tilde{m}_{t+i} \\
\mathcal{E}_t s_t &= F_1 \mathcal{E}_t s_{t-1} - \frac{F_1}{\alpha}\sum_{i=0}^{\infty}\left(\frac{1}{F_2}\right)^i \mathcal{E}_t \tilde{m}_{t+i} \qquad (4.66)
\end{aligned}$$

4.C.1 An Emprical Application

Woo (1985) estimates (4.66) jointly with (4.67) for the USD/DEM exchange rate over the period 1974.03–1981.10.[10] He does not reject the restrictions imposed by the rational expectations hypothesis, and his money demand parameters have the expected signs and plausible magnitudes. Although his estimated money demand parameters were not always statistically significant, his work was taken to be very supportive of the monetary approach. The following table reports his parameter estimates with and without an autocorrelation correction.

Woo (1985) Results

	Uncorrected	Corrected
λ	1.2964	1.2223
ϕ^G	0.5924	0.4713
ϕ^{US}	0.3466	0.2867
α	0.7991	0.8570

4.D The Data Generating Process: Detailed Analysis

A typical way of closing a rational expectations model is by assuming that the n exogenous variables follow a k^{th}-order vector autoregressive process (VAR) after differencing. The following treatment provides some general tools for implementing this closure.[11] Let X_t be the n-vector of exogenous variables at time t.

$$\Delta X_t = \sum_{j=1}^{k} B_j \Delta X_{t-j} + v_t \qquad (4.67)$$

[10]In his VAR for the exogenous variables, Woo includes six lags on relative money supplies and domestic income but five lags on foreign income. He runs the VAR in levels rather than in differences, and he detrends the data to achieve stationarity.

[11]Much of the development below follows Driskill et al. (1992) fairly closely.

The B_js are the $(n \times n)$ matrices $\{b_{ii',j}\}$, and v_t is an n-vector of serially uncorrelated errors. Equation (4.67) can be rewritten as a first-order VAR as follows.

$$\Delta Z_t = A\Delta Z_{t-1} + \delta_t \tag{4.68}$$

where,

$$Z_t^\top = (X_{1,t}, \ldots, X_{1,t-k+1}, X_{2,t}, \ldots, X_{2,t-k+1}, \quad \ldots \quad, X_{n,t}, \ldots, X_{n,t-k+1})$$
$$\delta_t = (v_{1,t}, 0, \ldots, 0, \underbrace{v_{2,t}}_{\substack{\text{element}\\ k+1}}, 0, \ldots, 0, \quad \ldots \quad, \underbrace{v_{n,t}}_{\substack{\text{element}\\ (n-1)k+1}}, 0, \ldots, 0)$$

and, A is the $nk \times nk$ matrix whose $(i-1)k + 1^{\text{th}}$ row is the column vectorization of the matrix formed by vertically concatenating the i^{th} row of all the B_js, with the rest of the elements zero except for the $k-1$ identity matrices beginning below each $b_{ii,1}$.

$$A = \begin{bmatrix}
b_{11,1} & . & . & . & b_{11,k} & b_{12,1} & . & . & . & b_{12,k} & \cdots & b_{1n,1} & . & . & . & b_{1n,k} \\
1 & 0 & . & . & 0 & 0 & . & . & . & 0 & \cdots & 0 & . & . & . & 0 \\
0 & 1 & . & . & 0 & 0 & . & . & . & 0 & \cdots & 0 & . & . & . & 0 \\
. & . & . & . & . & . & . & . & . & . & \cdots & . & . & . & . & . \\
0 & . & . & 1 & 0 & 0 & . & . & . & 0 & \cdots & 0 & . & . & . & 0 \\
b_{21,1} & . & . & . & b_{21,k} & b_{22,1} & . & . & . & b_{22,k} & \cdots & b_{2n,1} & . & . & . & b_{2n,k} \\
0 & . & . & . & 0 & 1 & . & . & . & 0 & \cdots & 0 & . & . & . & 0 \\
0 & . & . & . & 0 & 0 & 1 & . & . & 0 & \cdots & 0 & . & . & . & 0 \\
. & . & . & . & . & . & . & . & . & . & \cdots & . & . & . & . & . \\
0 & . & . & . & 0 & 0 & . & . & 1 & 0 & \cdots & 0 & 0 & . & . & 0 \\
 & & & & & & & \vdots & & & & & & & & \\
b_{n1,1} & . & . & . & b_{n1,k} & b_{n2,1} & . & . & . & b_{n2,k} & \cdots & b_{nn,1} & . & . & . & b_{nn,k} \\
0 & . & . & . & 0 & 0 & . & . & . & 0 & \cdots & 1 & . & . & . & 0 \\
0 & . & . & . & 0 & 0 & . & . & . & 0 & \cdots & 0 & 1 & . & . & 0 \\
. & . & . & . & . & . & . & . & . & . & \cdots & . & . & . & . & . \\
0 & . & . & . & 0 & 0 & . & . & . & 0 & \cdots & . & . & 0 & 1 & 0
\end{bmatrix}$$

We then calculate the j-step ahead linear least squares predictor $\mathcal{E}_t Z_{t+j}$ as follows. We can write,

$$\begin{aligned}
\mathcal{E}_t Z_{t+j} &= \mathcal{E}_t[Z_{t+j} - (Z_{t+j-1} + \ldots + Z_t) + (Z_{t+j-1} + \ldots + Z_t)] \\
&= \mathcal{E}_t[\Delta Z_{t+j} + \Delta Z_{t+j-1} + \ldots + \Delta Z_{t+1} + Z_t] \\
&= \sum_{i=0}^{j-1} \mathcal{E}_t \Delta Z_{t+j-i} + Z_t
\end{aligned} \tag{4.69}$$

Now from the data generating process (4.68), we know

$$\begin{aligned}\mathcal{E}_t\Delta Z_{t+j} &= A\mathcal{E}_t\Delta Z_{t-1+j} \\ &= A^2\mathcal{E}_t\Delta Z_{t-2+j} \\ &= \dots \\ &= A^j\Delta Z_t\end{aligned} \tag{4.70}$$

Combining (4.69) with equation (4.70) we get;

$$\begin{aligned}\mathcal{E}_t Z_{t+j} &= Z_t + \sum_{i=0}^{j-1} A^{j-i}\Delta Z_t \\ &= Z_t + \sum_{i=1}^{j} A^i\Delta Z_t\end{aligned}$$

Now [12]

$$\sum_{i=1}^{j} A^i = A(I - A^j)(I - A)^{-1}$$

$$\therefore\ \mathcal{E}_t Z_{t+j} = Z_t + A(I - A^j)(I - A)^{-1}\Delta Z_t \tag{4.71}$$

Equation (4.71) is the j-step ahead linear least squares predictor of Z_t.

Let G be a $(n \times nk)$ matrix of zeros, except for the elements g_{11}, $g_{2,k+1}$, $g_{3,2k+1}$, etc. which are equal to one, then $X_t = GZ_t$. Hence, using equation (4.71) we get the j-step ahead linear least squares predictor of X_t as follows.

$$\mathcal{E}_t(X_{t+j}) = GZ_t + GA(I_{nk} - A^j)(I_{nk} - A)^{-1}\Delta Z_t \tag{4.72}$$

[12] This is the analogue to the scalar result. Recall

$$\sum_{i=0}^{n} a^i = \frac{1 - a^{n+1}}{1 - a}$$

The sum from 1 to j is a times the sum from 0 to $j - 1$.

One more preliminary. It will be useful to note that

$$\begin{aligned}
A\sum_{j=0}^{\infty}\mu^j(I-A^j)(I-A)^{-1} &= A\left(\sum_{j=0}^{\infty}\mu^j I-\sum_{j=0}^{\infty}\mu^j A^j\right)(I-A)^{-1}\\
&= \left[\frac{1}{1-\mu}I-(I-\mu A)^{-1}\right]A(I-A)^{-1}\\
&= \frac{1}{1-\mu}(I-\mu A)^{-1}[(I-\mu A)-(1-\mu)I]A(I-A)^{-1}\\
&= \frac{1}{1-\mu}(I-\mu A)^{-1}[\mu(I-A)]A(I-A)^{-1}\\
&= \frac{\mu}{1-\mu}A(I-\mu A)^{-1}
\end{aligned} \tag{4.73}$$

Letting

$$C=\mu A(I-\mu A)^{-1} \tag{4.74}$$

we can express this result as

$$A\sum_{j=0}^{\infty}\mu^j(I-A^j)(I-A)^{-1}=\frac{1}{1-\mu}C \tag{4.75}$$

4.D.1 Observable Reduced Form

Revisis (4.54), the semi-reduced form exchange rate equation.

$$s_t=(1-\mu)\sum_{i=0}^{\infty}\mu^i\mathcal{E}_t\tilde{m}_{t+i}$$

Recall $\tilde{m}$ holds the exchange-rate fundamentals, especially money supplies and incomes, and $\mu=\lambda/(1+\lambda)$. fundamentals follow a k-th order VAR, as discussed above. In the notation of the previous section, the exchange rate solution is

$$s_t=(1-\mu)a'\sum_{i=0}^{\infty}\mu^i\mathcal{E}_t X_{t+i}$$

where

$$a=\begin{bmatrix}1\\-\phi\end{bmatrix}\qquad\text{and}\qquad X_t=\begin{bmatrix}h-h^*\\y-y^*\end{bmatrix}$$

From our results (4.72) and (4.75), we have

$$\begin{aligned}\sum_{i=0}^{\infty}\mu^i\mathcal{E}_t X_{t+i} &= \sum_{i=0}^{\infty}\mu^i[GZ_t + GA(I_{nk} - A^i)(I_{nk} - A)^{-1}\Delta Z_t] \\ &= \frac{1}{1-\mu}G(Z_t + C\Delta Z_t)\end{aligned}$$

We therefore have our solution for the exchange rate:

$$\begin{aligned}s_t &= a'G(Z_t + C\Delta Z_t) \\ &= a'X_t + a'GC\Delta Z_t\end{aligned} \tag{4.76}$$

This can be estimated simultaneously with (4.67). Note that while the equation for the spot rate is linear in the exogenous variables, it is non-linear in the structural parameters (because of C). So if you wish to estimate the structural parameters, you will need to use a method that can account for this non-linearity.

Lecture 5

Portfolio Balance Models of Floating Exchange Rates

The portfolio balance approach encompasses a diverse set of models. This chaper focuses on some common features shared by these models. In common with the monetary approach, portfolio balance models of flexible exchange rates focus on the role of asset stocks in the determination of exchange rates: short-run exchange-rate adjustments are determined in asset markets. There is an additional theme among these models: attention to the links between stocks of assets and saving flows.

The link between the stock of wealth (Ω) and the flow saving ($\mathcal{S}$) may be represented as follows.

$$\dot{\Omega}/P = \mathcal{S} \tag{5.1}$$

This step is generally taken in combination with a second link between wealth and saving (see Kenen (1985, p.672) for references).

$$\mathcal{S} = \mathcal{S}[\Omega/P] \qquad \mathcal{S}' < 0 \tag{5.2}$$

The open economy aspect derives from the presence of foreign assets in the domestic portfolio.[1] To keep the model simple, we will ignore terms of trade considerations by assuming purchasing-power parity.

The monetary approach to flexible exchange rates did not predict two salient events in the late 1970s.

- large deviations from purchasing-power parity
- the "stylized fact" that current account surplus countries had appreciating exchange rates

[1]The role of asset accumulation through the balance of payments was central to the monetary approach to the balance of payments. The concept of saving that we will use in this chapter is similar to the concept of "hoarding" used in the monetary approach to the balance of payments. In fact, except for the tendency of the portfolio balance tradition to treat interest rates and multiple assets explicitly, there is little to separate the portfolio balance and monetary approaches to the balance of payments. However, when the monetary approach was applied to flexible exchange rates, the emphasis on stock flow links was abandoned. These were re-introduced only with the portfolio balance models of flexible exchange rates.

We will give a separate analysis of the Dornbusch overshooting model, which can account for the large deviations from purchasing-power parity. That model can also accommodate to some extent the second stylized fact. For example, starting from a zero trade balance a monetary shock generates a large depreciation that throws the trade balance into surplus, and the surplus persists as the real exchange rate appreciates toward the new equilibrium. However if we start from a current account deficit, a positive monetary shock may simply reduce the deficit in the short run without affecting it in the long run. In the simple overshooting model, asset accumulation through the current account has no cumulative effect on the economy. (The steady state of this model can therefore include a current account balance of any magnitude.)

Portfolio balance models attempt to give a more robust explanation of the observed link between the current account and exchange rate movements. We will develop a particularly simple version of the model, which focuses on this issue. As in the simple monetary approach model to flexible exchange rates, we will assume constant PPP.[2] We introduce one key deviation from our monetary approach model: we assume that money demand depends on wealth as well as income. The resulting model predicts that full equilibrium requires current account balance. This extends the predictions of the simple monetary approach model, which makes no such prediction.

This portfolio balance model lends new emphasis to the current account by introducing the influence of asset accumulation on asset demand. (Under rational expectations, expected future current account balances should also affect S because they affect expected future asset accumulation.)

5.1 A Partial Equilibrium Model

This section introduces a simple partial equilibrium approach due to Kouri (1982). Let Ω be nominal wealth in domestic currency units, and let α be the fraction of wealth demanded as foreign assets. Then $\alpha\Omega$ is the domestic demand for foreign assets. Asset markets clear only if this demand equals the available supply, SNFA. Given i, i^*, and Ω, (5.3) characterizes exchange rate determination.

$$S\,\mathrm{NFA} = \alpha\Omega \tag{5.3}$$

In Figure 5.2, we draw a rectangular hyperbola in (S, NFA)-space. Along this curve, the domestic currency value of net foreign asset holdings is constant ($S\mathrm{NFA} = \alpha\Omega$). For each level of foreign assets, we can see the market clearing exchange rate. Here we see the role of the assets markets in exchange rate determination illustrated very clearly. The exchange rate adjusts so that the current portfolio is willingly held. Current net foreign assets are therefore a primary determinant of the spot exchange rate.

Portfolio balance provides a reasonable description of exchange rate determination in the short-run, when asset accumulation is negligible compared to the stock of existing assets. Over time, however, asset stocks change. A dynamic model must account for the

[2] Dornbusch and Fischer (1980) and Isaac (1989) discuss a variant with endogenous terms of trade.

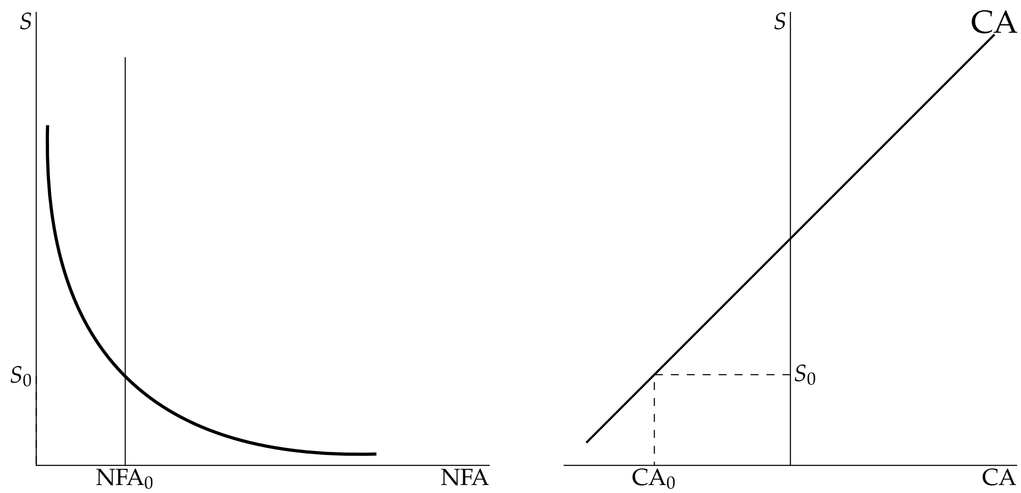


Figure 5.1: Portfolio Balance Model (Partial Equilibrium)

influence of these changes. For now, focus on accumulation of net foreign assets through the balance of payments.

The exchange rate is a determinant of the current account. The second chart in Figure 5.2 plots the current account as a function of the exchange rate.[3] The upward sloping line in (CA, S)-space represents the level of the current account for each level of the exchange rate. In combination with the first chart, this implies a link between net foreign assets and the current account. Current asset holdings determine the exchange rate, which influences the current account. Since asset accumulation takes place through the current account, this results in a dynamic interaction between the current account and the exchange rate. For example, a current account deficit draws down net foreign assets, depreciating the exchange rate, and thereby eventually eliminating the deficit.

5.2 General Equilibrium Model

The partial-equilibrium framework of the previous section illustrates some core considerations in a portfolio balance model of exchange rate determination. This section repairs some of the most obvious shortcomings of a partial-equilibrium approach. For example, it explicitly incorporates exchange-rate effects on the nominal value of wealth. More critically, it recognizes that the asset market equilibrium locus of Figure 5.2 should shift over time in response to any accumulation or decumulation of net foreign assets. The central task of the present section is to incorporate these changes in a more complete portfolio-balance model of exchange-rate determination.

In the short run, the portfolio balance model of this chapter is essentially the same as the monetary approach model of Lecture3. There is one crucial change: real money

[3] For the moment, ignore changes in prices, which the next section addresses. The upward slope may reflect satisfaction of the Marshall-Lerner condition; see Lecture7 for details. However, the mechanism in this chapter involves asset-pricing effects, as described in the next section.

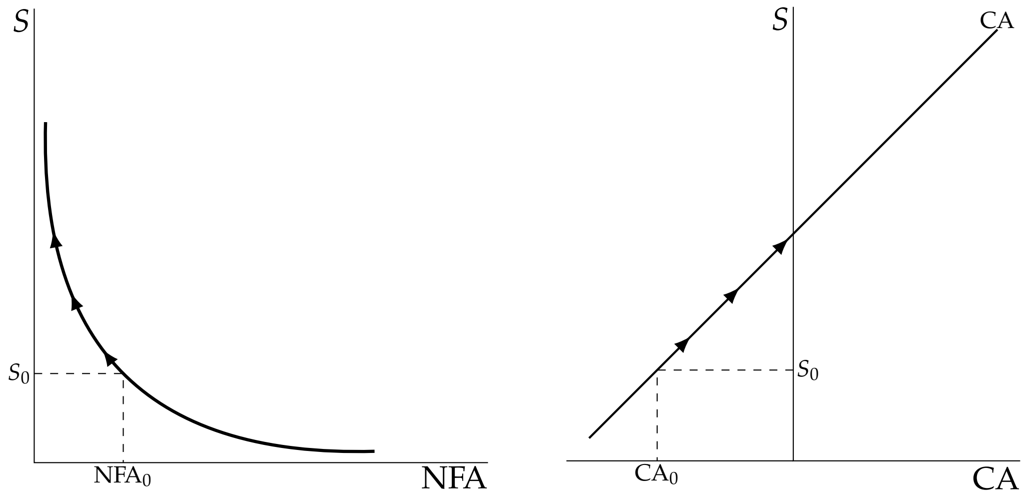


Figure 5.2: Dynamic Adjustment in the Partial-Equilibrium Model

demand depends on real wealth (Ω/P).

$$\frac{H}{P} = L\left[i, Y, \frac{\Omega}{P}\right] \qquad L_i < 0,\, L_Y > 0,\, 1 > L_{\Omega/P} > 0 \tag{5.4}$$

Aside from (5.4), this portfolio-balance model uses the standard components of a monetary approach model: full employment, interest parity (perfect capital mobility, and even perfect capital substitutability), and purchasing-power parity. This chapter focuses on a small open economy with perfect capital mobility. To simplify the notation, assume perfect capital substitutability as well, to that uncovered interest parity holds.

In this portfolio-balance model, the assets available to domestic residents are two: domestic money, and an internationally traded bond. Since this model is explicit about the alternative to holding money, it must specify its characteristics. For now, consider a risk-free real ***perpetuity***, which pays one unit of foreign output forever.

Domestic real wealth equals real money balances plus the real value of net foreign assets. If r is the real interest rate, then each perpetuity is worth $1/r$. Recall that purchasing-power parity ensures real interest parity. Letting a be the number of such perpetuities owned by domestic residents, domestic real wealth can be written as follows.

$$\frac{\Omega}{P} = \frac{H}{P} + \frac{a}{r} \tag{5.5}$$

Substituting this into the money market equilibrium condition produces (5.6).

$$\frac{H}{P} = L\left[i^* + \Delta s^e, Y, \frac{H}{P} + \frac{a}{r}\right] \tag{5.6}$$

For notational simplicity, since the real exchange rate is not the focus of this model, assume purchasing-power parity holds and is expected to continue to hold. Also to keep the model presentation simple, set foreign inflation to zero.

$$S = QP/P^* \tag{5.7}$$
$$\Delta s^e = \pi^e \tag{5.8}$$

Define the real interest rate by $r = i - \pi^e$; then $r = i - \Delta s^e$ by ***expected*** PPP. (Here i is the nominal interest rate.) We will let r be the exogenously given real interest rate. This allows us to characterize the nominal interest rate as follows.

$$\begin{aligned} i &= r + \pi^e \\ &= r + \Delta s^e \end{aligned} \tag{5.9}$$

Referring again to (5.6), this implies that money market equilibrium condition can be expressed as follows.

$$\frac{H}{P} = L\left[r + \Delta s^e, Y, \frac{H}{P} + \frac{a}{r}\right] \tag{5.10}$$

Equation (5.10) implicitly defines a functional dependence of real balances on r, Δs^e, and a. As a convenience in the following analysis, partially reduce the money-market equilibrium (5.10) to isolate real balances. Let the resulting function be $x[r, \Delta s^e, Y, a]$, and henceforth, refer to x as *real money demand*.

Determine the responses of x as follows. Consider the effects of a rise in expected depreciation, starting from a situation of money market equilibrium. This unambiguously creates excess supply in the money market (i.e., lowers money demand) ceteris paribus. This can be offset by a fall in H/P *as long as* an additional dollar of wealth generates less than an additional dollar of real money demand. Similarly, the excess demand created by an increase in a can be offset by an increase in H/P. Ceteris paribus, a rise in r involves both of the previous effects: a fall in money demand due to higher interest rates, and a fall in money demand due to lower real wealth. (The real value of the perpetuities is reduced by higher interest rates.)

Equation (5.11) summarizes these arguments and presents the money market equilibrium condition in a simpler way. Let subscript indicate the responsiveness to a single variable, ceteris paribus. For notational simplicity, x_e denotes the response of x to a change in expected depreciation.

$$\frac{H}{P} = x[r, \Delta s^e, Y, a] \qquad x_r < 0, x_e < 0, x_Y > 0, x_a > 0 \tag{5.11}$$

Recalling that goods market equilibrium is determined simply by purchasing power parity, $P = SP^*$, we have a simple static determination of the exchange rate in Figure 5.3.

In Figure 5.3, the LM curve represents the money market equilibrium (5.11). It is vertical, because no matter what the current exchange rate is, there is a unique price level that can clear the money market. (Of course this would change if the current level of the exchange rate had implications for its expected future level, as under the regressive expectations hypothesis, but for now we treat Δs^e as exogenous.) The purchasing-power parity locus is a ray with slope $1/P^*$.

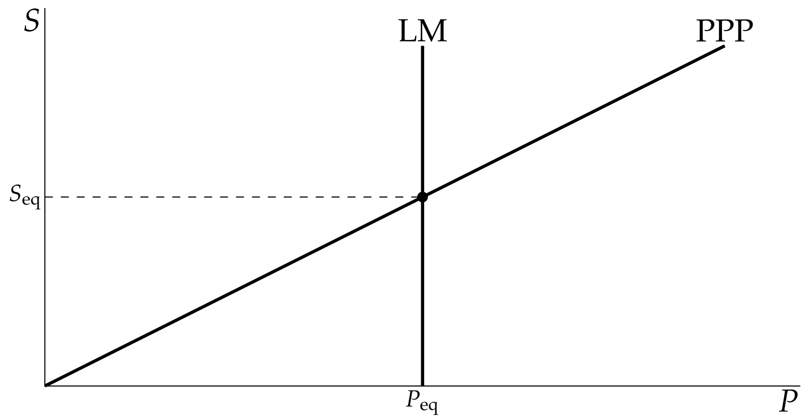


Figure 5.3: Static Equilibrium
Static equilibrium in the simple portfolio balance model.
LM: $H/P = x[r, \Delta s^e, Y, a]$; PPP: $S = P/P^*$.

5.3 Static Predictions

The basic predictions of our model are determined by comparative statics experiments. Graphically, our comparative statics experiments will be represented by shifts of the LM curve. Consider a one-time permanent increase in the money supply, as represented in Figure 5.4. An increase in H increases the equilibrium price level proportionately. This is represented by a rightward shift in the LM curve. In the new equilibrium, the price level and the exchange rate have risen proportionately. This result is familiar to us from our work with the monetary approach.

Other comparative statics experiments involve money demand instead of money supply. Any reduction in money demand raises the equilibrium price level, and can therefore also be represented by Figure 5.4. For example, a rise in Δs^e increases the domestic interest rate and thereby decreases money demand. The effects are a rise S and P proportional to the fall in money demand. Of course this is also compatible with the monetary approach.

Also compatible with our monetary approach analysis is the interest rate effect of a change in r: a rise in r raises the domestic interest rate and reduces real money demand. However, the effect of a change in r has been slightly complicated by its role in determining the real value of net foreign assets. The rise in r reduces the real value of net foreign assets. Since this again reduces real money demand, there is no qualitative difference from the monetary approach model.

Finally, we have a new influence on money demand: a. If there is an decrease in our holding of net foreign assets, this decreases the real demand for money (by decreasing wealth). As always, the price level and exchange rate must increase in proportion to the fall in real money demand. Thus this experiment can also be represented by Figure 5.4.

We can also proceed algebraically. Recall that given the foreign price level, P^*, under purchasing-power parity the rate of change of the price level equals the rate of change of the exchange rate: $\pi = \Delta s$. For notational simplicity, assume absolute purchasing-power parity: the real exchange rate is constant at 1.0, so that $P = SP^*$. Combine this with our

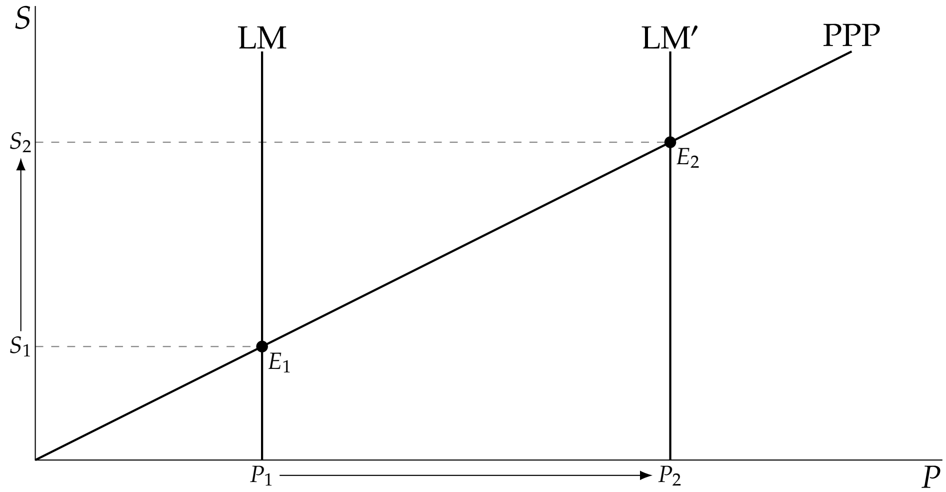


Figure 5.4: Comparative Statics of the Portfolio Balance Model
PPP line: $S = P/P^*$. LM curves: $H/P = x[r, \Delta s^e, Y, a]$. An increase in H, r, or Δs^e, or a decrease in a, shifts the LM curve to the right.

simplified money market equilibrium condition to get the following.

$$\frac{H}{SP^*} = x[r, \Delta s^e, Y, a] \tag{5.12}$$

Then solve for the exchange rate.

$$S = \frac{H}{x[r, \Delta s^e, Y, a]\, P^*} \tag{5.13}$$

This is the basic portfolio-balance story about exchange rate determination at each point in time. (The next section pays more attention to expectations, so Δs^e will become endogenous.) Again, note the dependence of S on a. This provides the dynamic link between the exchange rate and the current account.

Response to Net Foreign Assets

As net foreign assets accumulate, money demand rises, and correspondingly prices must fall. By purchasing-power parity, falling prices must be accompanied by a falling spot rate: the domestic currency must appreciate. Figure 5.5 illustrates these effects in the PPP-LM framework used above. Additionally, it separately illustrates the implied negative relationship between accumulated net foreign assets (a) and the equilibrium exchange rate. This is the PB curve, which represents portfolio balance in (a, S)-space.

5.4 Dynamics

This section introduces the model dynamics. As suggested above, these will be driven by savings behavior. This chapter adopts a ***target-wealth*** saving function (Metzler, 1951).

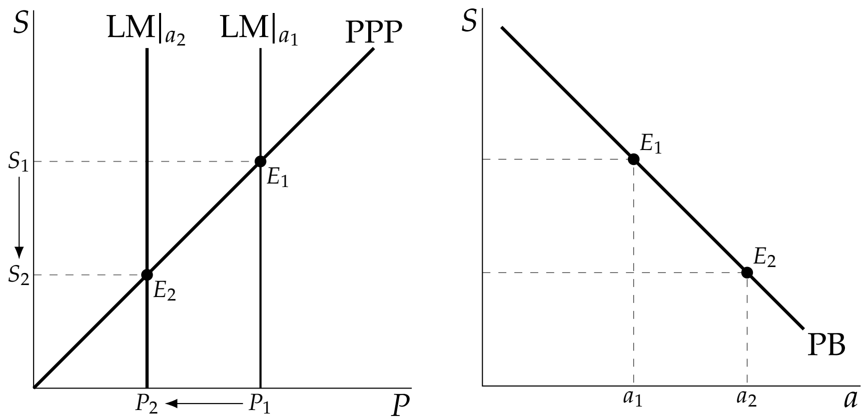


Figure 5.5: Exchange-Rate Effects of Net Foreign Assets
An increase in net foreign assets ($a_1 > a_2$) raises money demand, causing the price level to fall and (via PPP) depreciating the currency. The PB curve summarizes this.

In this conveniently simple framework, desired saving flows are not adjusted for any anticipated capital gains or losses on existing assets.

Recall that the only financial assets available in this simple portfolio-balance model are domestic money and the internationally traded bond. The definition of wealth along with absolute purchasing-power parity produces (5.14).

$$\begin{aligned} \mathcal{S}\left[\frac{\Omega}{P}\right] &= \mathcal{S}\left[\frac{H}{P}+\frac{a}{r}\right] \\ &= \mathcal{S}\left[\frac{H}{SP^*}+\frac{a}{r}\right] \end{aligned} \tag{5.14}$$

The following analysis takes place with a constant money supply. The lack of money growth implies that desired saving can be satisfied only by the accumulation of of the internationally tradable asset. Recall that this asset is a real perpetuity, leading to the following accounting relation between actual saving the current account.

$$\mathcal{S} = \frac{\dot{a}}{r} \tag{5.15}$$

Equating actual to desired saving yields the dependence of asset accumulation on H, S, P^*, a, and r.

$$\frac{\dot{a}}{r} = \mathcal{S}\left[\frac{H}{SP^*}+\frac{a}{r}\right] \tag{5.16}$$

Given the other variables, (5.16) establishes a relationship between $\dot{a}$ and a. This relationship is a first-order differential equation: it relates asset accumulation to the current level of assets. Figure 5.6 explores this dynamic relationship graphically. To do so, follow the usual rule in thinking about dynamics: start by characterizing the nullclines. (A nullcline is a locus of points where dynamic adjustment stops.)

The following graphical analysis takes place in (a, S)-space. The a-nullcline comprises

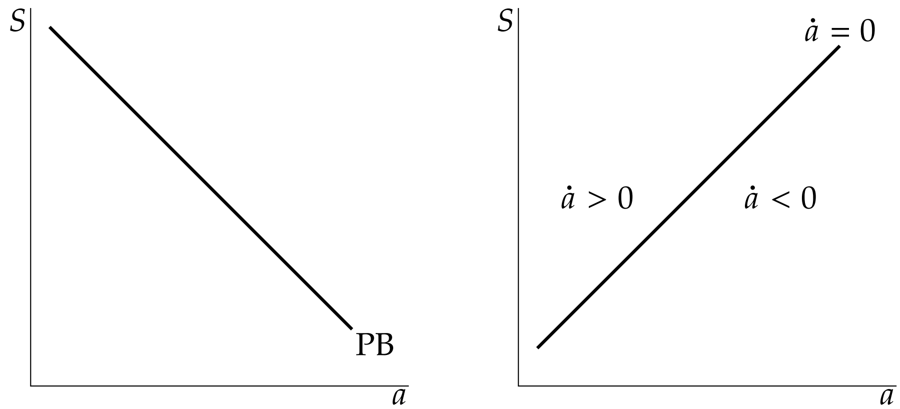


Figure 5.6: Portfolio Balance and Asset Dynamics
PB: $H/(SP^*) = x[r, \Delta s^e, Y, a]$, with exogenous expected depreciation. $\dot{a} = \mathcal{S}[H/(SP^*) + a/r]$. The economy is always in portfolio balance, but saving causes dynamic portfolio adjustments.

the combinations of a and S such that $\dot{a} = 0$. Since $\dot{a}$ is determined by saving which depends on wealth, the a-nullcline is a constant wealth locus. Increases in a increase wealth, while increases in S decrease wealth (by reducing real balances). Therefore the a-nullcline is upward sloping in (a, S)-space.

Once we have determined the nullcline, we can easily address the dynamic adjustments to its right or left. Pick any point on the nullcline, where saving is zero, and move to the right. This increases wealth, so saving must fall, leading to asset declines. (Remember that $\mathcal{S}' < 0$!) So to the right we have $\dot{a} < 0$. Next start at the same point on the nullcline, but this time move to the left. This decreases wealth, so saving must rise, leading to asset increases. So to the left of the nullcline we find $\dot{a} > 0$.

This is because a rise in a will raise wealth, and this will reduce saving. To keep saving at zero, we must have an offsetting rise in P, which will reduce wealth by reducing real balances. Given purchasing-power parity, we therefore require increases in S to offset increases in a so as to maintain $\dot{a} = 0$. This relationship is represented in Figure 5.6: it is $\dot{a} = 0$ locus seen in this figure. Together these two loci summarize the dynamic behavior of the economy in this portfolio balance model.

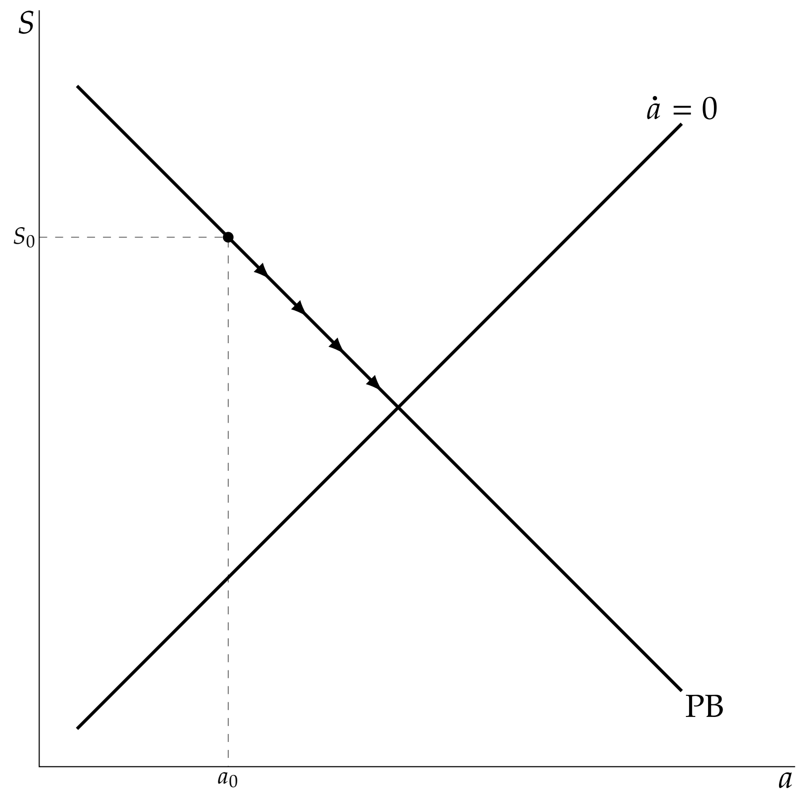


Figure 5.7: Exchange-Rate Dynamics in Portfolio Balance
PB: $H/(SP^*) = x[r, \Delta s^e, Y, a]$, with exogenous expected depreciation. $\dot{a} = \mathcal{S}[H/(SP^*) + a/r]$. The economy is always in portfolio balance, but saving causes dynamic portfolio adjustments.

We begin our dynamic story with the historically given level of net foreign assets. In Figure 5.10 this is labelled a_0. At this level of net foreign assets, there is a unique exchange rate that clears the assets markets. This is the exchange rate we determined in our LM–PPP analysis of Figure 5.3, and in Figure 5.10 we label it S_0. Our discussion of asset dynamics tells us that the low real wealth at point a_0, S_0 implies that this is a point of net asset accumulation. As we accumulate net foreign assets through current account surplusses, a increases and the exchange rate appreciates. Thus the model predicts the negative correlation between the current account and exchange rate depreciation that emerged as a stylized fact in the 1970s. The adjustment continues along the PB curve until we reach the $\dot{a} = 0$ locus.

Comment: Note that the stability of the model dynamics can easily determined by combining (5.18) and (5.19) to get

$$\frac{\dot{a}}{r} = \mathcal{S}\left[x[r, \Delta s^e, Y, a] + \frac{a}{r}\right] \tag{5.17}$$

Since $\mathcal{S}' < 0$, we know $d\dot{a}/da < 0$, and the model is stable.

We can also algebraically examine the effects of changes in the foreign interest rate, the expected rate of depreciation, or the money supply. Here (5.18) of our PB curve, and we use (5.19) to determine the $\dot{a} = 0$ locus.

$$S = \frac{H}{x[r, Y, \Delta s^e, a]P^*} \qquad \text{PB curve} \tag{5.18}$$

$$\frac{\dot{a}}{r} = \mathcal{S}\left[\frac{H}{SP^*} + \frac{a}{r}\right] \tag{5.19}$$

5.4.1 Some Thought Experiments

Consider the following experiments (starting from long-run equilibrium):

Receive a one-time wealth transfer (increase a): Neither curve shifts, but we move to a new position on PB and then slowly adjust back to old position.

Monetary expansion (increase H): Both curves shift up proportionately; see the PB-aa chart in Figure 5.9. There is complete monetary neutrality (no real changes).

This should look familiar from our work on the monetary approach model. Review Figure 5.4 for the effect in the PPP-LM framework. Since prices adjust flexibly, there is no further adjustment needed.

Fiscal expansion: We do not have an explicit fiscal variable, but we can treat this as a fall in national saving. So where we used to have $\dot{a} = 0$, we now have $\dot{a} < 0$. The aa-curve shifts left. We move along the old PB curve until we reach the new equilibrium at a reduced level of a.

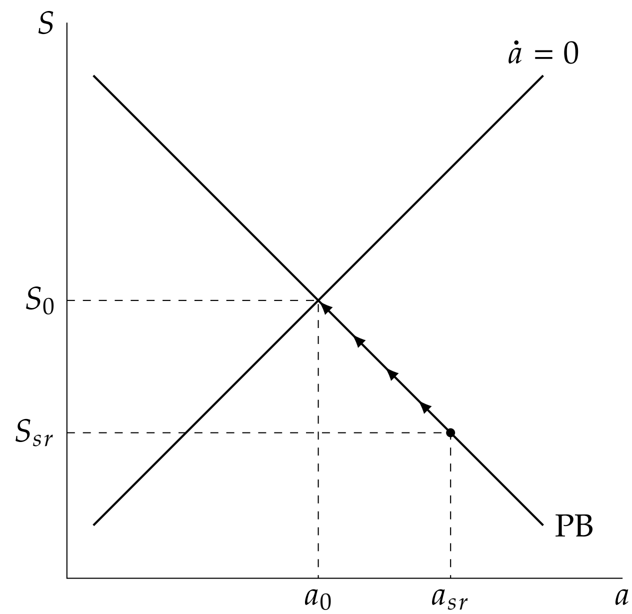


Figure 5.8: Short-Run and Long-Run Effects of Wealth Transfer
Simple flexprice portfolio balance model under static expectations.
$\dot{a} = \mathcal{S}[H/(SP^*) + a/r]$; PB: $H/(SP^*) = x[r, \Delta s^e, Y, a]$.

Increase Δs^e: An expected depreciation generates an actual depreciation.

Start with the PPP-LM representation in Figure 5.11. The second experiment should look familiar at the beginning: our work on the monetary approach model also showed us that an expected deprecitation produces an actual depreciation.

However, we now get a subsequent effect on the current account. We have lower real wealth due to the lower real balances caused by the rise an prices (an corresponding depreciation). We therefore save toward our target wealth. As we accumulate wealth money demand rises, driving prices back down. This continues until our original level of wealth is restored. However, the composition of wealth has changed: we now have more a and somewhat less $\frac{H}{P}$.

Note that the current account improves: the depreciation reduces wealth (via real balances), which increases saving (and thus the current account). A period of appreciation and declining current account balance follows.

One thing that may make us uncomfortable with the second experiment is the idea that expected depreciation will remain constant as the exchange rate continues to change over time. We will address this with a rational expectations analysis.

5.5 Portfolio Balance with Rational Expectations

When working with a deterministic model, economists interpret rational expectations as exact knowledge of the path implied by the rules of motion. This pins down expected

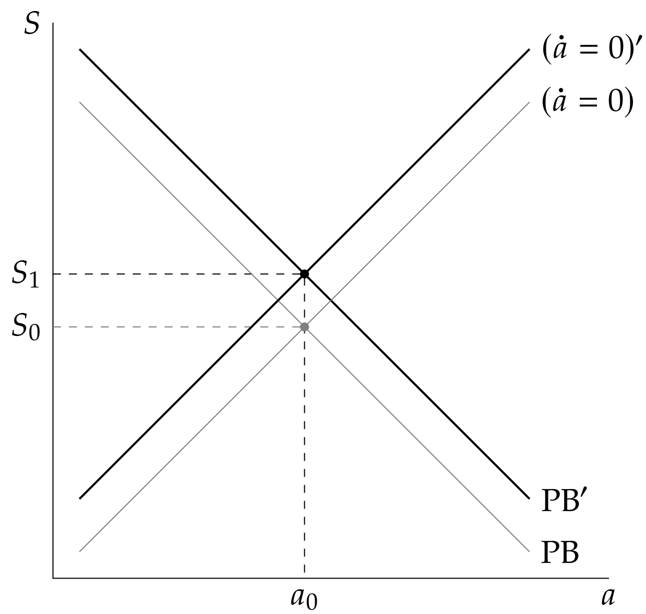


Figure 5.9: Increase in H

depreciation under rational expectations:

$$\Delta s^e = \dot{S}/S \tag{5.20}$$

This changes the dynamic system, as seen in (5.21).

$$\begin{aligned} \frac{H}{SP^*} &= x\left[r, \frac{\dot{S}}{S}, Y, a\right] \\ \frac{\dot{a}}{r} &= \mathcal{S}\left[\frac{H}{SP^*} + \frac{a}{r}\right] \end{aligned} \tag{5.21}$$

The graphical representation looks just like Figure 5.10. (See problem 4.)

To approach the algebra, we will construct linear approximation to our dynamic system around the steady state. This yields the following linear first-order differential equation system in the variables δa and δS.

$$\begin{aligned} \frac{H}{SP^*} &= x\left[r, \frac{\dot{S}}{S}, Y, a\right] \\ \frac{\dot{a}}{r} &= \mathcal{S}\left[\frac{H}{SP^*} + \frac{a}{r}\right] \end{aligned} \tag{5.21}$$

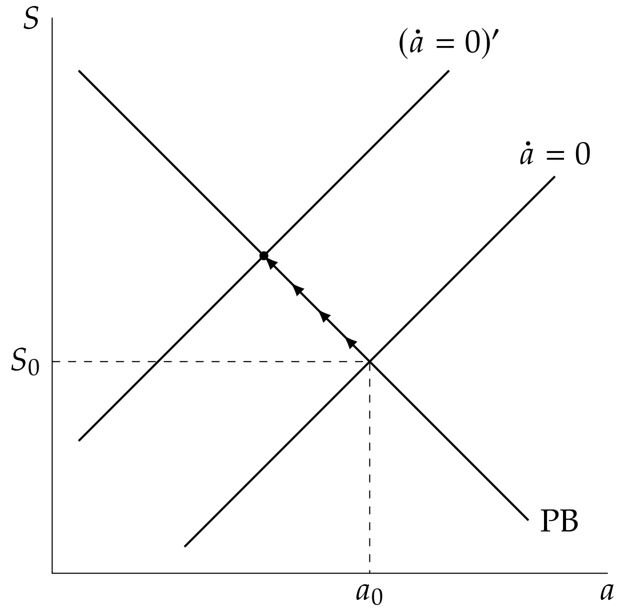


Figure 5.10: Short-Run and Long-Run Effects of Fiscal Expansion
Simple flexprice portfolio balance model under static expectations.
$\dot{a} = \mathcal{S}[H/(SP^*) + a/r]$; PB: $H/(SP^*) = x[r, \Delta s^e, Y, a]$.

$$
\begin{aligned}
-\frac{H}{S^2P^*}\,\delta S &= x_e\left[\frac{1}{S}\,\delta\dot{S} - \frac{\dot{S}}{S^2}\,\delta S\right] + x_a\,\delta a \\
\frac{1}{r}\,\delta\dot{a} &= -\mathcal{S}_w\frac{H}{S^2P^*}\,\delta S + \mathcal{S}_w\frac{1}{r}\,\delta a
\end{aligned}
\tag{5.22}
$$

Recall that we are linearizing around the steady state. Since the money supply is constant, we know $\dot{S} = 0$ at the steady state. This slightly simplifies our expression of the system. We will get a further simplification by using the differential operator, so that we can write $\dot{\delta a}$ as $D\,\delta a$ and $\dot{\delta S}$ as $D\,\delta S$. This allows us to write our dynamic system as

$$
\begin{aligned}
-\left(\frac{H}{S^2P^*} + x_e\frac{1}{S}D\right)\,\delta S - x_a\,\delta a &= 0 \\
(D - \mathcal{S}_w)\,\delta a + \mathcal{S}_w\frac{rH}{S^2P^*}\,\delta S &= 0
\end{aligned}
$$

We get a slight additional simplification by giving this system a matrix representation.

$$
\begin{bmatrix} -\frac{H}{S^2P^*} - x_e\frac{1}{S}D & -x_a \\ \mathcal{S}_w\frac{rH}{S^2P^*} & D - \mathcal{S}_w \end{bmatrix}\begin{bmatrix} \delta S \\ \delta a \end{bmatrix} = \mathbf{0}
\tag{5.23}
$$

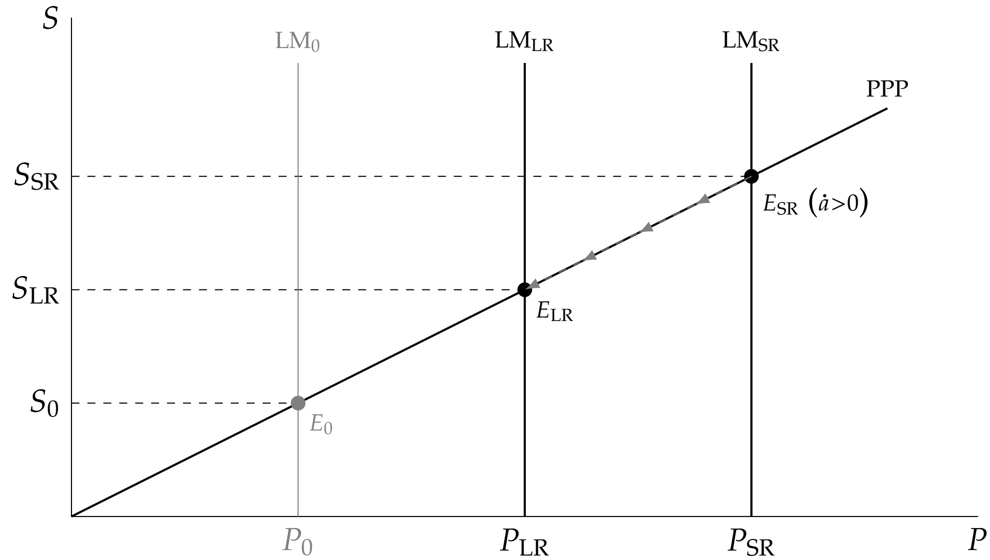


Figure 5.11: Increase in Δs^e (static expectations)

In summary, we have

$$\boldsymbol{P}[D]\begin{bmatrix}\delta S\\ \delta a\end{bmatrix} = \mathbf{0} \tag{5.24}$$

We are going to solve this system with the adjoint matrix technique. The first step is to find the characteristic roots.

In forming the characteristic equation of this system, we will slightly abuse notation: we now treat D as a variable. This allows us to write the characteristic equation as

$$|\boldsymbol{P}[D]| = 0 \tag{5.25}$$

In more detail, this is

$$-\frac{H}{S^2P^*}D + \frac{H}{S^2P^*}\mathcal{S}_w - x_e\frac{1}{S}D^2 + x_e\frac{1}{S}\mathcal{S}_w D + x_a\mathcal{S}_w\frac{rH}{S^2P^*} = 0 \tag{5.26}$$

Rearrange to get

$$D^2 + \left(\frac{H}{x_eSP^*} - \mathcal{S}_w\right)D - \mathcal{S}_w\frac{H}{SP^*}\frac{1+x_ar}{x_e} = 0 \tag{5.27}$$

This is a quadratic equation, so we will find two characteristic roots. Since the last term is negative, we know we our two characteristic roots are real and opposite in sign. That is, we have a convergent saddle-path. Let $D_1 < 0 < D_2$ denote these roots.

Recall that we are working with the matrix operator $\boldsymbol{P}[D]$ where

$$\boldsymbol{P}[D] = \begin{bmatrix}-\frac{H}{S^2P^*} - x_e\frac{1}{S}D & -x_a\\ \mathcal{S}_w\frac{rH}{S^2P^*} & D - \mathcal{S}_w\end{bmatrix} \tag{5.28}$$

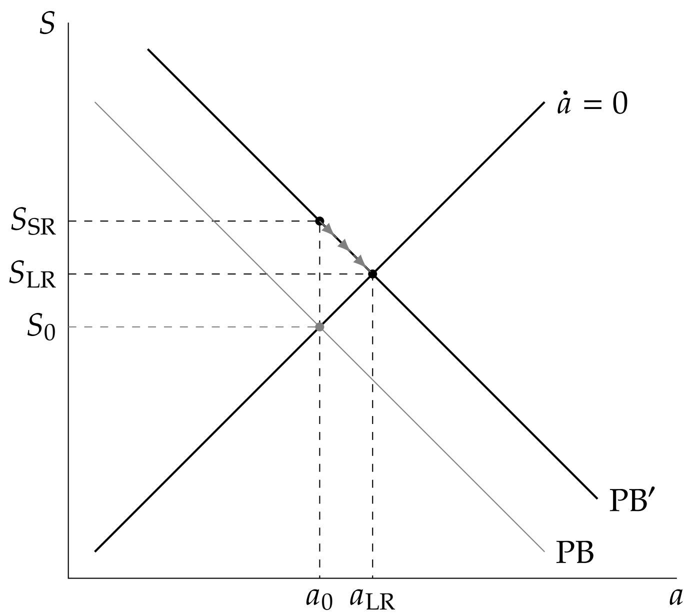


Figure 5.12: Increase in Δs^e

The adjugate (or "adjoint") matrix is therefore

$$\boldsymbol{P}^{\#}(D) = \begin{bmatrix} D - \mathcal{S}_w & x_a \\ -\mathcal{S}_w \frac{rH}{S^2 P^*} & -\frac{H}{S^2 P^*} - x_e \frac{1}{S} D \end{bmatrix} \tag{5.29}$$

The adjoint-matrix technique states the solution to this system in terms of the characteristic roots and a column of the adjugate of $\boldsymbol{P}[D]$.[4]

$$\begin{bmatrix} \delta S \\ \delta a \end{bmatrix} = \eta_1 \begin{bmatrix} D_1 - \mathcal{S}_w \\ -\mathcal{S}_w \frac{rH}{S^2 P^*} \end{bmatrix} e^{D_1 t} + \eta_2 \begin{bmatrix} D_2 - \mathcal{S}_w \\ -\mathcal{S}_w \frac{rH}{S^2 P^*} \end{bmatrix} e^{D_2 t} \tag{5.30}$$

Recall that the values of η_1 and η_2 can be determined if we are given the initial state of the system $(\delta a_0, \delta S_0)$. But since $D_2 > 0$, this solution will diverge unless we set $\eta_2 = 0$. This is our transversality condition, and the resulting solution is

$$\begin{bmatrix} \delta S \\ \delta a \end{bmatrix} = \eta_1 \begin{bmatrix} D_1 - \mathcal{S}_w \\ -\mathcal{S}_w \frac{rH}{S^2 P^*} \end{bmatrix} e^{D_1 t} \tag{5.31}$$

Now we have only one constant to determine, so we only need one initial condition. We use the value of δa_0 to determine the value of η_1. This is motivated by the economic interpretation of the model: only the value of a is given to us as an historically predetermined variable. In contrast, S is a jump variable, and the transversality condition ensures that it jumps to the convergent arm of our dynamic system.

[4]Since $D_1 < 0$ and $\mathcal{S}_w < 0$, it may seem that we have failed to sign $D_1 - \mathcal{S}_w$. However, a little algebra shows this is negative.

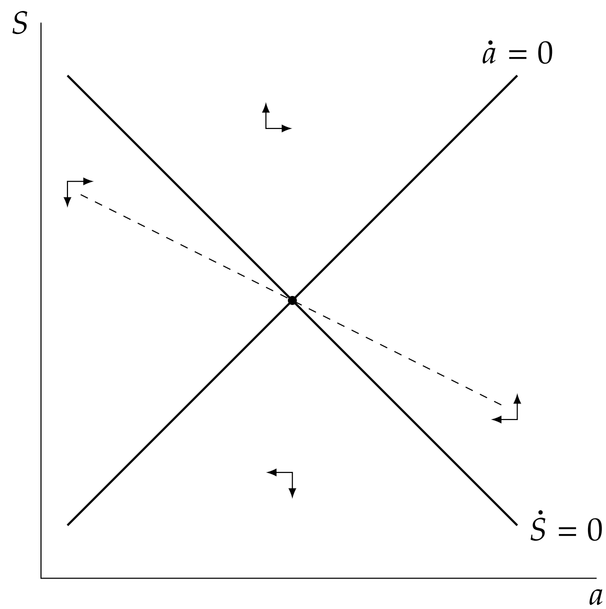


Figure 5.13: Rational Expectations Dynamics

5.6 Portfolio Balance with Endogenous Risk Premia

Like the monetary approach, the portfolio balance model has up to this point treated the risk premium as exogenously given. However some portfolio balance models link the risk premium to asset supplies. The portfolio balance approach treats the risk premium as endogenous, related to the supply of domestic and foreign assets.

$$\text{rp} = \frac{1}{\gamma}(b_t - s_t - b_t^*)$$

The covered interest rate parity condition can then be rewritten as the portfolio balance equation (5.32).

$$b_t - s_t - b_t^* = \gamma(i_t - i_t^* - \Delta s_t^e) \tag{5.32}$$

If we follow the Frankel solution procedure (imposing purchasing-power parity only in the long run) using the portfolio balance equation instead of the interest parity condition, we solve for the exchange rate as

$$s_t = \frac{\gamma\theta}{1+\gamma\theta}[h_t - h_t^* - \phi(y_t - y_t^*) + (\lambda + \frac{1}{\theta})(\overline{dlPde} - \overline{dlPfe}) - \frac{1}{\theta}(i_t - i_t^*) + \frac{1}{\gamma\theta}(b_t - b_t^*)]$$

The term in brackets is just our previous solution for the exchange rate, except for the new term in the relative bond supply. So portfolio balance models differ from the monetary approach models in predicting that relative bond supplies are an important determinant of the exchange rate. Additionally, the predicted coefficient on relative money supplies is now less than unity.

Problems for Review

1. Derive the signs of the partial derivatives of $x[\cdot,\cdot,\cdot,\cdot]$ from the money market equilibrium condition (5.10).

2. Provide graphs and intuition for all of our comparative static experiments under static expectations.

3. Analyze the stability dynamic adjustment process under static expectations by substituting
$$\frac{H}{SP^*} = x[r, \Delta s^e, Y, a]$$
into the following equation
$$\frac{\dot{a}}{r} = \mathcal{S}\left[\frac{H}{SP^*} + \frac{a}{r}\right]$$

4. Under rational expectations, we will have $\Delta s^e = \dot{S}/S$. Draw the phase diagram, carefully explaining the slope of the convergent saddle-path. Apply the adjoint matrix technique to
$$\frac{H}{SP^*} = x\left[r, \frac{\dot{S}}{S}, Y, a\right]$$
$$\frac{\dot{a}}{r} = \mathcal{S}\left[\frac{H}{SP^*} + \frac{a}{r}\right]$$
in order to analyze the model dynamics under rational expectations.

5. Do the long-run comparative statics algebra for a change in H.
Hint: use our equations of motion, but set $\dot{S} = 0$ and $\dot{a} = 0$.

- Branson, William, “Asset Markets and Relative Prices in Exchange Rate Determination,” *Sozialwissenschaftliche Annalen* 1, 1977, 69–89.

- Frankel, Jeffrey, “Monetary and Portfolio Balance Models of Exchange Rate Determination,” in J. Frankel, *On Exchange Rates* (Cambridge, MA: MIT PRess, 1993).

- Frankel, J.A., 1983, “Monetary and Portfolio-Balance Models of Exchange Rate Determination,” in J. Bhandari and B. Putnam (eds), *Economic Interdependence and Flexible Exchange Rates* (Cambridge, MA: MIT Press)

5.A Other Aspects of the Portfolio-Balance Model

5.A.1 Why Might UIP Fail

The results of Eichenbaum and Evans (1995) suggest that uncovered interest-rate parity model does not always work well. In fact, there sometimes appears to be systematic deviations from its predictions for long periods of time.

One reason for its failure may be its underlying assumption that investors are indifferent between, say, a 3% return in their home currency and a 2% return in a foreign currency combined with a 1% appreciation of that currency. There are a few reasons to imagine investors may prefer the 3% return in their home currency:

- Risk Aversion: Suppose the returns above are on risk-free bonds, then the 3% domestic currency investment is risk-free, while the 3% total return (including expected capital gain) from the foreign currency is risky. We are exposed to exchange rate risk.

- Transaction Costs: Buying foreign currency and using it to buy foreign bonds may involve greater transaction costs.

Therefore consider a model in which home and foreign currency assets are not perfect substitutes. Additionally, allow for *home bias*: investors prefer to have a majority of their assets in their home currency. This produces a simple portfolio balance model of exchange rates. The model is a simplified version of Blanchard et al. (2005).

- two countries (the US and the Rest of the World)

- domestic and foreign assets are not perfect substitutes

- home bias

We want the model to explore connections between exchange rates, trade deficits and net foreign asset holdings. For example, we want to predict the short-run and long-run effects of a shift in world portfolio demands toward dollar-denominated assets.

Notation:

- X = Value in dollars of dollar-denominated assets.

- X^* = Value in foreign currency of non-dollar-denominated assets.

- F = Net asset position of ROW relative to the US in dollars: dollar assets owned by ROW minus foreign assets owned by US.

- S = Amount of foreign current bought by one dollar

- $W = X - F$ = US Wealth denominated in dollars

- $W^* = X^* + F$ = Rest of world wealth denominated in foreign currency.

- α = Desired share of US assets in US portfolios.

- α^* = Desired share of ROW assets in ROW portfolios.

5.A.2 Desired Portfolios and the Exchange Rate

Once again, the exchange rate adjusts to equilibrate the asset markets. Equating the total supply (X) to the total demand for dollar-denominated assets, we get

$$X = \alpha(X - F) + (1 - \alpha^*)(X^* S + F) \tag{5.33}$$

$$(1 - \alpha)(X - F) + \alpha^* F = (1 - \alpha^*) S X^* \tag{5.34}$$

This means S is determined by

$$S = \frac{(1 - \alpha)X + (\alpha + \alpha^* - 1)F}{(1 - \alpha^*)X^*} \tag{5.35}$$

Our solution implies that changes in the net international investment position—i.e., change in F through the current account—alter the exchange rate. For example, an increase in F causes a depreciation of the domestic currency as long as both countries have home bias in their asset demands (so that $\alpha + \alpha^* > 1$).

Of course, our desired portfolio composition (represented by α and α^*) can also affect S, as can changes in domestic and foreign asset supplies. These changes may all be inconsistent with uncovered interest parity.

For example, consider an increase in demand for US assets, represented by a rise in α^*. This can happen if US returns are perceived to increase: α^* may fall but a gap in expected returns may remain. Or α^* may fall because of lower transaction costs in holding US assets, say due to the financial liberalizations of the 1990s. Indeed, we saw during this period an increase in demand for dollar-denominated assets, particularly after financial disasters in Russia, Asia, and South America. Our solution (5.35) suggests that this decline in α^*? increases the denominator and reduces numerator F is positive. That is, if the US is a net debtor to rest of the world, as it has been by many measures since the mid-1980s. So a decline in α^* appreciates the domestic current. This may to explain the dollar appreciation in the late 1990s and the late noughts.

5.A.3 Exchange Rates, Trade Deficits and Wealth

Exchange rates play a role beyond equating supply and demand for assets denominated in various currencies. They affect trade flows. Consider the effect of an increase in the value of the dollar relative to the euro:

- European firms get more euros from exporting their goods to the US. Or they can keep the amount of euros obtained the same but boost sales by cutting their dollar prices in the US.

- US firms get fewer dollars back for each Euro earned from exporting to Europe, making exporting less attractive. Thus, an appreciation of the dollar (increase in $1/S$) will tend to increase imports to the US and reduce exports from the US, thus increases the US trade deficit.

But trade deficits are related to wealth: If the US buys more from the world than it sells back then this implies an worsening of the net asset position of the US, i.e. an increase in F.

5.A.4 Wealth Shifts, Home Bias and the Exchange Rate

An increase in the exchange rate thus leads to a trade deficit which increases F. How does this affect the exchange rate? Our model

$$1/S = \frac{1-\alpha^*)X^*}{(1-\alpha)X + (\alpha + \alpha^* - 1)F} \tag{5.36}$$

tells us that it depends on whether $\alpha + \alpha^* - 1$ is positive or negative. Studies show, however, that investors tend to display home bias such that most people want most of their assets to be denominated in their home currency.

This means that $\alpha > 0.5$ and $\alpha^* > 0.5$ so $\alpha + \alpha^* - 1 > 0$. Trade deficits lead to increases in F and this leads to a decline in the value of the dollar.

So, a shift in desired portfolios that initially raises the value of the dollar, then triggers a redistribution of wealth that leads to the dollar subsequently declining. This pattern of trade deficits and a declining value of the dollar did indeed follow the appreciation of the late 1990s.

5.A.5 The Exchange Rate in the Long-Run

So an increase in foreign demand for US assets leads to an appreciation of the dollar followed by a depreciation. Which effect wins in the end? Consider the evolution over time of US net asset position. Let r be the average interest rate on the US debt and let D be the US trade deficit. Then over time we have

$$F_t = (1+r)F_{t-1} + D_t \tag{5.37}$$

The debt can only be stabilized ($\Delta F_t = 0$) if $D_t = -r\,F_t$.

In other words, to stop US net debt going off to infinity, in the long run the US will have to run a trade surplus that pays off the interest payments on the debt.

In the current US case, years of large trade deficits are leading to a large net debt position (positive F) so trade surpluses will be required. Thus, the positive shift in foreign demand for US assets, while initially causing the dollar to appreciate will, in the longer-run, lead to a larger depreciation.

Lecture 6

Purchasing Power Parity

A key ingredient of the flexprice monetary approach and related models of flexible exchange rates is the assumption that the real exchange rate (Q) is exogenous. Under this exogeneity assumption, relative price levels determine a corresponding exchange rate rate according to (6.1). In this chapter we explore the underpinnings of (6.1) and consider some empirical tests.

$$S = Q\frac{P}{P^*} \tag{6.1}$$

Learning Goals

After reading this chapter, you will understand:

- the real exchange rate and its relationship to purchasing power parity.
- ways in which supply and demand influence the real exchange rate even in the long run.
- the relationship between commodity price parity and purchasing power parity.
- how prices and exchange rates are related in the long run.

6.1 Commodity Price Parity

Recall that ***arbitrage*** is the simultaneous purchase of something where it is cheap and sale of it where it is dear. Chapter Lecture2 discusses how spatial arbitrage ensures that exchange rates are essentially identical in geographically dispersed markets. Similarly, when it is not costly to spatially arbitrage a commodity, that commodity should be equally affordable in geographically dispersed markets.

For example, U.S. dollars into euros and then buying gold in Paris should not yield more gold than simply buying it in the U.S. If spatial arbitrage were costless for all commodities, the country of residence would have no effect on the purchasing power of a given income, regardless of the currency of denomination.

Begin by considering an extreme case. Imagine a good that is truly costless to transport.

Domestically this means that it must sell for the same price at every location. That is, the ***law of one price*** must hold for this good. This must also be true internationally. If the good is entirely costless to transport, and if there are no barriers to international trade in the good, then it must be equally costly to acquire at every location. This implies a link between the price of the good in different currencies and the exchange rates between those currencies.

Suppose good i can be costlessly arbitraged, and it can be bought and sold for P_i domestically and P_i^* abroad. Then the value of the exchange rate is implicit in these prices. For unless

$$P_i = SP_i^* \tag{6.2}$$

there will be an opportunity for profitable arbitrage. For example, if $P_i < SP_i^*$, an arbitrageur could buy the good domestically for P_i in domestic currency, simultaneously sell it abroad for P_i^* in foreign currency, and finally sell the foreign currency for SP^*. This would yield a risk-free profit in the domestic currency.

Spatial arbitrage would add to demand where the good is cheap and to supply where the good is dear. Arbitrage activity should continue until the equality (6.1) holds internationally across all of the markets in this commodity. Thus, arbitrage activity enforces the equality (); we say that prices differences have been arbitraged away.

Domestically, spatial arbitrage ensures that commodities obey the *law of one price*. Internationally, when arbitrage is costless and currencies are freely traded, arbitrage ensures that the equality (6.1) holds continually. This is just an international version of the law of one price; it does not matter that different currency denominations are involved. Equation (6.1) is known as ***commodity price parity*** (CPP). Commodity price parity is just the law of one price applied internationally.

If commodity price parity applied to all commodities, a given income would buy the same goods in any location. In this sense, a consumer's country of residence would not affect her consumption opportunities. Changes in exchange rates would not imply changes in consumption possibilities. Equivalently, under universal CPP, consumers face no exchange rate risk.

Example:
Date: 27 March 2018
Cost of one ounce of gold in New York: USD 1351
Cost of one ounce of gold in London: GBP 955
Cost of one GBP (in both locations): USD 1.41
Gold satisfies CPP, since $1351 \approx 1.41 * 955$.

6.1.1 Barriers to Commodity Arbitrage

Even within a single country, arbitrage activity is not costless, so the law of one price never holds exactly.[1] Transportation costs and restraints of trade will mean that the

[1]See O'Connell and Wei (2002) for evidence on the tendency toward satisfaction of the law of one price in the United States.

costs of delivering a commodity will differ at different geographical locations. These cost differentials will influence pricing, and therefore transportation costs and restraints of trade are two likely sources of deviations from the law of one price. This may be even more important internationally: transportation costs and restraints of trade can lead to substantial deviation from CPP (Engel and Rogers, 2001).

Transportation Costs

Transportation costs are a natural barrier to trade. Large countries and countries with poorly developed transportation infrastructure may experience large deviations in prices across spatial locations. Even in the presence of transportation costs, such as shipping and insurance fees, arbitrage limits the divergence of prices. The prices can differ by as much as the transportation costs without consequence, but when the price difference exceeds the transportation costs, incentives arise for arbitrage activity.

In order to compare foreign and domestic prices, we use a common currency. For example, the U.S. dollar roughly doubled in value against the German mark in the early 1980s, but automobile prices in the U.S. and Germany did not fully reflect this exchange-rate movement. As a result, people in the U.S. began to buy automobiles in Germany and ship them home. However the extent of such arbitrage was modest, and it failed to eliminate the price differential.[2]

Some goods and services are subject to extremely high transportation costs relative to their value. Consider trying to transport haircuts or housing services over any significant distance.[3] Even large international price discrepancies will not offset the transactions costs required for arbitrage. These goods are considered to be *non-traded*.

Non-Traded Inputs

Some non-traded goods are inputs into the production of other goods. In addition, some factors of production are not cheaply transportable across locations. This means that the costs of inputs into the production of traded goods will diverge between geographical locations, even within a country. (For example, land in New York city is going to be costly.)

It does not follow that prices of traded goods will also differ. If the traded goods are easily arbitraged, price differences at different locations will be limited. This may affect the profitability of producing goods at different locations, when input costs vary geographically.

Restraints of Trade

Many different kinds of trade restraints affect the arbitrage of goods and services. Any institution that retards the spatial arbitrage of commodities may be viewed as a restraint

[2]See Caves, Frankel, and Jones (1996, p.429) for additional discussion.

[3]Haircuts are a traditional example of a non-traded good, but in 2002 a student told me that some Austrians were crossing into the Czech Republic to visit (much cheaper) hair stylists.

on trade. Within a country this includes prohibitions on the provision of professional services across regional boundaries (as in the state-specific licensing of lawyers in the U.S.) as well as laws designed to allow differing regional rates of commodity taxation. International trade barriers include quotas, import tariffs, and export duties: when successfully enforced, these allow domestic and foreign prices to diverge. As with transportation costs, tariffs create a band of possible divergence between the domestic and foreign prices (Samuelson, 1964). In the extreme case, when trade barriers or transportation costs are prohibitively high, the band of possible divergence becomes so wide that geographical arbitrage never takes place. Once again, such commodities will be *non-traded*, and large divergences between the domestic and foreign prices may arise.[4] Successfully enforced quotas prevent arbitrage of more than a fixed quantity, again allowing large international price differentials. Agricultural products are a classic example of goods whose arbitrage is prevented by high tariff barriers.

Some laws are specifically designed to limit entry or to segment markets. Just as these can prevent arbitrage within a country, they can prevent international arbitrage. Licensing laws are an example. For example, foreign lawyers cannot provide domestic legal services unless they acquire domestic credentials. When domestic and foreign markets are segmented, so that a firm faces differing degrees of market power in different markets, a firm may price its product differently at home and abroad.

Imperfect Competition

When a firm produces a commodity for which there are no close substitutes, it may exercise market power. It is no longer a price-taker: it can affect the price of its product. However, by itself this does not mean that the firm can force deviations from CPP. In order to price the product differently in different geographical locations, the firm must not face arbitrage activity that would move the good from low price locations to high price locations. Therefore transportation costs and barriers to trade will remain the key determinants of deviations from CPP, even in the presence of market power.

For example, in the mid-1990s New Zealand decided to permit parallel imports. A ***parallel import*** occurs when an imported good is legally purchased in one country and then imported into another market. Parallel imports are a method of circumventing the exporter's intended distribution channels. New Zealand's new policy allowed importers to bring in brand name goods for which they did not have a franchise, thus undermining the ability purveyors of these goods to segment markets in order to maintain high prices. At the time, the United States Trade Representative voiced strong opposition to this move, taking the stance that parallel imports facilitate piracy and complicate the enforcement of intellectual property rights.

[4] In principle, even the prices of non-traded goods can show some tendency to equality, since they may be substitutes for traded goods in consumption and may be produced with common or closely substitutable factors of production.

6.2 Purchasing Power

What is the real ***purchasing power*** of a given nominal income? For a consumer of only a single good, there is be a simple answer to this question: *deflate* the consumer's nominal income by dividing it by the price of that good, thereby determining the number of units of the good that income will buy. Even in the oversimplified setting, a departure from commodity price parity implies that a consumer's geographical location may affect her purchasing power.

More realistically, consumers purchase many goods. Economists therefore approach the question of purchasing-power by constructing a ***price index***. A price index is essentially a weighted average of many different prices. Economists assess the purchasing power of consumers by constructing a price index based on a typical consumption basket. For example, to determine the purchasing power of nominal personal income, we may deflate it by the consumer price index (*CPI*). This measures the ability of consumers to purchase the collection of goods that was used to construct this *CPI*.

6.2.1 Price Indexes

The result of deflating nominal income by an appropriate price index is called ***real income***. Very roughly, the price index is the domestic currency cost of a typical consumption basket, so the purchasing power of a given nominal income is measured as the number of these baskets that income can purchase.

Any price index should satisfy certain basic ***properties***. Consider a price index based on a basket of n commodities, as follows.

$$CPI = f[P_1, \dots, P_n] \tag{6.3}$$

If the price of any commodity in the basket increases, the price index should increase. We call this property ***monotonicity***. Another very important property such a price index should have is ***homogeneity***: if *every* commodity in the basket doubles in price, then the price index should double. Similarly, if all prices are cut in half, the price index should be halved.[5]

Homogeneity means that equiproportional movements in all prices lead to a proportional movement in the price index. Given CPP, so that $P_i = SP_i^*$ holds for every commodity in the basket, homogeneity may tightly link foreign and domestic price indexes.

$$\begin{aligned} CPI &\stackrel{\text{def}}{=} f[P_1, \dots, P_n] \\ &\stackrel{\text{CPP}}{=} f[SP_1^*, \dots, SP_n^*] \\ &\stackrel{\text{hom}}{=} Sf[P_1^*, \dots, P_n^*] \end{aligned} \tag{6.4}$$

Construct the foreign price index in the identical fashion as the domestic price index, so

[5]More precisely, the price index is homogeneous of degree one in the commodity prices. A good consumer price index should also be concave, representing the ability of consumers to substitute among commodities, but not all price indexes satisfy concavity.

that

$$CPI^* = f[P_1^*, \ldots, P_n^*] \tag{6.5}$$

Then (under CPP) we discover a relationship that looks very much like CPP:

$$CPI = S\ CPI^* \tag{6.6}$$

We call this relationship ***absolute*** purchasing power parity. Absolute purchasing power parity says that countries have equal price *levels* when expressed in a common currency. Equivalently, absolute purchasing power parity implies that the spot rate equals the relative price level.

$$S = CPI/CPI^* \tag{6.7}$$

6.3 Absolute Purchasing Power Parity

The idea that exchange rates should be linked to national price levels has a long history (Einzig, 1970). The classic statement, however, was given by Cassel (1918, p.413).

> The general inflation which has taken place during the war has lowered this purchasing power in all countries, though in a very different degree, and the rates of exchanges should accordingly be expected to deviate from their old parity in proportion to the inflation of each country.
>
> At every moment the real parity between two countries is represented by this quotient between the purchasing power of the money in the one country and the other. I propose to call this parity "the purchasing power parity." As long as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries takes place, the actual rate of exchange cannot deviate very much from this purchasing power parity.

Cassel is suggesting that the spot rate between two currencies should equal the relative price level between the two countries, a proposition we have called absolute purchasing power parity.[6] The previous section demonstrates that absolute PPP is implied when foreign and domestic price indexes are identically constructed and commodity price parity holds for all commodities in the "market basket" on which the price indexes are based. These are very restrictive conditions. For many commodities, commodity price parity does not hold. In addition, price index construction differs widely among countries.

6.3.1 The Big Mac Standard

Since 1986, *The Economist* has published yearly international price comparisons for McDonald's "Big Mac" sandwich. The composition of the Big Mac is generally uniform, making it an internationally comparable "basket of commodities" that is an attractive candidate for PPP comparisons. In addition, most of the ingredients of the Big Mac are individually

[6] As noted by Isard (1995, p.58), Cassel later allowed for short-run deviations from PPP.

traded as standardized commodities on international markets. For such commodities, we expect the law of one price to hold at least approximately.

Furthermore, Pakko and Pollard (1996) show that relative Big Mac prices are highly correlated with the PWT measure. This is illustrated by in table 6.1. The Penn World Table (PWT) constructs price indexes based on a common market basket of about 150 commodities (Summers and Heston, 1991), which should address the first of these concerns. Nevertheless, as seen in the second column of table 6.1, considerable deviations from purchasing power parity are evident.

Country	P/SP^{US} (PWT)	P/SP^{US} (Big Mac)
Australia	0.98	0.86
Belgium	1.13	1.29
Britain	1.10	1.32
Canada	1.04	0.90
Denmark	1.40	1.85
France	1.19	1.42
Germany	1.27	1.14
Hong Kong	0.75	0.51
Ireland	1.06	1.00
Italy	1.22	1.29
Japan	1.41	1.25
Netherlands	1.14	1.24
Singapore	0.91	0.70
Spain	1.08	1.50
Sweden	1.51	1.91

Source: Pakko and Pollard (1996, p.5)

Table 6.1: Two PPP Indicators (1991)

6.3.2 Explaining the Deviations

We have seen that there are two basic sources of deviation from absolute PPP: deviations from CPP (trade barriers, including transportation costs), and differences in the construction of domestic and foreign price indexes. By focusing on the Big Mac, we have tried to limit the relevance of the composition of the price index. (We will return to this.) Further, transportation costs do not immediately suggest themselves as an explanation of the Big Mac parity deviations, for as noted above the Big Mac ingredients are internationally traded standardized commodities. However it is a mistake to identify brand name products with their ingredients: what is being sold is really the product bundled with a reputation. If the Big Mac has no close substitutes, the ability to trade the ingredients may not offset the difficulty in transporting the final product. McDonald's may be able to "price to market" because of the lack of competition in some markets. Such explanations based on imperfect competition have gained in popularity. For example, Feenstra and

Kendall (1994) find significant PPP deviations trace to the incomplete pass-through of exchange rate changes into final goods prices. Finally, as pointed out by O'Connell and Wei (2002), franchise restaurant meal have a significant input of nontraded intermediate inputs. Indeed, citetong-1997-jimf estimates that non-traded goods constitute 94% of the prices of a Big Mac.

6.3.3 Commodity Price Parity

Isard (1977) examines the law of one price using wholesale and export price data on relatively disaggregated groups of manufactured goods. He looks at the U.S., Japan, and Germany for 1970–1975. Perhaps surprisingly, even at his level of disaggregation the law of one price fails, and nominal exchange rate changes affect relative prices. Kravis et al. (1978a) disaggregate even further and obtain similar results. The finding that nominal exchange rate changes affect disaggregated relative prices for traded goods should make us pessimistic about PPP for aggregate price indexes. And indeed, the correlation between real and nominal exchange rates is extremely strong.

Further, the ingredients may be subject to legal restrictions on trade. For example, most countries restrict agricultural imports with tariffs and quotas. Pakko and Pollard point out that Korea looks consistently overvalued on the Big Mac standard, and it maintains high barriers against beef imports. Such import restrictions will tend to raise prices in the importing country, making its currency look overvalued. Similarly, differing tax practices are another possible source of deviation: the Big Mac prices reported by *The Economist* are inclusive of taxes. Countries with higher taxes will therefore appear to have overvalued currencies. Pakko and Pollard give an example of this effect: when Canada imposed its national seven percent sales tax in 1992, the price of a Big Mac rose by about the same percentage.

6.4 Real Exchange Rates and Purchasing Power

In an ordinary year you exchange domestic money for goods, and as a result your annual expenditures are naturally measured in units of the national currency. If you spend a year abroad, you use foreign money to make your daily purchases. In this case your annual expenditures are naturally measured in units of that national currency. How might we compare your expenditures at home and abroad?

We might simply take your foreign expenditures and multiply them by the exchange rate, S, in order to find out the domestic currency value of your spending. However, this procedure will not tell us much about your standard of living while you were abroad. Although individuals must make monetary exchanges, their central concern is their control over goods and services. It would be nice to have a simple way to represent the material standard of living that your expenditures in each country represent. For this, we need a way to transform your *nominal* expenditures (measured in currency units) into *real* expenditures. While there is no perfect way to do this, economists generally turn nominal into real expenditures by *deflating* nominal expenditures with a consumer price index.

We will make our first approach to the real exchange rate in a very special setting. Suppose you are considering spending a year abroad. Let's construct your domestic consumer price index P as the cost in domestic currency of your current consumption basket. Let us then construct your foreign consumer price index P^* as the cost abroad (in foreign currency) you would have to incur to be *just as happy* as you would be with your current consumption basket. So each price index is simply the monetary cost of a specific consumption basket. While the consumption basket is different in each country, you would be equally happy with either basket.

Suppose you are a U.S. student considering a year abroad, and that you have USD 15k for living expenses. In the U.S., this will afford you a material standard of living of USD 15k$/P$. If you spend the year abroad, you will have USD 15k$/S$ in foreign currency to spend. Of course you are likely to buy different things abroad than at home: these things constitute your foreign consumption basket. Your material standard of living abroad will therefore be (USD 15k$/S$)$/P^*$. Your *relative* material standard of living, Q, is therefore given by (6.8).

$$Q = \frac{SP^*}{P} \tag{6.8}$$

This is just the ratio of your material standard of living at home to your material standard of living abroad. If Q rises, it becomes more of a sacrifice for you to live abroad.

6.5 The Purchasing Power Parity Doctrine

The relationship (6.6) between domestic and foreign price levels is known as absolute PPP. It should be clear that absolute PPP depends on very special circumstances: we arrived at (6.6) by assuming CPP (for every commodity) *and* that the domestic and foreign price indexes were constructed identically. Since both of these assumptions seriously misrepresent the relationship between the price indexes of different nations, we cannot expect absolute PPP to hold between actual price indexes.

However the real exchange rate may nevertheless be stable. Recall our definition of the real exchange rate from chapter Lecture3.

$$Q = S\frac{P^*}{P} \tag{6.9}$$

The *purchasing power parity doctrine* holds that, in the long run, Q is a constant determined by real economic activity (along with any trade barriers or transportation costs). While Q may fluctuate in the short-run, the PPP doctrine says that these fluctuations will be corrected in the long run. The nominal exchange rate therefore has a fixed long-run relationship to relative national price levels via the constant equilibrium real exchange rate.

$$S = Q\frac{P}{P^*} \tag{6.10}$$

Note that we are not saying that relative price levels *cause* the exchange rate, but rather that there is an equilibrium relationship between relative price levels and the exchange rate.

6.6 Policy Applications

Two prominent policy applications of purchasing power parities are the setting of exchange rate parities and the international comparison of real incomes. The appropriate price index may be quite different in the two cases. For the setting of exchange rate parities, we want to find a purchasing power parity that is a good measure of the equilibrium exchange rate. It is quite plausible that the underlying price indexes for this purpose should heavily weight traded goods. For the international comparison of real incomes, we are interested in price indexes that will appropriately reflect differences in the cost of living across countries. We expect that the underlying price indexes for this purpose should heavily weight non-traded goods and services.

6.6.1 Setting Parities

Consider a country that is about to adopt a fixed parity for its exchange rate. Perhaps it has had a floating rate, or perhaps it is simply changing its fixed parity. What parity should it adopt? It could just use the spot rate that is determined by the market in the absence of exchange rate intervention. However, we know that spot rates are subject to large swings. Pegging to the current spot rate may therefore lead to a large overvaluation or undervaluation of the currency relative to its long-run equilibrium value. Purchasing power parity has been proposed as a measure of the long-run equilibrium exchange rate.

Purchasing power parity was used in interwar discussions of the return to the gold standard (Young, 1925). Britain and France used purchasing power parity reasoning in deciding upon the par values of their currencies. This led to possibly the most famous example of how PPP arguments can seriously mislead policy makers: the April 1925 return of Britain to gold standard at the prewar parity. John Maynard Keynes argued at the time that although wholesale price indexes suggested that the prewar parity would appropriately value the pound sterling (against the dollar), the truth was quite different.[7] He argued that the wholesale price indexes were dominated by raw materials prices, which tended to satisfy the Law of One Price. Taking into account changes in nominal wages and in the cost of living indicated that the prewar parity would be a serious overvaluation of the pound sterling. Events proved Keynes right.

After the suspensions of trade and currency convertibility associated with WWII, the move back toward fixed exchange rates generated another round of interest in PPP. Discussions of the sustainability of the Breton Woods System were also influenced by PPP reasoning. Houthakker (1962) used PPP calculations to argue that the dollar had become overvalued.

Purchasing power parities have been used in other exchange rate policy decisions. For example, high inflation countries sometimes adopt a "crawling peg," where the exchange rate is allowed to track the inflation rate but not to float freely. A rate of devaluation

[7] Keynes (1925); Moggridge (1972, pp.105–6).

equal to the inflation rate will help prevent the exchange rate from becoming severely overvalued, but it does not take into account foreign inflation. Instead, one might devalue so as to maintain the real exchange rate constant at the PPP level (Williamson, 1965). Some countries have even sought to maintain an undervalued exchange rate in order to stimulate exports or used overvaluation as part of a disinflationary package.

6.6.2 Comparing Real Incomes

Purchasing power parity also influences the international comparisons of real incomes and growth. With strict CPP, relative real incomes could be compared across countries by comparing relative nominal incomes, after using current spot rates are used to measure the incomes in a common currency. However, we have seen that there are many violations of CPP, and evidence has accumulated that these cause CPP based approaches to generate systematic underestimation of the real incomes of developing countries (Kravis et al., 1978b,a). It appears to be particularly important that nontraded goods have a much lower price in developing countries (see section 6.8.1); simple exchange-rate-based income comparisons ignore this.

To get more valid international income comparisons, we compute a PPP exchange rate as the ratio of two appropriately constructed price indexes. The price indexes are generated based on a basket of goods that is specifically selected to facilitate these international comparisons. The prices of nontraded goods and services play an important role.

Some countries construct their own PPP exchange rate measures. For example, Statistics Canada constructs bilateral PPP exchange rates for comparison with the US. The OECD constructs a number of multilateral PPP exchange rates, annually for European OECD countries and intermittently for other countries. Generally, PPP exchange rates show smaller fluctuations than floating spot rates.

In 1968, the United Nations International Comparison Project (ICP) began the first large-scale attempt to develop a data set that would allow consistent cross country comparisons of real income (Kravis et al., 1982). Purchasing power parity reasoning plays a key role in this effort. As expected, relative to PPP-based real income comparisons, the project found that exchange-rate-based real income comparisons produced serious understatements of the real income of developing countries. Per capita GDP in the poorest countries proved to be two to three times greater under the PPP-based comparisons. The Penn World Table (PWT), which extends the ICP effort to additional countries, uses PPP reasoning to develop real income series back to 1950 (Summers and Heston, 1991). Heston and Summers (1996) observe that

> After considerable resistance, international agencies now seem to be persuaded that PPP-based estimates of GDP are superior to exchange-rate-based ones for most if not all of their purposes.

As poor countries have recognized, there are political implications to this improved method. Comparisons of real income are often the basis for the determination of international aid and burden sharing. The differences can be large. For example, in September 1997, the Organization for Economic Co-operation and Development (OECD) released research applying PPP based comparisons to China's economy.[8] The report containing two startling claims: China's economy is much bigger than previously thought, and annual economic growth in China between 1986 and 1994 has been dramatically overstated. The new estimate lent support to those who argue that China is already leaving the ranks of the developing countries, while for the period 1986–1994, the study puts the annual growth rate at six percent, which is much lower than the generally accepted figure of 9.8 percent. The economist responsible for the OECD report said that the much lower estimate of the Chinese government and of the World Bank derived from the use of exchange rates as price converters, resulting in misleading data. To achieve his conclusions, the report's author eliminated exchange rates as price converters, giving greater importance to purchasing power parities. By 2000, the World Bank was reporting China's GDP at about \$1T at market prices but at more than \$4T after a PPP conversion!

6.6.3 Choice of Price Index

If all relative prices were constant, then the choice of price index would not matter for purchasing power parity considerations. By homogeneity of the price indexes, we could normalize on any good satisfying CPP.[9] To see this, let f and f^* be the functions calculating the domestic and foreign price indexes, and let $P_i(= SP_i^*)$ be the price of the internationally arbitraged good.

$$\begin{aligned} CPI &\overset{\text{def}}{=} f[P_1, \dots, P_n] \\ &\overset{\text{hom}}{=} P_i f[P_1/P_i, \dots, P_n/P_i] \end{aligned} \tag{6.11}$$

$$\begin{aligned} CPI^* &\overset{\text{def}}{=} f^*[P_1^*, \dots, P_n^*] \\ &\overset{\text{hom}}{=} P_i^* f^*[P_1^*/P_i^*, \dots, P_n^*/P_i^*] \end{aligned} \tag{6.12}$$

$$\begin{aligned} \therefore \quad Q &\overset{\text{def}}{=} S\frac{CPI^*}{CPI} \\ &\overset{\text{sub}}{=} S\frac{P_i^* f^*[P_1^*/P_i^*, \dots, P_n^*/P_i^*]}{P_i f[P_1/P_i, \dots, P_n/P_i]} \\ &\overset{\text{cpp}}{=} \frac{f^*[P_1^*/P_i^*, \dots, P_n^*/P_i^*]}{f[P_1/P_i, \dots, P_n/P_i]} \end{aligned} \tag{6.13}$$

When relative prices are constant, the real exchange rate will be constant. But if relative prices change, the real exchange rate will change unless f and f^* respond proportionality: realistically, identical construction is required for this. If price index construction is identical at home and abroad, changes in relative prices have no effect on the PPP relationship. In this case, (6.13) implies the real exchange rate is constant at $Q = 1$. That is, absolute

[8](The OECD report, "China's Economic Performance in an International Perspective," is discussed in *International Trade Reporter*, September 1997, "OECD finds Chinese data distortions mask bigger economy, slower growth."

[9]The indexes themselves could even involve only non-traded goods. For notational simplicity, we will put goods consumed in either country in each price index, so that different goods will generally have zero weight in each price index.

purchasing power parity is implied by CPP *plus* identical price index construction.

Of course, relative prices do change, and domestic and foreign price index construction differs. As a result, our choice of domestic and foreign price indexes can be crucial to our results. (See Officer (1976) for a detailed discussion.) For example, from 1963–1972, the U.S. CPI rose by less than the Japanese CPI, but the U.S. WPI rose by more than the Japanese WPI. As a result, the CPIs implied a growing overvaluation of the yen, while the WPIs implied increasing undervaluation. The devaluation of the dollar in the December 1971 Smithsonian Agreement was seen by some as lending support to the WPI measure. This has motivated some authors to abandon the potential empirical content of PPP, claiming that the relevant price indexes are not observable (Bilson, 1981; Hodrick, 1978).

In reality, focusing on the constancy of relative prices is a distraction. Relative prices change all the time. Price indexes respond to changes in relative prices. But how important is the contribution of changing relative prices to price index changes? Large changes in price indexes tend to be highly correlated, and the core idea behind the purchasing power parity doctrine is the same as the core idea behind the quantity theory: the long-run neutrality of money. This is the underpinning of the purchasing power parity doctrine. From a Classical perspective, monetary policy is the source such proportional movements in prices. When we shift our attention to monetary neutrality as the underpinning of the purchasing power parity doctrine three conclusions emerge. First, the ideal price index for PPP comparisons will be that price index most reliably linked to the money supply. Second, since large price movements are highly correlated, the choice of price index should not matter much as long as the changes in price levels have been large. And third, since monetary neutrality is seen by most economists as a long-run proposition, purchasing power parity should also be considered a long-run proposition.

6.7 Expected Purchasing Power Parity

During the 1980s, economists came to view the real exchange rate as subject to permanent shocks. While undermining the purchasing power parity doctrine in any strict sense, important questions remained. In particular, was the real exchange best characterized as subject *only* to permanent shocks? In such circumstances we might write

$$Q_t = Q_{t-1} + u_t \tag{6.14}$$

where u_t is a period t shock to the real exchange rate that is zero on average. An example is when the real exchange rate follows *random walk*, where it is just as likely to rise as to fall each period. In this case, $\mathcal{E}_t u_{t+1} = 0$, so

$$\mathcal{E}_t Q_{t+1} = \mathcal{E}_t Q_t + \mathcal{E}_t u_{t+1} = Q_t \tag{6.15}$$

In this sense, we can say that the real exchange rate satisfies *expected purchasing power parity*.

However, the real exchange rate may be subject to both temporary and permanent shocks. The real interest of the economist who is examining purchasing power parity is the relative contribution of the two types of shocks. Huizinga (1987) takes up precisely this question and finds that temporary shocks make an important contribution to real exchange rate behavior. This is what would be expected in sticky-price models of the exchange rate, as will be seen in Lecture8. Lastrapes (1992) asked the same question in a slightly different way: are nominal shocks (such as money supply shocks) or real shocks (such as technology shocks) more important to the behavior of the real exchange rate? Lastrapes (1992) isolates the nominal and real shocks by looking at the nominal and real exchange rate simultaneously: nominal shocks should affect only the nominal exchange rate in the long run, while real shocks can affect both the nominal and real exchange rate in the long run.[10] While nominal shocks were important determinants of both real and nominal exchange rate variability, real shocks proved more important even in the short run.

6.8 Long-Run Purchasing Power Parity

Empirically, the real exchange rate is not constant, but perhaps it tends over time to a constant value. For example, perhaps there is a constant long-run equilibrium real exchange rate. This hypothesis is known as long-run purchasing power parity (LRPPP). LRPPP allows for transitory deviations of the real exchange rate, but it characterizes the real exchange rate as moving over time toward its long-run equilibrium value.

Long-run purchasing power parity is intended as an approximation of reality, as a useful guide. It is meant essentially to embody an underlying notion of the neutrality of money, so that large price level changes are superimposed on a relatively unchanging real economy. Still, we can ask, "How long is the long run?" The more time that elapses, the more likely it is that large real changes will effect relative price levels. Before considering some specific critiques based on this observation, consider figure 6.1.

[10]That is, Lastrapes (1992) identifies the nominal shocks in a bivariate VAR (in the nominal and real exchange rates) by imposing long-run neutrality of the nominal shocks.

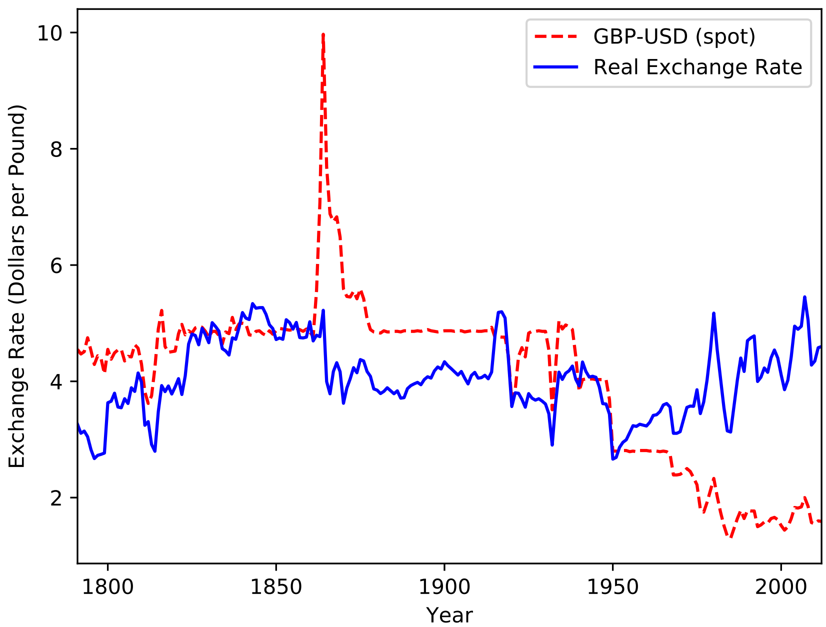


Figure 6.1: US/UK Real Exchange Rate over 200 Years

6.8.1 The Balassa-Samuelson Critique

The Balassa-Samuelson critique focuses on the inclusion in national price indexes of the prices of both traded and non-traded goods. Changes in price indexes deriving from purely monetary sources need not create problems for PPP, as traded and non-traded goods prices may be expected to rise proportionately. But any events that shift the relative prices of traded and non-traded goods do pose problems for PPP. The obvious candidates are sectorally asymmetric changes in production technology or expenditure patterns. If these changes differ across countries, then PPP will fail even if price index construction is identical.

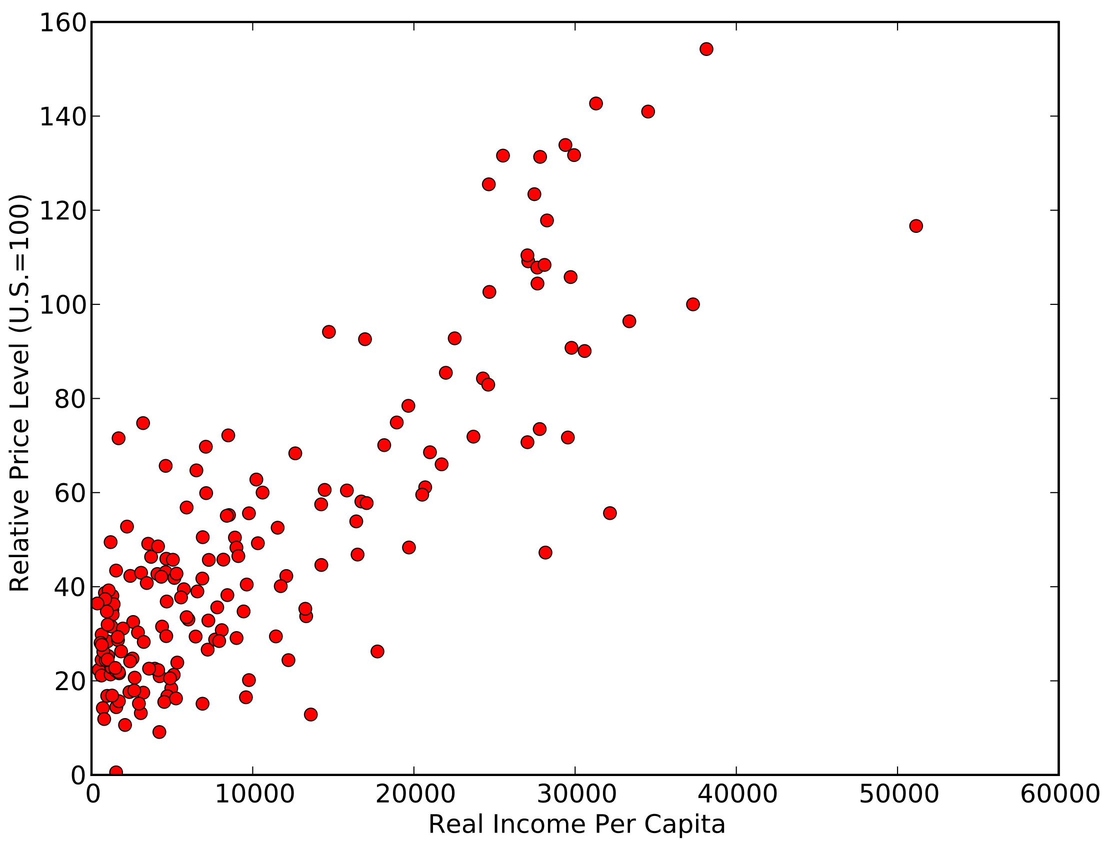


Figure 6.2: Price Level vs. Real Income Per Captia in 2000

Price levels tend to be higher in high income countries, as predicted by the Balassa-Samuelson hypothesis. (Each observation represents a single country in the year 2000. Axes are logarithmically scaled.)
Source: Penn World Table 6.1

Ricardo (1817) argued that countries with high manufacturing productivity will also tend to have relatively costly non-traded goods. Samuelson (1964) emphasized that the postwar experience was one of disparate productivity growth rates across countries. Further, income growth appears correlated with productivity increases in the production of traded goods. The relative price of non-traded goods can therefore be expected to rise fastest in the fastest growing countries based just on supply considerations. But demand may contribute as well, if non-traded goods are "superior" goods. If a country's traded goods are geographically arbitraged and its non-traded are rising in relative price, then its real exchange rate will tend to appreciate.

How relevant is this to the Big Mac parity deviations? Certainly Big Macs are produced with non-traded inputs: real estate, utilities, labor services. While it does not seem likely that labor productivity differences would be large, wage differences certainly are.

According to the Balassa-Samuelson theory, we ought to expect overvaluation of the dollar against the lowest productivity countries. Kravis and Lipsey (1983) offer evidence that higher relative income implies a higher relative price level. Pakko and Pollard (1996) provide evidence that real income per capita is positively correlated with relative price levels, and thus developing countries and economies in transition usually appear undervalued against the dollar. Of course we also observe PPP deviations among developed countries, and for these productivity differences do not offer a plausible explanation.

The Balassa-Samuelson critique may be of less relevance in the very long run, if knowledge, physical capital, and human capital are eventually mobile enough to offset international differences in sectoral productivity (Froot and Rogoff, 1995).

Algebra for the Balassa-Samuelson Critique

We will begin with commodity price parity for internationally traded goods, so that our (identically constructed) indexes of traded goods prices, P_T and P_T^*, bear the relationship

$$P_T = SP_T^* \tag{6.16}$$

In addition to traded goods prices, the non-traded goods prices, P_{NT} and P_{NT}^*, contribute to the overall price level.

$$P = f[P_T, P_{NT}] \tag{6.17}$$
$$P^* = f^*[P_T^*, P_{NT}^*] \tag{6.18}$$

Take our standard definition of the real exchange rate

$$Q \stackrel{\text{def}}{=} S\frac{P^*}{P} \tag{6.19}$$

and substitute for the price indexes to get the following.

$$\begin{aligned}
Q &\overset{\text{sub}}{=} S\frac{f^*[P_T^*, P_{NT}^*]}{f[P_T, P_{NT}]} \\
&\overset{\text{cpp}}{=} \frac{P_T}{P_T^*}\frac{f^*[P_T^*, P_{NT}^*]}{f[P_T, P_{NT}]} \\
&= \frac{f^*[P_T^*, P_{NT}^*]/P_T^*}{f[P_T, P_{NT}]/P_T} \\
&\overset{\text{hom}}{=} \frac{f^*[1, P_{NT}^*/P_T^*]}{f[1, P_{NT}/P_T]}
\end{aligned} \tag{6.20}$$

So changes in the relative price of non-traded goods affect the exchange rate. For example, if the price of the non-traded good rises domestically, the real exchange rate appreciates.

6.8.2 The Houthakker-Magee Critique

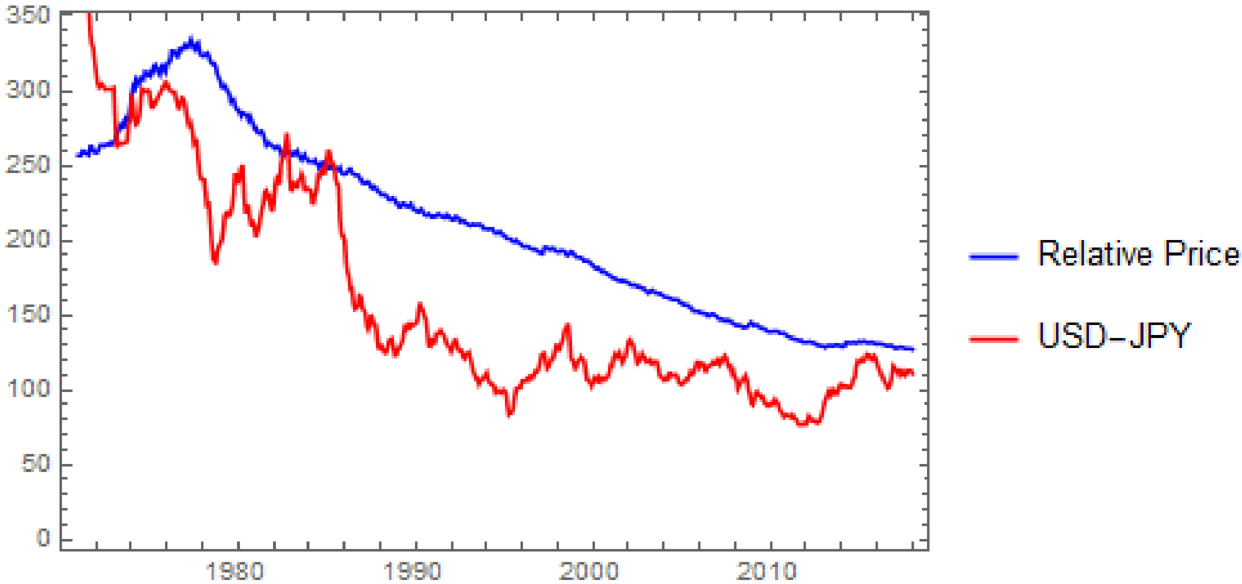
Houthakker and Magee (1969) considered the implications of the empirical divergence of income elasticities of exports and imports in many countries. In such circumstances, it would seem that real exchange rate changes would be necessary to balance trade over time as world income growth on its own will lead to larger and larger trade imbalances. Interestingly, the fastest growing countries in the postwar period are estimated to have relatively low income elasticities of imports, offsetting this effect to some extent. Furthermore, there has been some subsequent evidence that these elasticities are more aligned than Houthakker and Magee believed.

6.9 Testing the Purchasing Power Parity Hypothesis

In the 1980s and 1990s, purchasing power parity was subject to intense empirical scrutiny. In the 1980s, the conclusion was roughly that purchasing power parity does not hold even in the long run. By the end of the 1990s, opinion had reversed: long-run purchasing power parity believed to be supported by the best evidence, although it was also found that the tendency toward PPP was rather sluggish.

How might we go about testing PPP in any of its various forms? There are several popular approaches. We can look at individual commodities (or at least at low levels of aggregation) to determine whether the law of one price holds for traded goods. We can ask whether the real exchange rate is constant over time or at least independent of the nominal exchange rate. We can use regression analysis to ask whether PPP is supported as a statistical relationship.

It is clear at this point that PPP does not hold in the short run, even for goods that appear quite similar. It's status as a long-run relationship is more promising but still controversial. Almost every aspect of existing tests—including choice of countries, price indexes, and sample period—appears to influence the results.



Source: FRED (monthly data)

Figure 6.3: Dollar-Yen Exchange Rate vs. Relative Price Level

Frenkel (1978) and Genberg (1978) find some support for PPP, using monthly data from the 1920s and 1970s. But the fact that any choice of price index indicates clearly that large fluctuations in real exchange rates have characterized the generalized float certainly calls into question any claims of real exchange rate constancy. (See plots in Isard (1995, ch.4).)

In a world of relative price changes, incommensurate domestic and foreign price indexes, and documented failures even of commodity price parity, one may begin to wonder why there should be any expectation that PPP would be a useful characterization of reality. However the core intuition behind the PPP doctrine is the neutrality of money. Price levels move around a lot; real variables, including the real exchange rate, are expected to be fairly independent of movements in the overall price level, especially in the long run.

Choice of Exchange Rate

Hakkio (1985) argues that considering several exchange rates simultaneously can improve empirical work on PPP. (Essentially, the information in the correlations in the shocks affecting different exchange rates can be exploited.) Officer (1980) proposes testing PPP with an effective exchange rate, which is trade-weighted bilateral exchange rates, and a similarly constructed foreign price level. This puts more weight on the PPP relationship between major trading partners, which has some intuitive plausibility. However this approach also undermines the key intuition behind PPP: the neutrality of money.

Perhaps as a result, tests of PPP generally use bilateral exchange rates.

6.9.1 Regression Tests of PPP

For the conduct of regression tests, relative purchasing power parity is often restated in logs, so that (6.10) becomes (6.21).[11]

$$s = q + p - p^* \tag{6.21}$$

Simultaneity

Exchange rates and prices are endogenous, which can lead to simultaneity bias in the estimates. Krugman (1978) argues that this can lead to false rejections of PPP. Frenkel (1981) finds that prices help predict ("Granger cause") exchange rates, and he suggests reversing the usual regression.

Expected PPP

PPP is very often given in rates of change. Differencing (6.21) yields (6.22).

$$\Delta s = \Delta q + \Delta p - \Delta p^* \tag{6.22}$$

Since purchasing power parity treats the real exchange rate as constant, we should impose $\Delta q = 0$. Under this restriction, the percentage change in the exchange rate, Δs, must equal the inflation differential, $\Delta p - \Delta p^*$. However, the data clearly show under any measure of relative prices that large changes in the real exchange rate are common. It may well be that—although the real exchange rate is constantly changing—none of the changes can be anticipated. In this case, we can still invoke the ex ante relationship

$$\Delta s^e = \Delta p^e - \Delta p^{*e} \tag{6.23}$$

We will refer to this relationship as *expected PPP*. For example, if the real exchange rate is believed to follow a random walk—as some empirical evidence has suggested—then expected PPP holds.

The standard tests of expected PPP consider whether the real exchange rate follows a random walk. Initial empirical evidence was supportive of expected PPP, in the sense that the hypothesis that the real exchange rate followed a random walk was not rejected by the data (Roll, 1979; Adler and Lehmann, 1983; Cumby and Obstfeld, 1984). From the end of the 1980s, however, evidence accumulated that real exchange rates display "mean reversion", which is another way of saying that exchange rates do have a long-run tendency to PPP. Abauf and Jorion (1990) provides evidence that the low power of earlier tests is the source of their failure to distinguish slow adjustment from no adjustment, showing that testing many countries at once (i.e., in "cross section") proves more supportive of PPP.

[11] Sometimes PPP is discussed under the strong assumption that $q = 0$. This is absolute PPP, which we have seen holds only under very unrealistic conditions. As a relationship between the price indexes of different countries (which may even have different base years), the only real justification for absolute PPP is presentational simplicity.

Flood and Taylor (1995) have even found PPP style correlations between national price levels and exchange rates in cross-section data covering 10- and 20-year horizons.

6.9.2 Long-Run Purchasing Power Parity

The death of PPP as a short-run proposition does not directly undercut the notion of PPP as a long-run tendency. Nor is it evidence against expected PPP.

The basic idea behind LRPPP is that over time we should expect changes in exchange rates to reflect changes in national price levels. We can get a sense of the extent to which this is broadly correct by looking at the behavior of prices and exchange rates over an extended period of time. For example, consider table 6.2.

Country	P_{89}/P_{73}	PPP_{89}/PPP_{73}	S_{89}/S_{73}
Australia	4.4	1.6	1.8
Austria	2.1	0.8	0.7
Canada	3.2	1.1	1.2
France	3.3	1.2	1.4
Germany	1.7	0.6	0.7
Greece	14.4	5.1	5.3
Italy	4.9	1.8	2.4
Japan	2.2	0.8	0.5
Korea	5.8	2.1	1.7
Sweden	3.7	1.3	1.5
Switzerland	1.7	0.6	0.5
Turkey	278.0	99.3	151.6
UK	4.9	1.8	1.5
US	2.8	1.0	1.0

Source: Argy (1994, p.332)

Table 6.2: Inflation and Exchange Rates, 1973–89

The purchasing power parities, $PPP = P/P^*$, are calculated against the dollar, and the exchange rates are the domestic currency cost of a dollar. In brief, table 6.2 supports the qualitative prediction of PPP. Countries with more inflation than the U.S. had depreciating currencies; those with less inflation had appreciating currencies. At the same time it is clear that the *quantitative* prediction of PPP is not fully supported: exchange rate changes deviate from the changes in purchasing power parities, so there were real exchange rate changes even over a sixteen year horizon.

6.9.3 Regression Tests of Long-Run PPP

Suppose we run the regression

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p_t^* + \epsilon_t \tag{6.24}$$

where ϵ_t is the regression error. If PPP holds, we should find that the coefficient restrictions $\beta_1 = 1$ and $\beta_2 = -1$ are satisfied. This is referred to as the "homogeneity" restriction. Similarly for

$$\Delta s_t = \tilde{\beta}_0 + \beta_1 \Delta p_t + \beta_2 \Delta p_t^* + \tilde{\epsilon}_t \tag{6.25}$$

Early tests of PPP took this approach. For example, Frenkel (1978) estimated (6.24) and (6.25) for the interwar floating exchange rates of the 1920s. He found support for both forms of PPP for the pound, franc, and dollar over the period 1921.02–1925.05. However Krugman (1978) did not find support for PPP over a longer interwar period. And when Frenkel (1981) tried the same tests for the 1970s, PPP was strongly rejected by the data. Similarly, Pigott and Sweeney (1985) estimate PPP with pooled data and can't find a relationship between Δs and $\Delta(p - p^*)$.

In the mid-1980s there was a shift in emphasis. Some economists began to search for any long-run relationship between prices and exchange rates, and often referred to this search as testing PPP Patel (1990); MacDonald (1993). From this perspective, empirical evidence of *any* stable long-run relationship in the form of (6.24) to support a "weak-form" PPP.[12] MacDonald (1993) cites arguments that transportation costs and price level measurement errors can lead to violations of the homogeneity restriction Taylor (1988); Patel (1990).[13] In fact, he finds evidence of a long-run relationship between exchange rates and price levels while firmly rejecting the homogeneity restriction. But whatever relationships violating the coefficient restrictions do represent, they do not represent PPP under any standard interpretation. Other economists began to treat PPP as a relationship between tradable goods prices, allowing real exchange rates to be affected by the relative price of non-traded goods.

Even in considering the prices of traded goods, the evidence for PPP in the short-run is not supportive. For example, Isard (1977) finds that changes in nominal exchange rates lead to large, persistent changes in the relative price of foreign manufactured goods. It appears that prices of such goods are too "sticky" for PPP to hold in the short-run.[14] This suggests that the macroeconomic implications of models that assume perfect price flexibility must be considered cautiously.

The evidence seems overwhelming that PPP is violated in the short run, and that the violations persist. This has shifted discussion to LRPPP. In the 1980s, many economists found evidence that the real exchange rate follows a random walk.[15] On the one hand, this means that expected PPP was supported. On the other hand, it means that deviations from PPP are never corrected. These tests were very weak, however, and the available time series are fairly short. Frankel (1990) argues that failure to reject a random walk for the real exchange rate should be expected on the basis of the short data sets that had

[12]That is, weak-form PPP is equivalent to the existence of a cointegrating relationship between exchange rates and price levels. In this case, although the exchange rate and price levels are nonstationary, the regression residual in (6.24) is stationary.

[13]Taylor (1988) simply postulates that measurement errors or transportation costs contribute multiplicatively to the measured logarithm of prices, which implies the oddity that they contribute exponentially to the levels. Realistically, transportation costs and measurement error contribute at most multiplicatively to the *level* of prices, which permits homogeneity.

[14]This need not violate the CPP, as foreign and domestic manufactured goods are not perfect substitutes.

[15]See Roll (1979) for an early discussion of expected PPP. Frankel and Froot (1987) are less supportive.

been used for these tests. Using data for the pound-dollar real exchange rate over the period 1869–1987, he finds clear evidence in favor of PPP. Note how odd this is, however: over such long periods we would be most inclined to anticipate large structural shifts that would have permanent effects on the real exchange rate.

Lothian and Taylor (1996) found that two centuries of dollar-sterling and franc-sterling exchange rate data favored a simple stationary AR(1) formulation, and Froot and Rogoff (1995) suggest that for major industrialized countries, economists believe that about half the deviation from PPP disappears after four years. This seems roughly in accord with the current consensus for industrialized countries. For example, Choi et al. (2006) find for 21 OECD countries (1973–1998) that the CPI-based real exchange rates have a half-life of 2.3–4.2 years. Similarly, Burstein and Gopinath (2014) find most countries display a deviation half-life of 3–9 years, with Switzerland being an exception with an estimated half-life of 1.6 years. Given the high short-run volatility of real exchange rates, the apparently slow convergence to purchasing power parity is often considered to be a puzzle. Rogoff (1996) refers to this as the purchasing-power-parity puzzle.[16]

Nonstationarity

Standard regression analysis assumes stationarity in the time-series data. This just means that the individual time series do not get extremely difficult to predict as the forecast horizon increases. (See Granger 1986 for a formal definition.) If this assumption is not fulfilled, the standard regression procedures for statistical inference can be misleading.

Exchange rates and price levels often appear to be nonstationary. Shocks to the series appear to be permanent: they don't fade away with time. This means that shocks to the series appear to add up over time (instead of dying out), causing the accuracy of our predictions about the future to decay terribly as the time horizon increases. When shocks are permanent, so that they accumulate over time, we say the series is "integrated".

Purchasing power parity is often interpreted as saying that despite the nonstationarity of the price levels and the exchange rate, they should move together over time. That is, although we may not have much to say individually about the level of either exchange rates and price levels in the long run, we do have something to say about their long-run relationship. We say that although the series are integrated, they are also cointegrated.

When series are cointegrated, they have a long-run relationship. Deviations from this long-run relationship are temporary. That is, the deviations tend to die out over time.

Cointegration offsets our concerns about simultaneity bias. Cointegration also offsets our concerns about running regressions of integrated data in levels. In fact, when series are cointegrated, regressions run in differences are misleading. (Technically, the MA process representing the differenced variables is not invertible, so we cannot construct an AR process from it.)

---

[16]For Rogoff the puzzle is actually multi-faceted. He argues that convergence is too slow to be attributed to nominal rigidities, but that models relying on real shocks cannot generate realistic short-term volatility.

6.9.4 Some Studies

In the 1980s, most research failed to find evidence of long-run PPP. However, the time series used in early tests of long-run PPP were too short to persuasively discriminate permanent shocks from transient but persistent shocks Frankel (1986b). Research on larger datasets—60 to 700 years—has produced more favorable results, suggesting that deviations from PPP have half-lives of between 2.8 and 7.3 years. The long time-series often rely on data drawn from both fixed-and floating-rate periods, although . . . Some economists also expressed concern that long time-series for real exchange rates are only available for industrialized countries, which might produce selection bias Froot and Rogoff (1995). In response, some researchers turned to "panel" data: looking simultaneously at a collection of countries in order to add to the size of their data sets Frankel and Rose (1996); Papell (1997); Oh (1996); Wu (1996). Generally, these panel-data studies found mean-reversion to PPP at rates similar those found in the time series literature. By the end of the 1990s, most economists had accepted that the evidence favored long-run purchasing power parity, completely reversing the consensus of the 1980s.[17]

Problems for Review

1. What is the difference between CPP and PPP?

2. In the U.S., Virginia and Maryland are bordering states that have imposed very different taxes on cigarettes. For example, in 1998 Maryland's tax per pack of \$0.36 was about twelve times the Virginia tax. Since transportation costs cannot be the answer for these bordering states, how are markets being segmented? As an economist, are you surprised to learn that Maryland arrests people found with large quantities of cigarettes lacking the Maryland state tax stamps?

3. Northern Brands Inc. operated in the U.S. and Canadian cigarette markets from 1992–1997. It sold "Export A" cigarettes, manufactured by RJR's Canadian affiliate (RJR-Macdonald Inc.) to two other companies: Baltic Imports and LBL Importing, Inc. These two companies brought the cigarettes into the U.S. after filing statements (with the U.S. Customs Service) saying the cigarettes were to be exported. This allowed the importation without paying millions of dollars U.S. excise taxes. The cigarettes were then brought back into Canada via the St. Regis Mohawk Indian

[17]An important dissent is O'Connell (1998), who exploits the insight of Abauf and Jorion (1990) that panels of real exchange rates against a common currency will exhibit cross-sectional dependence. Most of the panel-data studies ignore this, and O'Connell (1998) show it matters. Cross-sectional dependence can easily be seen if we calculate the real exchange rate for each country i at time t as

$$q_{i,t} = s_{i,t} + p_{US,t} - p^*_{i,t} \tag{6.26}$$

where $s_{i,t}$ is the log of the domestic currency cost of the U.S. dollar, then any two real exchange rates will have in common the movement in the U.S. price level plus any common movement in the U.S. nominal exchange rates. Ignoring this produces faulty statistics (the tests are incorrectly sized); correcting for it alters the conclusions. For example, O'Connell (1998) found no evidence in favor of the PPP in broad panels of CPI exchange rates over the 1973–1995 period.

Reservation in northern New York, carried in bass boats, and resold in Canada. Canada's taxes were thereby evaded as well. Relate this activity to the PPP doctrine.

4. On 24 May 2002, the NYT reported two brothers had smuggled $7.5 million in cigarettes from North Carolina to Michigan, in a scheme to raise money for Lebanon's Hezbollah. Why is such geographical arbitrage profitable?

5. Find a recent issue of *The Economist* and calculate the commodity price parities implied by the prices on the front cover. Then compare these with the exchange rates listed in the financial indicators (at the end of the magazine). Does CPP hold?

6. Consider the following table. There are two countries and two goods.

Time	t	t+1
S	100	125
P_1	\$8	\$10
P_2	\$4	\$5

Assume commodity price parity. What is the foreign currency price of the two goods at the two points in time? What is the domestic inflation rate? What is the foreign inflation rate?

7. Suppose PPP is known to hold as is covered interest parity between two countries. What determines any differences between the expected *real* returns on risk free interest bearing assets in the two countries?

8. Suppose there are two goods with domestic prices P_1 and P_2. Let the domestic and foreign price indexes be

$$P = {P_1}^{\beta} {P_2}^{1-\beta}$$
$$P^* = {P_1^*}^{\beta^*} {P_2^*}^{1-\beta^*}$$

Show that the real exchange rate is

$$Q = \frac{(SP_1^*)^{\beta^*}(SP_2^*)^{1-\beta^*}}{{P_1}^{\beta}{P_2}^{1-\beta}} \tag{6.27}$$

Clearly if the exchange rate and the commodity prices can move independently, then the real exchange rate will fluctuate. Suppose however that CPP holds. Show that the real exchange rate is then

$$Q = \left(\frac{P_1}{P_2}\right)^{\beta^*-\beta} \tag{6.28}$$

Now as long as P_1 and P_2 change proportionately, so that the relative price P_2/P_1 is unchanged, there is no change in the real exchange rate. but changes in the relative price still cause changes in the real exchange rate. Finally, if set $\beta = \beta^*$ so that price index construction is identical at home and abroad. Now what is the effect of changes in relative prices on the PPP relationship?

9. Suppose that we use the same weights and the same goods to construct the domestic and foreign price indexes: $CPI = f[P_1, \ldots, P_N]$ and $CPI^* = f[P_1^*, \ldots, P_N^*]$. Vanek (1962, p.84) notes that our choice of f will not affect our purchasing power parity calculation if $P_i/P_i^* = k\, \forall i$. Show this, using the first degree homogeneity of f.

Balassa-Samuelson

Define the foreign and domestic prices levels as weighted averages of traded and non-traded goods prices:

$$p = (1 - \beta p_T + \beta p_N \tag{6.29}$$
$$p^* = (1 - \beta^*)p_T^* + \beta^* p_N^* \tag{6.30}$$

Define the real exchange rate for tradeables:

$$q^T = e + p_T^* - p_T \tag{6.31}$$

If absolute purchasing power parity held for all tradeables and the indexes were identical, we would have $q^T = 0$.

Define the real exchange rate:

$q = e + p^* - p$

Work on the real exchange rate has found it to be non-stationary. There may still be a LR relationship between the exchange rate and price levels, however, if we don't impose homogeneity. Further, we may need to account for permanent shifts in the relative price of tradeables.

$$\begin{aligned} q &= e + p^* - p \\ &= (q^T - p_T^* + p_T) + p^* - p \\ &= q^T + \beta(p_T - p_N) - \beta^*(p_T^* - p_N^*) \end{aligned}$$

Thus a rise in the domestic relative price of tradeable goods tends to depreciate the real exchange rate.

Balassa (JPE,1964) notes that productivity grows faster in tradeable than non-tradable goods sectors, and this is most true in faster growing countries. In light of our equation for the real exchange rate, this suggests that faster growing countries will tend to experience real appreciation. Other influences include the price of oil and the size and composition (i.e., is the spending on traded or non-traded goods) of government expenditure. Zhou (SEJ, April 1995) uses these observations to write

$rer = f(P_{oil}, G, G^*, y - y^*, m - m^*)$

Here $y - y^*$ is a productivity differential and $m - m^*$ is the monetary base differential.

Cointegration analysis (1973.1-1993.2 for Finland, Japan, US) found the monetary differential unimportant but the real shocks important determinants of the LR real exchange rates. Oil price shocks were especially important; fiscal variables fairly important; productivity differentials less so. Strauss (SEJ, April 1995) finds larger effects of the productivity differential for 10 countries with annual data (1960-1990), but omits oil prices. He also tests directly for relative price effects, which generally are also important. However the productivity effects do not appear to be working throught the relative price channel, as the two are not cointegrated! An error correction model indicates that the relative price of non-tradeables is an important determinant of short run movements in the real exchange rate. Note: Strauss actually constructs relative productivity measures! (Most people just use industrial productivity.)

Appendix on Balassa-Samuelson

We will begin with commodity price parity for internationally traded goods, so that our indexes of traded goods prices, P_T and P_T^*, bear the relationship

$$P_T = KSP_T^* \tag{6.32}$$

The constant K can deviate from unity to allow for real factors and price index construction. Traded goods prices and non-traded goods prices, P_{NT} and P_{NT}^*, contribute to the overall price level.

$$P = f[P_T, P_{NT}] \tag{6.33}$$
$$P^* = f^*[P_T^*, P_{NT}^*] \tag{6.34}$$

Take our standard representation of the real exchange rate

$$Q = S\frac{P^*}{P} \tag{6.35}$$

and substitute for the price indexes to get the following.

$$\begin{aligned} Q &= S\frac{f^*[P_T^*, P_{NT}^*]}{f[P_T, P_{NT}]} \\ &= K\frac{P_T}{P_T^*}\frac{f^*[P_T^*, P_{NT}^*]}{f[P_T, P_{NT}]} \\ &= K\frac{f^*[1, P_{NT}^*/P_T^*]}{f[1, P_{NT}/P_T]} \end{aligned} \tag{6.36}$$

Thus changes in the relative price of non-traded goods affect the exchange rate.

To get a sense of the contribution of wages and productivities to real exchange rates, model price determination with a markup pricing model.

$$P_T = \mu_T \frac{W_T}{A_T} \quad (6.37)$$

$$P_{NT} = \mu_N \frac{W_{NT}}{A_{NT}} \quad (6.38)$$

$$P_T^* = \mu_T^* \frac{W_T^*}{A_T^*} \quad (6.39)$$

$$P_{NT}^* = \mu_N^* \frac{W_{NT}^*}{A_{NT}^*} \quad (6.40)$$

This gives us relative prices:

$$\frac{P_{NT}}{P_T} = \mu \frac{W_{NT}/A_{NT}}{W_T/A_T} = \frac{W_{NT}/W_T}{A_{NT}/A_T}$$

$$\frac{P_{NT}^*}{P_T^*} = \mu^* \frac{W_{NT}^*/A_{NT}^*}{W_T^*/A_T^*} = \frac{W_{NT}^*/W_T^*}{A_{NT}^*/A_T^*}$$

where $\mu = \mu_{NT}/\mu_T$.

Therefore

$$Q\ K \frac{f^*[1, \frac{W_{NT}^*/W_T^*}{A_{NT}^*/A_T^*}]}{f[1, \frac{W_{NT}/W_T}{A_{NT}/A_T}]} \quad (6.41)$$

For example, if

$$P = {P_T}^{1-\alpha} {P_{NT}}^{\alpha} \quad (6.42)$$

$$P^* = {P_T^*}^{1-\alpha^*} {P_{NT}^*}^{\alpha^*} \quad (6.43)$$

so that our real exchange rate solution implies

$$Q = K \left(\frac{P_{NT}^*}{P_T^*} \right)^{\alpha^*} \left(\frac{P_T}{P_{NT}} \right)^{\alpha} \quad (6.44)$$

Then we end up with

$$Q = K \frac{\mu^{*\alpha^*}}{\mu^{\alpha}} \left(\frac{W_{NT}^*/W_T^*}{A_{NT}^*/A_T^*} \right)^{\alpha^*} \left(\frac{W_{NT}/W_T}{A_{NT}/A_T} \right)^{-\alpha} \quad (6.45)$$

Thus we see that deviations in relative wages or in relative productivities can cause deviations from PPP.

The Balassa-Samuelson model traditionally has been interpreted as linking the real exchange rate to relative productivities. However, the real exchange is empirically subject to large short-run fluctuations, while relative productivities evolve very slowly. With this background, it is clear that the Balassa-Samuelson link must apply only to long-run real exchange rate determination. See Asea and Corden (1994) and Froot and Rogoff (1995) for recent surveys of the literature.

Canzoneri et al. (1996) find evidence in a panel of 13 OECD countries that the relative price of nontraded goods reflects relative labor productivities, but they assume a common wage across traded and nontraded sectors. Strauss and Ferris construct GDP price indexes, productivity measures, and real wage compensation rates for the traded and nontraded sectors of 14 OECD countries for the years 1970–1990. They provide evidence that productivity growth is higher in the traded goods sector, and that real (product) wage growth in each sector tends to match that sector's productivity growth. They find intersectoral wage growth appears to influence the relative price of nontradables more than differences in productivity growth.

Much empirical work has relied on total factor productivity measures of labor producitivity. Canzoneri et al. (1996) argue for using the average product of labor as this gets rid of the need for data on sectoral capital stocks or labor's share in production. They also note that for a broad range of technology, marginal and average products are proportional. For example, if both capital and labor are mobile, CES production functions with constant returns to scale imply this proportionality for perfectly competitive, profit-maximizing firms. If capital is not mobile, then let traded and nontraded goods be produced according to

$$T = F[K_T]{L_T}^{\mu_T}, \qquad N = G[K_N]{L_N}^{\mu_N} \tag{6.46}$$

This technology obviously includes the Cobb-Douglas technology as a special case. An implication is the proportionality between average and marginal products.

$$\partial T/\partial L_T = \mu_T F[K_T]{L_T}^{\mu_T - 1} = \mu_T T/L_T = \mu_T A_T \tag{6.47}$$

$$\partial N/\partial L_N = \mu_N F[K_N]{L_N}^{\mu_N - 1} = \mu_N N/L_N = \mu_N A_N \tag{6.48}$$

For perfectly competitive profit maximizing firms, the marginal product of labor equals the real product wage. Thus technology leading to equations (6.47) and (6.48) implies for competitive firms the relationships (6.37)–(6.40).

Since tastes, technology, and factor endowments can all influence the relative price of traded and nontraded goods, all offer themselves a potential explanantions. Kravis and Lipsey (1983,1988) and Bhagwati (1984) emphasize the role of factor endowments, and Dornbusch, Fischer, and Samuelson (1977) emphasize the role of technology.

Lecture 7

Real Exchange Rates and the Trade Balance

Our discussion of purchasing power parity in Lecture6 documented the large short-run fluctuations in the real exchange rate. In this chapter we will consider the effect of such fluctuations on the balance of trade. Since exchange rates are much more volatile than prices, nominal exchange-rate movements lead to changes in the relative price of foreign goods. This change in relative prices affects the foreign demand for domestic goods as well as the domestic demand for imports.

Learning Goals

After reading this chapter, you will understand:

- how the real exchange rate affects imports and exports
- the Marshall-Lerner condition
- the J-curve
- how trade-balance effects can be incorporated into the monetary approach

7.1 Real Exchange Rates and the Trade Balance

We begin our analysis of the effects of the exchange rate on the balance of trade with two simplifying assumptions. First, let us momentarily hold constant real income. Second, let us initially hold constant all prices except the nominal exchange rate. Specifically, the domestic currency prices of domestically produced goods will remain unchanged, as will the foreign currency price of foreign produced goods. This is equivalent to saying that the supply of any good is infinitely elastic at its home country price. Exogenous income and prices are "ceteris paribus" assumptions that will simplify our initial analysis.

Consider our demand for imports, $\mathrm{M}^d(SP^*/P, Y)$. As illustrated in Figure 7.1, the demand depends negatively on the relative price of the foreign good, which is the real exchange rate. (It also depends positively on real income, an effect we ignore for now but will return to in the future.) Thus if the domestic currency depreciates, there is a rise in the relative cost of imports and a decline in import demand. This reduces our total expenditures on imports, measured in foreign exchange.

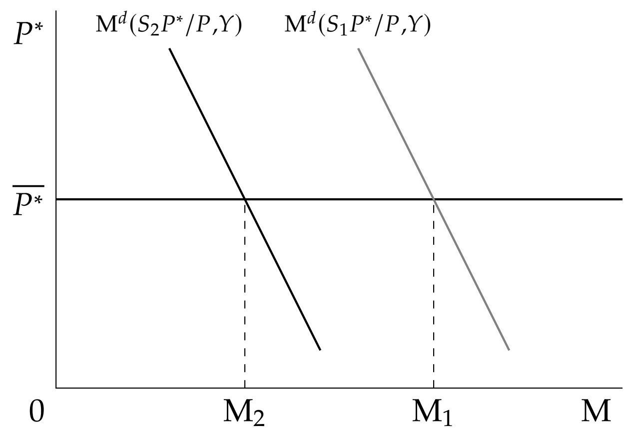


Figure 7.1: Imports and the Demand for Foreign Exchange ($S_2 > S_1$)

Now consider foreign demand for our goods, $X^d(SP^*/P, Y^*)$. This demand depends negatively on the relative price of the domestic good, and therefore depends positively on the real exchange rate. (It also depends positively on real foreign income, an effect we ignore for now.) Thus if the domestic currency depreciates, foreigners experience a fall in the relative cost of their imports (domestic exports), and we see a rise in the demand for domestic exports.

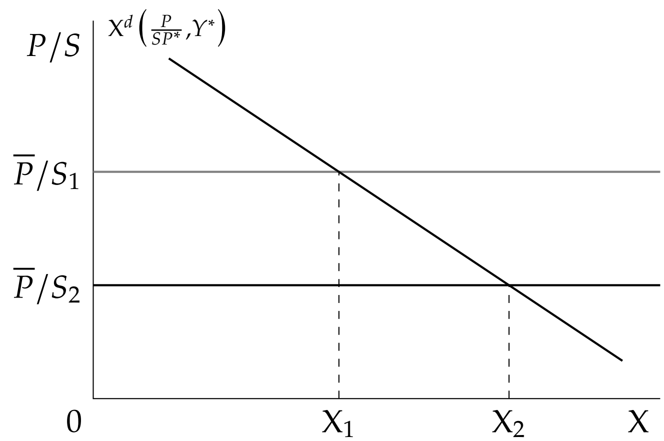


Figure 7.2: Exports and the Supply of Foreign Exchange ($S_2 > S_1$)

Since a depreciation of the domestic currency leads to a rise in the quantity of exports and a fall in the quantity of imports, it may seem natural that such a depreciation improves the balance of trade. However, that is not necessarily the case. The balance of trade is a measure of the value of exports relative to the value of imports, not of the quantity of exports relative to the quantity of imports. A change in the exchange rate has a price effect on relative values that can offset the changes in quantities.

To see this, let us focus on a common policy situation for countries that have pegged their exchange rate to another currency. When foreign exchange reserves are running low, it is often proposed that a devaluation of the domestic currency can help replenish these reserves. The idea is that selling more abroad while importing less will raise net foreign exchange receipts. But figure 7.2 shows us that a devaluation of the domestic currency can reduce the value of exports. While it is true that there is an increase in the quantity of exports, there is also a reduction in the amount of foreign exchange earned from each unit of exports. The net effect is a rise in the foreign exchange *value* of exports only if the quantity of exports increases faster than the price falls. That is, exports are earning more foreign exchange only if export demand is elastic.

Of course an improvement in net foreign exchange earnings does not depend entirely on the increase in exports; the reduction in imports also contributes. Consider the domestic trade balance measured in foreign currency:

$$\mathrm{TB}^{FX} = \frac{P}{S}\mathrm{X} - P^{*}\mathrm{M} \tag{7.1}$$

Given the foreign and domestic prices, we see that a change in the exchange rate has three effects on the trade balance: the quantity of imports changes, the quantity of exports changes, and the foreign exchange value of each unit of exports changes. The last effect is called the valuation effect or the price effect: P/S is the foreign exchange earned by each unit of exports, and a change in S will change this quantity directly.

7.1.1 Marshall-Lerner Condition

For a depreciation of the domestic currency to increase foreign exchange earnings, we need quantity effects of falling imports and rising exports together to be larger than the valuation effect of the rise in S. The Marshall-Lerner condition is a precise statement of this requirement: given an initial position of balance trade, a depreciation will improve the trade balance if the export and import elasticities of demand sum to more than unity.

$$\epsilon_X + \epsilon_M > 1 \tag{7.2}$$

Here ϵ_X is the real exchange rate elasticity of demand for exports and ϵ_M is the real exchange rate elasticity of demand for imports.

The intuition for the Marshall-Lerner condition is straightforward. Suppose X and M were completely unresponsive to changes in relative prices. Then a 1% rise in the exchange rate would lead to a 1% fall in the value of our exports, deteriorating the trade balance. This fall can be obviously be offset by a 1% rise in the quantity of exports. If we begin with balanced trade, so that the value of imports equals the value of exports, then for the same reason the price effect could be offset by a 1% decrease in the value of imports, which can be achieved by a 1% decrease in the quantity of imports.

The policy application for countries with pegged exchange rates is not immediate however. Devaluation is generally considered when the trade balance is in deficit, not when it is in balance. In these circumstances, the value of imports is initially larger than the value of exports. While a 1% increase in the exchange rate may still be offset by a 1%

increase in the quantity of exports, it can now be offset by *less* than a 1% decrease in the value of imports. The condition for an improvement is relaxed: the sum of the elasticities may be smaller. A 1% fall in the quantity of imports, which surely reduces the value of imports by 1%, has a larger effect on the trade balance than a one percent rise in S, which reduces the (smaller) value of exports by 1%.

Algebra

To make the algebra as simple as possible, we will work with the trade balance measured in terms of the domestic good: $\mathrm{TB} = \mathrm{X} - Q\mathrm{M}$.

$$\begin{aligned} \mathrm{TB}[Q, Y, Y^*] &= \frac{S}{P}\mathrm{TB}^{FX} \\ &= \mathrm{X}[Q, Y^*] - Q\mathrm{M}[Q, Y] \end{aligned} \tag{7.3}$$

Differentiating with respect to the real exchange rate Q, we get

$$\begin{aligned} \frac{\partial \mathrm{TB}}{\partial Q} &= \mathrm{X}_Q - Q\mathrm{M}_Q - \mathrm{M} \\ &= \mathrm{M}\left(\mathrm{X}_Q \frac{Q}{\mathrm{X}} \frac{\mathrm{X}}{Q\mathrm{M}} - \mathrm{M}_Q \frac{Q}{\mathrm{M}} - 1\right) \\ &= \mathrm{M}\left(\epsilon_x \frac{\mathrm{X}}{Q\mathrm{M}} + \epsilon_m - 1\right) \\ &= \mathrm{M}\left(\epsilon_x \frac{\mathrm{TB}}{Q\mathrm{M}} + \epsilon_x + \epsilon_m - 1\right) \end{aligned} \tag{7.4}$$

where ϵ_x is the real exchange rate elasticity of exports and ϵ_m is the real exchange rate elasticity of imports. From an initial position of trade balance (so that $\mathrm{X} = Q\mathrm{M}$), this simplifies to

$$\frac{\partial \mathrm{TB}}{\partial Q} = \mathrm{M}(\epsilon_x + \epsilon_m - 1) \tag{7.5}$$

which is positive iff the Marshall-Lerner condition is satisfied.

When we attempt to apply this analysis to the devaluation of a pegged exchange rate, we must recall that devaluation is most commonly considered in situations of balance of payments difficulties. This means that the more general condition (7.4) is of considerable interest. Note that when the trade balance is in deficit (so that $\mathrm{X} < Q\mathrm{M}$) satisfaction of the Marshall-Lerner condition is not sufficient to improve the trade balance. The intuition lies in the larger price effect, which in this setting falls on import prices.

Link to Houthakker-Magee Critique

Consider two countries with growing income and an initial balance of trade. Then

$$\mathrm{X}[Q, Y^*] = Q\mathrm{M}[Q, Y] \tag{7.6}$$

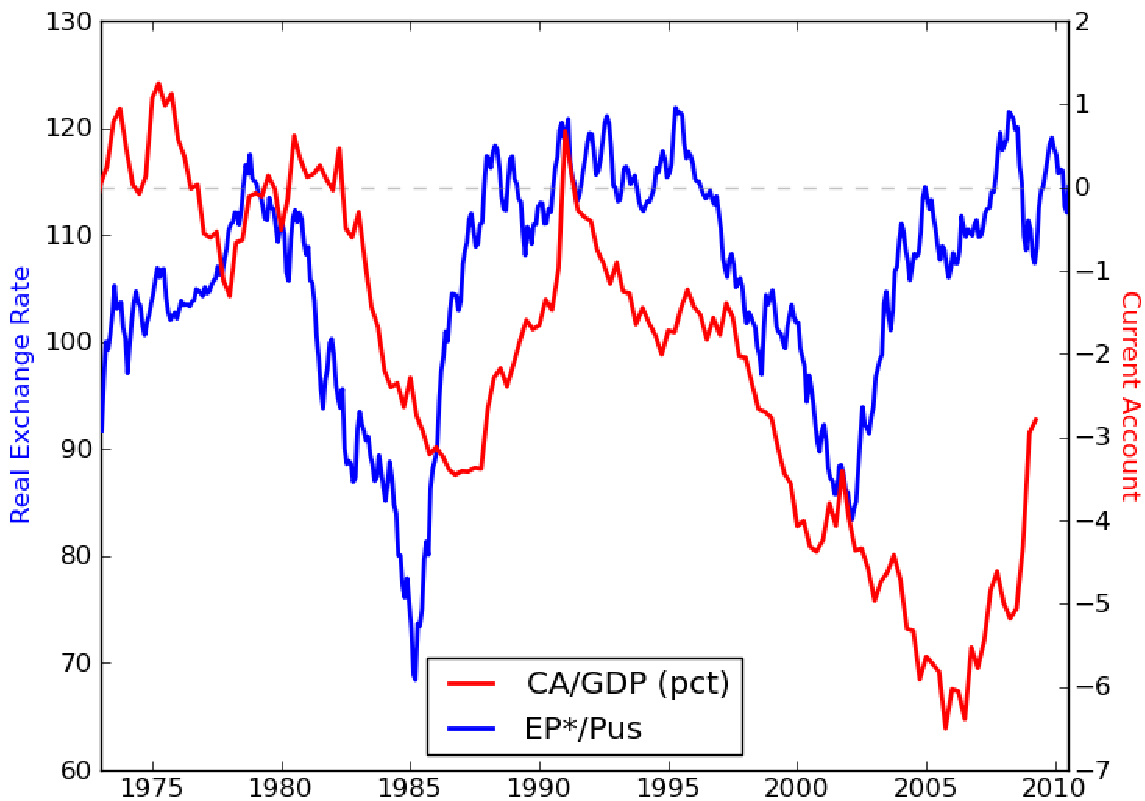
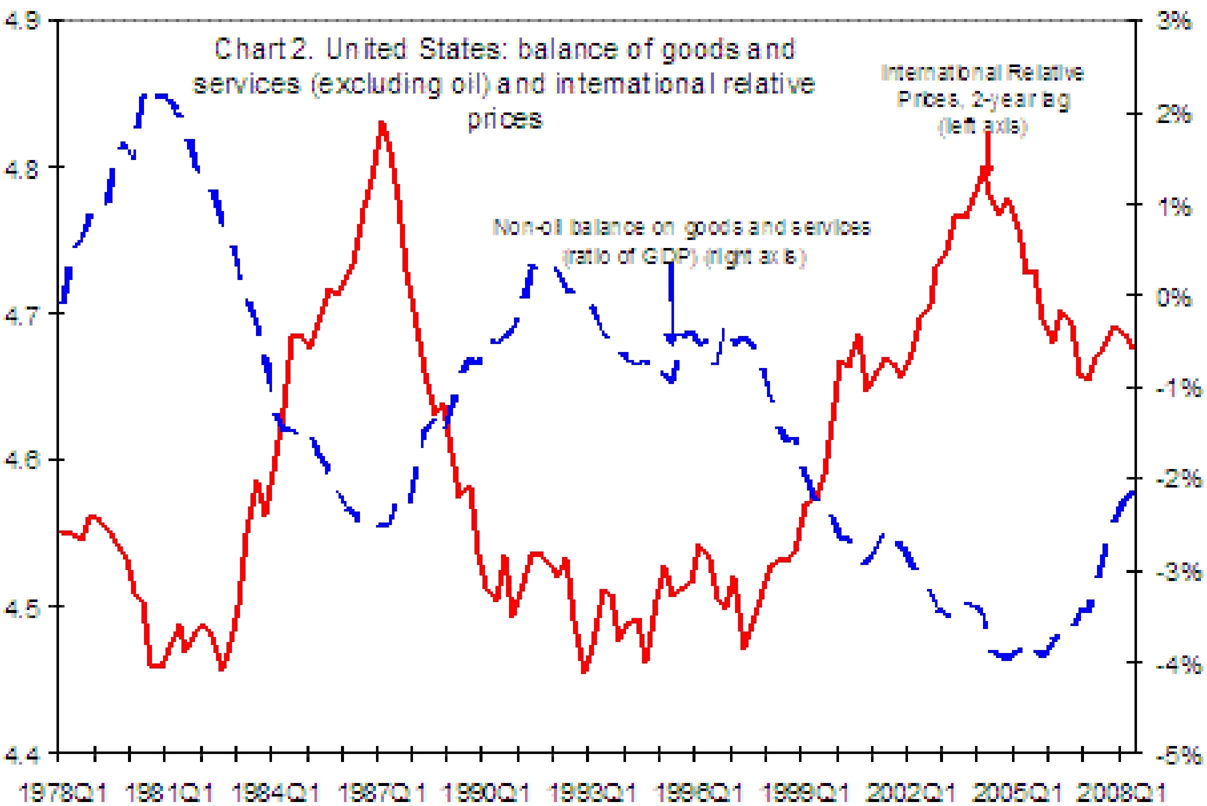


Figure 7.3: U.S. Current Account and Real Exchange Rate (Q)



Source: `http://www.voxeu.org/index.php?q=node/2684`

Figure 7.4: U.S. Trade Balance (No Oil) vs. Real Exchange Rate ($1/Q$)

In order to maintain trade balance, we need the left and right sides to grow at equal rates. That is, we need the rate of growth of exports to equal the rate of growth of the real value of imports. Letting ξ_x be the income elasticity of exports, the growth rate of the left side is

$$\hat{X} = \xi_x \hat{Y}^* + \epsilon_x \hat{Q} \tag{7.7}$$

Letting ξ_m be the income elasticity of imports, the growth rate of the right side is

$$\hat{M} = \xi_m \hat{Y} - \epsilon_m \hat{Q} \tag{7.8}$$

To maintain trade balance we need

$$\hat{X} = \hat{M} + \hat{Q} \tag{7.9}$$

or equivalently (as we saw above)

$$\xi_x \hat{Y}^* + \epsilon_x \hat{Q} = \xi_m \hat{Y} - \epsilon_m \hat{Q} + \hat{Q} \tag{7.10}$$

From this we can solve for the real exchange rate growth rate that is required to maintain trade balance:

$$\hat{Q} = \frac{\xi_x \hat{Y}^* - \xi_m \hat{Y}}{1 - \epsilon_x - \epsilon_m} \tag{7.11}$$

We see that there will be a trend in the real exchange rate unless

$$\frac{\xi_x}{\xi_m} = \frac{\hat{Y}}{\hat{Y}^*} \tag{7.12}$$

Johnson (1958) noted the implication that trade can become increasingly unbalanced at a given real exchange rate. Houthakker and Magee (1969) provide an early empirical analysis suggesting that income elasticities of some countries are indeed divergent enough for this to be the case. For example, they find Japan's income elasticity of exports to be nearly three times its import elasticity, while in the UK the income elasticity of exports is only half the import elasticity.

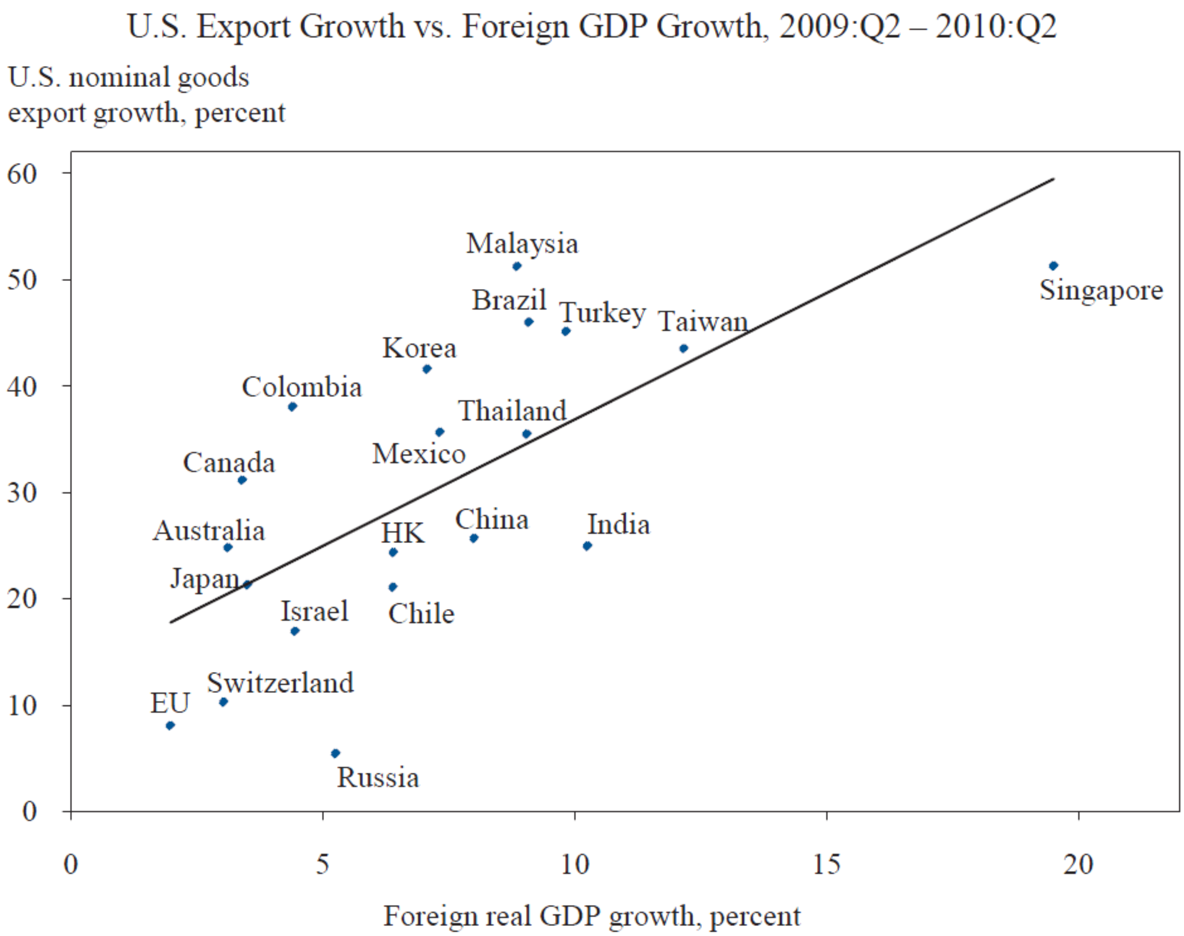
Foreign Exchange

We can use our derivation of the Marshall-Lerner condition to determine the impact of a domestic currency depreciation on our earnings of foreign exchange. Note that

$$\text{TB}^{FX} = P^* \frac{\text{TB}}{Q}$$

Given the foreign price level, it should be clear that with an initial trade balance of zero the Marshall-Lerner condition is again necessary and sufficient for a depreciation to improve foreign exchange earnings: $\text{TB} > 0$ implies $\text{TB}^{FX} > 0$.

However, an interesting difference emerges when we consider the impact of a domestic



Source: ERP 2011 (Fig 4-11)

Figure 7.5: U.S. Exports and Foreign Growth

currency depreciation in the presence of a balance of trade deficit: recall from our earlier discussion that the Marshall-Lerner condition *is* sufficient to ensure that a depreciation will raise our foreign exchange earnings.

7.2 Elasticity Dynamics and the J-Curve

Are actual trade balance elasticities high enough to satisfy the Marshall-Lerner condition? The view known as elasticity pessimism answers "no" (Prais, 1962). The view became popular in the 1940s when in many countries devaluation failed to initiate trade balance improvements. Countries dependent on oil imports lent some support to elasticity pessimism in the 1970s. Oil was generally priced in dollars, and the demand for oil is inelastic in the short run. Devaluation against the dollar by oil dependent economies led to little change in the dollar value of their oil bill. Measured in domestic currency, the trade balance deteriorated.

Further, some early econometric estimates of the demand elasticities were low, lending some support to this view (Prais, 1962). Houthakker and Magee (1969) provide an early empirical analysis suggesting that import and export responses to relative prices are much more difficult to detect than responses to income. They suggest that this traces to simultaneity problems and to inadequacies of the available import and export price indices. These estimates have since been criticized for many shortcomings including their partial equilibrium assumptions, neglect of endogenous price responses (BRM), measurement error (especially in price data, which are often misinvoiced), aggregation problems, and neglect of elasticity dynamics. The last critique has received special attention.

7.2.1 The J-Curve

Intuition suggests that given more time, consumers and producers will offer a larger response to a given price change. The evidence is supportive: import and export elasticities appear larger in the long run than in the short run. By the early 1970s, Junz and Rhomberg (1965), Magee (1973), and many others had shown empirically that trade balance elasticities were duration dependent. Both theoretical and empirical investigations have continued to support this conclusion. For example, given a real depreciation, some research suggests that only half of the eventual quantity adjustments are completed in the first three years. Even after five years the adjustment is not quite complete.

Think of the U.S. experience in the 1980s. The dollar began a massive appreciation in 1980 which continued until 1985, at which point an equally dramatic depreciation began. The trade balance did respond to these real exchange rate movements, but with a two to three year lag. Other countries exhibit similar *elasticity dynamics*.

In the presence of elasticity dynamics, we can imagine that the Marshall-Lerner condition is violated in the short run yet satisfied over a longer period of time. In such circumstances we would see a J-curve: a currency depreciation would initially cause a trade balance deterioration, but eventually the trade balance would recover and improve. Belief in the J-curve is widespread, but it has proved quite difficult to reliably document.

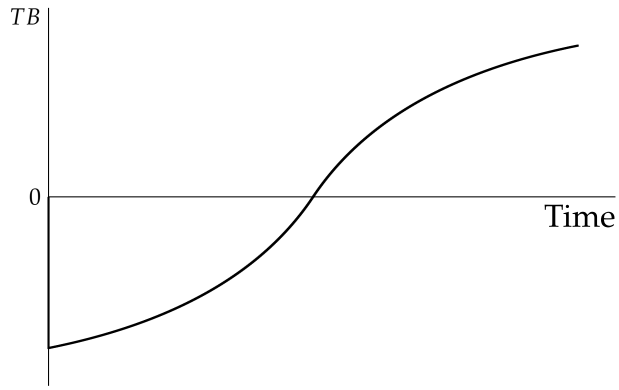


Figure 7.6: J-Curve Effects of Domestic Currency Depreciation

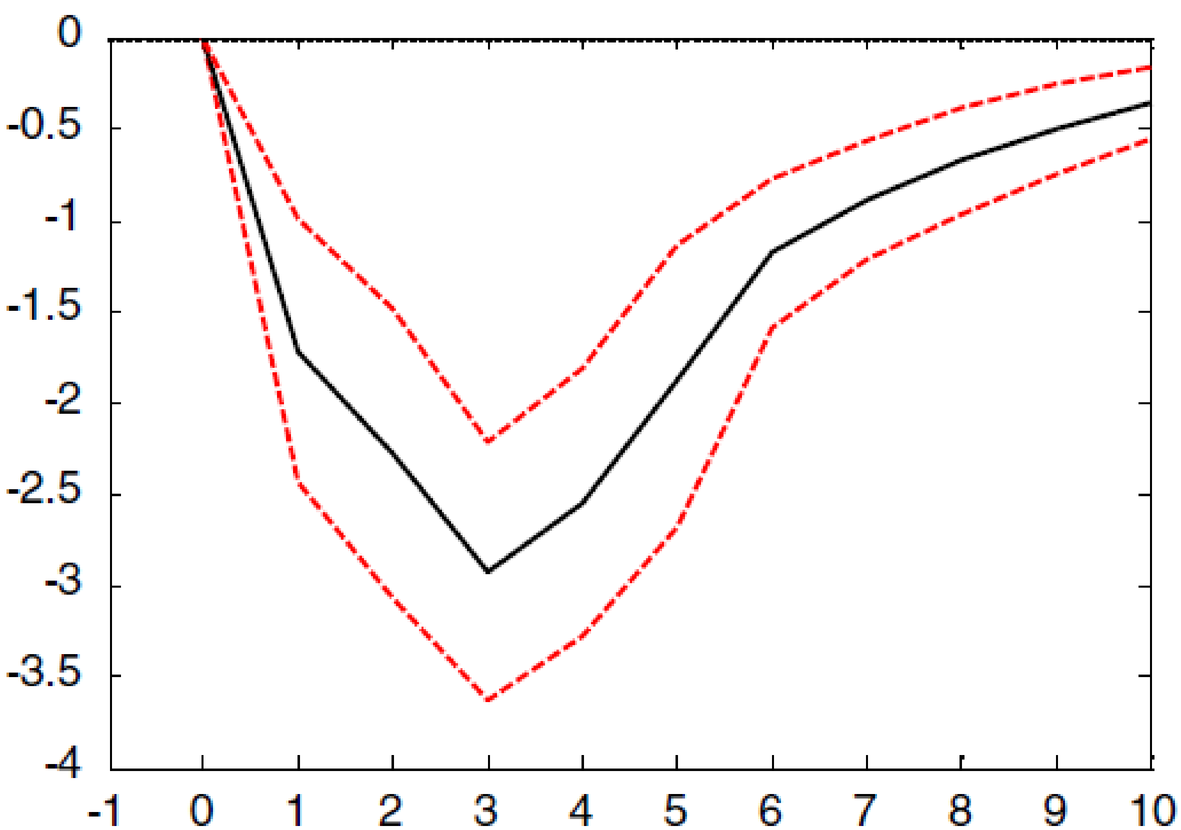


Figure 7.7: CA and a Real Appreciation Shock (Impulse Responses Across Years)
Source: Kappler et al. (2013)

Magee (1973) provides the basic theoretical framework. He distinguishes three periods following an exchange rate fluctuation: the currency contract period during which movement in the trade balance is largely determined by the currency denomination of previous contractual arrangements, the pass-through period during which the domestic price of foreign goods adjusts to the exchange rate changes, and the subsequent period of quantity responses to the changes in relative prices. Later theoretical elaborations highlight production and delivery lags, distribution bottlenecks, adjustment costs, decision lags, lags in distinguishing permanent and transitory changes in relative prices, intertemporal substitution effects resulting from anticipated price changes, and implicitly contracted long term trade relationships (Magee, 1973; Gerlach, 1989; Marquez, 1991). It is natural to expect habitual consumer expenditure patterns to play their role as well. Finally, firms are increasingly able to relocate their production facilites internationally.[1] The consistent prediction is that the long run real exchange rate elasticity of the trade balance exceeds the short run elasticity.

The unexpected adverse trade balance movements following the 1967 U.K. and 1971 U.S. devaluations led many early researchers to a more extreme position. For example, Dornbusch and Krugman (1976) argue that short-run elasticities may be small enough to violate the elasticity condition and generate a "J-curve": a perverse, negative short run response of the trade balance to a real depreciation, followed by an eventual improvement.

The events of the 1980s renewed interest in the J-curve, but the existence as well as the duration of any perverse effect remains in dispute. Moffett (1989) and Noland (1989) find protracted perverse responses of the U.S. and the Japanese trade balances. Marquez (1991) finds a perverse effect averaging five quarters for the U.S., while Artus (1975) finds only a one quarter perverse effect for the UK. Other work finds no indication of a perverse effect. Koch and Rosensweig (1990) do not detect an initial perverse response of the U.S. trade balance, although they suggest the possibility that delays in pass through may generate a delayed perverse effect. Felmingham (1988) confirms that the real exchange rate elasticity of the Australian trade balance increases over time, but he finds a normal trade balance response even in the first quarter following a real exchange rate change. Finally, Rose and Yellen (1989) argue that evidence of a perverse response of the U.S. trade balance stems from a neglect of aggregation, non-stationarity, and simultaneity problems.

Rose (1990) considers the elasticity issue for thirty developing countries. He tries to explain changes in the real trade balance for these countries with changes in the real exchange rate, domestic real income, and foreign real income.[2] He finds it impossible for most countries to reject the hypothesis that the real exchange rate has no effect whatsoever on the real multi-lateral trade balance.

---

[1]For example, during the dollar appreciation of the early 1980s, U.S. companies moved production abroad. With the subsequent depreciation, some companies returned to domestic production. In addition, foreign firms shifted production to the U.S. (Japanese auto manufacturing may be the best known example.) Note that if some U.S. companies may continue to operate their new foreign production, despite the dollar reversal. Such permanent change in response to temporary shocks is known as hysteresis.

[2]It is crucial to note that he uses an instrument for the real exchange rate and does not report results for the actual real exchange rate. Lags of the regressors are included to accommodate elasticity dynamics. Rose (1990) also includes an error correction term. He presents both annual (1970–1988) and quarterly (1977–1987) results. Real income is only available annually for these countries, so he proxies real income with real money.

TABLE 17A2-1 Estimated Price Elasticities for International Trade in Manufactured Goods

	η			η^*		
Country	**Impact**	**Short-run**	**Long-run**	**Impact**	**Short-run**	**Long-run**
Austria	0.39	0.71	1.37	0.03	0.36	0.80
Belgium	0.18	0.59	1.55	—	—	0.70
Britain	—	—	0.31	0.60	0.75	0.75
Canada	0.08	0.40	0.71	0.72	0.72	0.72
Denmark	0.82	1.13	1.13	0.55	0.93	1.14
France	0.20	0.48	1.25	—	0.49	0.60
Germany	—	—	1.41	0.57	0.77	0.77
Italy	—	0.56	0.64	0.94	0.94	0.94
Japan	0.59	1.01	1.61	0.16	0.72	0.97
Netherlands	0.24	0.49	0.89	0.71	1.22	1.22
Norway	0.40	0.74	1.49	—	0.01	0.71
Sweden	0.27	0.73	1.59	—	—	0.94
Switzerland	0.28	0.42	0.73	0.25	0.25	0.25
United States	0.18	0.48	1.67	—	1.06	1.06

Source: Estimates are taken from Jacques R. Artus and Malcolm D. Knight, *Issues in the Assessment of the Exchange Rates of Industrial Countries*. Occasional Paper 29. Washington, D.C.: International Monetary Fund, July 1984, table 4. Unavailable estimates are indicated by dashes.

Table 7.1: Import and Export Elasticities over Time

Table Source: Krugman, Obstfeld, and Melitz, Table 17A2-1

In sum, both empirical and theoretical investigations indicate that elasticity dynamics are an important determinant of the response of the trade balance to real exchange rate changes. Whether the short run elasticity is small enough to generate a J-curve is more controversial.

7.3 A Simple "Classical" Model

The flexprice monetary approach model treats the real exchange rate as exogenous, so it cannot help us understand variations in the real exchange rate. We have seen that absolute PPP is clearly violated, and that even relative PPP is violated at least in the short run. This suggests that if we wish to develop an exchange rate model capable of making short run predictions, we will need to accommodate PPP violations.

Our first approach to such violations will draw on the response of the trade balance to the real exchange rate, discussed in the previous section. We will incorporate this response into a model of exchange rate determination. In this section, we simply incorporate trade-balance effects into our simple Classical model. This produces the Classical version of the Mundell-Fleming model, where net exports are less than perfectly elastic with respect to variations in the real exchange rate. This allows real shocks, including fiscal policy shocks, to influence the real exchange rate.[3]

Let us begin with our simple Classical model. This treats nominal money, real income, and the real interest rate as exogenous constants. The money market is in constant equilibrium, once again represented by (7.13).

$$\frac{H}{P} = L[i, Y] \tag{7.13}$$

On the left is the real money supply. On the right is real money demand, also known as liquidity preference. In figure 7.8, we we draw the combinations of price level and spot rate such that (7.13) holds. As usual, since we are asking that *liquidity preference* equal the real *money supply*, we label this the "LM curve". For simplicity, we have left out any portfolio balance effects.[4]

In our simple monetary approach model, goods market equilibrium was simply a matter of purchasing power parity. Now however we recognize that the real exchange rate—as the relative price of foreign goods—influences the demand for a country's output. That is, aggregate demand is influenced by the real exchange rate, as well as by the interest rate, income, and fiscal policy.[5] Goods market equilibrium obtains when output equals the demand for output. In a Classical setting, output is exogenous, and the real exchange rate adjusts to achieve goods market equilibrium.

$$Y = \mathrm{AD}[i, Y, G, SP^*/P] \tag{7.14}$$

We will use (7.13) and (7.14) to determine the price level and the spot rate; all other variables are as exogenously fixed. Macroeconomic equilibrium is represented by Figure 7.8. Looking at (7.13), we see that there is a unique price level that clears the money market. So the LM curve is vertical. The IS curve represents goods market equilibrium The slope of the IS curve is determined by the real exchange rate that clears the goods market. That is, if Q_o is the real exchange rate that satisfies (7.14), then the slope of the IS curve is Q_o/P^*.

Due to the exogeneity of real income and the proportional response of P and S to H, this model offers a particularly simple characterization of flexible exchange rates. Changes in the money supply have no real effects. A fiscal expansion, in contrast, causes a real appreciation.

Intuitively, the story is fairly simple. Consider monetary policy. Money is neutral in this Classical economy: changes in the nominal money supply do not affect any real

[3] Additionally, Lecture8, takes a second step by allowing for *sticky* prices in the short run. Sticky prices imply that even monetary policy can influence the real exchange rate in the short run.

[4] Portfolio balance effects are added in question 6.

[5] For the moment, we will ignore the role of wealth as determinant of money demand or aggregate spending. In chapter Lecture5, we added wealth considerations to the monetary approach model in order to develop the "portfolio balance approach". We also momentarily ignore the role of expectations. This allows us to treat the interest rate i as exogenously given. In the background it depends on the real interest rate and inflation expectations. Further, the domestic interest rate i is linked to the foreign interest rate i^* by the international capital markets. We will explore these links later.

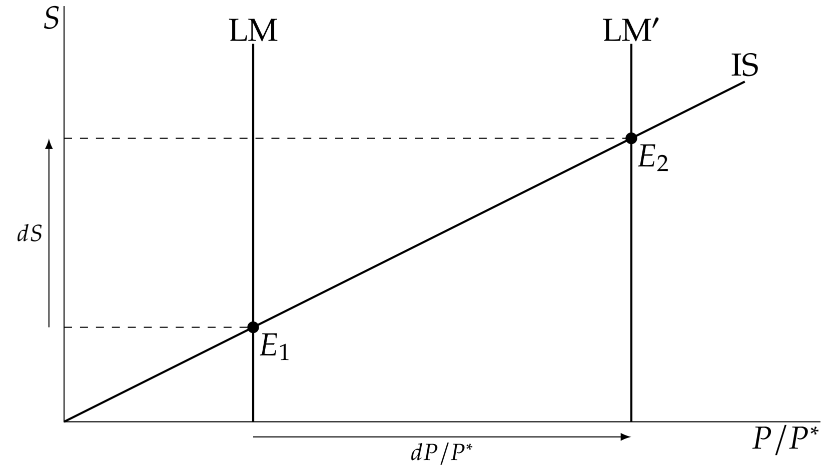


Figure 7.8: Money Supply Increase

variables. When we double the money supply in a Classical model the price level must double: since Y and i are given, the money supply increase shows up directly in the price level. Given the spot exchange rate, higher domestic prices reduce the demand for our goods, generating an excess supply in the goods market. The domestic currency must depreciate (the relative price of our goods must fall) to remove this excess supply and thereby restore equilibrium in the goods market.

If you are bothered by the lack of a more explicit causal story, which would clarify the forces behind the exchange rate movement, that is natural. In order to get a better feel for how such outcomes might be generated in a natural fashion, we need a model where the dynamics are explicit. We will introduce such a model in chapter Lecture8. Nevertheless, the comparative statics are readily represented in figure 7.8.

Now consider fiscal policy. Figure 7.9 represents the effects of a fiscal expansion. Expansion fiscal policy drives up demand in the goods market, but in this Classical model output is unchanged. You might expect this to drive up the price level, reducing real balances and increasing the interest rate. But since capital is highly mobile, such an interest rate differential would lead to immense capital inflows. Such capital inflows would appreciate the exchange rate and shift demand away from domestic production, removing the pressure on prices. So the effect of fiscal policy falls entirely on the exchange rate and the trade balance. A fiscal expansion causes a real exchange rate appreciation, which crowds out enough private demand to restore goods market equilibrium. This is another version of the *twin deficits* phenomenon, where the deterioration of the fiscal balance is accompanied by a deterioration in the trade balance.

As long as aggregate demand is unchanged, purchasing-power parity remains a good description of the model outcomes. However, even though this is a long-run model, it neither assumes nor implies purchasing power parity. As we have just seen, fiscal policy is one of the determinants of the long-run real exchange rate. Purchasing power parity requires that in the long run only relative prices are relevant to the determination of the exchange rate—a property that Edison (1987) calls ***exclusiveness***.[6] This simple Classical

[6]Edison (1987) reports violations of exclusiveness in almost a century of USD/GBP exchange rates.

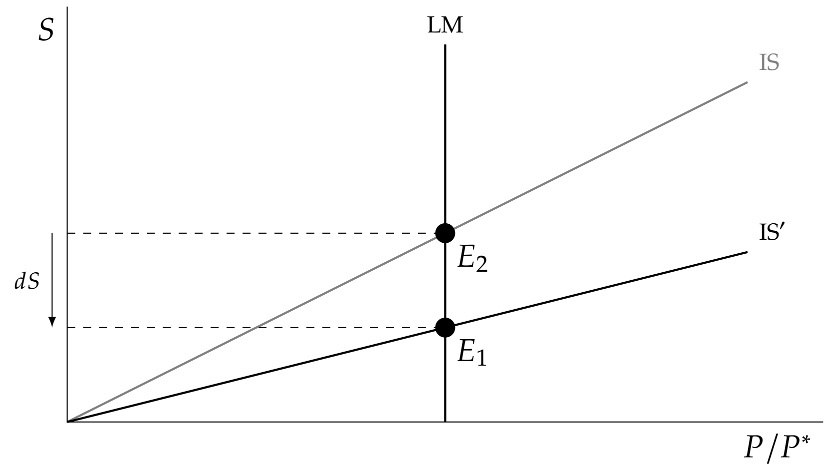


Figure 7.9: Fiscal Expansion: Classical M-F Model

model violates exclusiveness because aggregate demand is one determinant of the real exchange rate.

7.3.1 Algebra

Restate equations (7.13) and (7.14) in log-linear form as equations (7.15) and (7.16), which again represent the LM and IS curves in the simple Classical model.

$$h - p = \phi y - \lambda i \tag{7.15}$$

$$y = \rho(s + p^* - p) - \sigma i + g \tag{7.16}$$

The Classical version of the Mundell-Fleming model is based on the structural equations (7.15) and (7.16). Keep in mind that a model is more than a structure; it is also a specification of the endogenous variables. In the Classical Mundell-Fleming model, p is and s are endogenous.[7]

We can solve for s and p as follows.

$$p = h + \lambda i - \phi y \tag{7.17}$$

$$\begin{aligned} s &= h + \lambda i - \phi y - p^* + \frac{1}{\rho}(y - g + \sigma i) \\ &= h - p^* - \frac{1}{\rho}g + (\frac{\sigma}{\rho} + \lambda)i + (\frac{1}{\rho} - \phi)y \end{aligned} \tag{7.18}$$

Note the recursive structure of the model: to solve for the price level, we only need equation (7.15). We can then plug this solution for p into equation (7.16) to solve for the

Somewhat disturbingly, in one case (p.382) the violation is attributed to relative money supplies.

[7]Recall from footnote 5 we are putting off consideration of a number of factors: wealth effects, the role of expectations, and interest rate determination.

exchange rate.[8]

You can see that money remains neutral in this model: if we double the money supply, the price level and nominal exchange rate also double.[9]

End

7.4 Trade-Weighted Effective Exchange Rates

We like to speak of the effects of a change in "the" exchange rate. Of course every country has a multitude of exchange rates, one for every convertible currency in the world. Now changes in these exchange rates tend to be highly correlated, but they are not perfectly correlated. As a result, different exchange rates suggest different pictures of the change in the domestic currency's value. For example, the U.S. dollar may rise against the yen at the same time it falls against the Canadian dollar.

One reason for divergence among a country's exchange rates may be differing foreign inflation rates. That is, there may be a lot of nominal exchange rate movement without much real exchange rate movement. The purchasing power parity doctrine carries this observation to the extreme: nominal exchange rate movements do not cause any real exchange rate movements. As we have seen in our discussion of purchasing power parity, however, nominal exchange rate movements do in fact produce real exchange rate movements. So differing inflation rates is not the whole explanation of divergence among a country's exchange rates.

If all of a country's exchange rates behaved identically, there would be little harm in speaking of "the" exchange rate. Divergence makes this a riskier practice. Rather than pick a single exchange rate as representative of the value of the domestic currency, economists use a constructed measure known as the real effective exchange rate. The real effective exchange rate computes a weighted average of a country's exchange rates, where the weights are generally the foreign countries' trade shares. This is what we will generally mean by "the" real exchange rate.

For example, a widely cited index for the United States is published by the Board of Governors of the Federal Reserve System. It gives 10 nations' currencies exchange-rate weights based on the nations' importance in world trade. Is is therefore referred to as a *trade-weighted index*.

To construct an effective exchange rate, we need to select a currency basket, a set of weights, and a base year. The currency basket is the set of currencies that will be included in the effective exchange rate calculation. In a trade-weighted effective exchange rate, these will generally be the currencies of the country's most important trading partners. The

[8] This recursive structure is readily seen as a zero restriction in the matrix representation of this system:

$$\begin{bmatrix} 1 & 0 \\ -\rho & \rho \end{bmatrix} \begin{bmatrix} p \\ s \end{bmatrix} = \begin{bmatrix} h - \phi y + \lambda i \\ y + \sigma i - g - \rho p^* \end{bmatrix}$$

$$\begin{bmatrix} p \\ s \end{bmatrix} = \frac{1}{\rho} \begin{bmatrix} \rho & 0 \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h - \phi y + \lambda i \\ y + \sigma i - g - \rho p^* \end{bmatrix}$$

[9] Recall doubling H implies adding $\ln 2$ to h. Likewise for the exchange rate and price level.

weights will then reflect the relative importance of the currency basket members. There are two common choices: bilateral weights and multilateral weights. Bilateral weights are the most intuitive: a country in the currency basket receives a weight equal to its share of the home country's total trade with currency basket countries. A country's multilateral weight is based instead on its share of total trade among the currency basket countries.

Let S_i^b be the base year spot rate for country i. Let S_i^c be the current year spot rate for country i. Then the effective exchange rate in the current year is

$$EER_b^c = \sum_i w_i \frac{S_i^c}{S_i^b}$$

The effective exchange rate in the base year is always one. Similarly, the real effective exchange rate is

$$REER_b^c = \sum_i w_i \frac{Q_i^c}{Q_i^b}$$

Multilateral effective exchange rate indices are the most popular. The IMF publishes multilateral-weighted nominal and real effective exchange rates in *International Financial Statistics*. The *Federal Reserve Bulletin* publishes a multilateral-weighted nominal effective exchange rate. Another popular nominal effective exchange rate is the J.P. Morgan index, published by the *Financial Times* and by the *Wall Street Journal*. For the U.S., the Federal Reserve Bank of Atlanta publishes an index based on 1984 bilateral trade weights for 18 currencies, as well as subindices for Europe and for the Pacific Rim (also based on bilateral weights).

7.4.1 Properties of Effective Exchange Rates

Construction of an effective exchange rate involves many of the principles of price index construction. To construct a price index, we need to select a basket of goods, a set of weights, and a base year. Similarly, to construct an effective exchange rate, we need to select a currency basket, a set of weights, and a base year. Effective exchange rates also some basic properties with price indices. If we increase any single exchange rate, the index should increase. If we double every spot rate, the index should double.[10]

7.4.2 Calculating a Bilateral Effective Exchange Rate

Here is a simple example of the construction of a bilateral effective exchange rate. Suppose the home country trades with two other countries. Let the home country's exports from country A and country B be X_A and X_B, while its imports are M_A and M_B. The bilateral weight for country A will be

$$w_A = \frac{X_A + M_A}{X_A + M_A + X_B + M_B}$$

[10]That is, the effective exchange rate index should be homogeneous of degree one in the constituent exchange rates.

Similarly, the weight on country B will be

$$w_B = \frac{X_B + M_B}{X_A + M_A + X_B + M_B}$$

Note that the weights must sum to one. In a fixed-weight index, the weights are calculated only for the base year.

7.4.3 Composite Reserve Currencies

There are two well known artificial units of account based on baskets of national currency: the Special Drawing Right (SDR) of the International Monetary Fund, and the European Currency Unit (Ecu) of the European Monetary System. The construction of composite currencies closely resembles the construction of effective exchange rates. A fixed basket of currencies is given a set of weights based largely on trade flows.

Special Drawing Rights

For example, consider the SDR. From 1970–1980, the SDR was a weighted average of the currencies of the 16 largest trading countries. Basket composition and currency weights are reviewed twice per decade. A five-currency SDR was created in January 1981, at which point the SDR became a weighted sum of the USD, DEM, JPY, FFR, and GBP. Naturally the DEM and FFR were replaced once Germany and France adopted the EUR, leading officially to a four currency basket in 2001 (although the DEM and FFR had already been replaced by their EUR equivalents in 1999). The weights are based on the multilateral export volumes and relative importance as reserve currencies and are adjusted about twice a decade as ammendments to IMF Rule O-1. The USD has the largest weight in the basket, followed by the EUR, JPY, and GBP.

Currency	Amount	S^1	USD value
EUR	0.4100	1.35930	0.557313
JPY	18.400	93.12000	0.197595
GBP	0.0903	1.54220	0.139261
USD	0.6320	1.00000	0.632000
SDR			1.526169

[1] EUR-USD, USD-JPY, and GBP-USD.

Source: `http://www.imf.org/external/np/fin/data/rms_sdrv.aspx`

Table 7.2: SDR-USD Exchange Rate on 13 April 2010

Table 7.2 illustrates a typical SDR-USD exchange rate computation. There are four constituent currencies. The SDR is considered to contain a fixed amount of each constituent currency. Suppose we want to know the dollar value of an SDR: then we just need to add up how much it would cost to buy these amounts of all the constituent currencies. For example, if it takes USD 1.292 to buy EUR 1 then it must take USD 0.550392 to buy EUR

0.426. The dollar value of the SDR is just the sum of the dollar values of the fixed amounts of the constituent currencies.

European Currency Unit

The European Currency Unit, or ECU, was the precursor to the euro. In 1979, the European Monetary System introduced the ECU as the official unit of account for the European Union, with ISO currency code XEU. From 1979 until 1991 the currency basket and weights varied, but in 1991 the Maastricht treaty froze the composition and weights. Calculation of the USD-XEU exchange rate followed the same procedure as the calculation of the USD-SDR exchange rate. Note that the dollar wass not a constituent of the XEU, which as a result was seen as a hedge against the dollar. Further, EMS stabilization efforts keep the XEU relatively stable vis a vis the constituent national currencies. Perhaps as a result, the XEU won considerable private acceptance. Each XEU was replaced by one EUR in January 1999.

Problems for Review

1. Get the Federal Reserve Board's effective exchange rate for the dollar from the Federal Reserve Bank of St. Louis at `http://www.stls.frb.org/publ/usfd`. What is the base year? Has the dollar appreciated or depreciated since the base year?

2. Given constant income and price levels, graphically illustrate the effect of a rise in the exchange rate on the trade balance measured in foreign exchange.

3. Given constant income and price levels, what is the effect of an exchange rate depreciation on the "real" trade balance?

4. Update the exchange rates in table 7.2 to determine the current USD value of an SDR. Also, find the cost of an SDR in EUR, JPY, and GBP.

5. In the *Classical* revision of the Mundell-Fleming model, provide intuition and graphical analysis for the comparative static effects of an increase in each one of the following: y, i^*, and g.

6. In the *Classical* version of the Mundell-Fleming model, redo the analysis in question 5 after adding portfolio-balance effects to money demand (following chapter Lecture5).

7. In 1994 the two largest trading partners of the U.S. were Japan and Canada. Consider the following table:

	Canada	Japan
US exports	\$114,869	\$131,115
US imports	\$51,517	\$119,135

Given that the currency basket countries are Canada and Japan, show the bilateral trade weights for 1994 are 0.39 and 0.61.

8. Starting from a position of trade-balance deficit, satisfaction of the Marshall-Lerner condition is sufficient to improve the trade balance measured in foreign exchange but not to improve the real trade balance. Explain why.

Advanced Problems:

9. How do the comparative statics in problem 5 change if expenditures depend on real balances?

10. Refering to equation 7.4, provide a fully commented derivation of the Marshall-Lerner condition for a currency depreciation to improve the trade balance. Note that $\mathrm{TB}^{FX} = P^*\mathrm{TB}/Q$, and derive the related condition for the trade balance measured terms of foreign exchange. How do these differ if trade is not initially balanced?

11. In a "Keynesian" version of the Mundell-Fleming model, do the comparative statics algebra for a change in h and for a change in g. Provide the intuition behind your results.

Lecture 8

Exchange Rate Overshooting

Learning Goals

After reading this chapter, you will understand:

- some exchange-rate implications of sticky prices
- the effect of sticky prices on the neutrality of money
- why exchange rates may be more volatile than monetary policy
- how changes in monetary policy can cause exchange-rate overshooting

As developed in Lecture3, the flexprice monetary approach to flexible exchange rates relies on two key ingredients:

- the Classical model of price determination, and
- an exogenous real exchange rate.

This simple model met with some some notable early empirical successes, but these successes proved transient. As data accumulated under the post-Bretton Woods flexible exchange rate regimes, the simple monetary approach appeared increasingly unsatisfactory as a description of everyday exchange-rate behavior among countries with moderate inflation.

The present chapter explores the extent to which this empirical problem with the flexprice monetary approach model derives from its long-run orientation. Perhaps useful descriptions of short-run exchange-rate movements will require an approach that explicitly incorporates short-run considerations. As suggested by Lecture6, the imposition of continuous purchasing-power parity appears particularly suspect.

The widespread adoption of floating exchange rates in the early 1970s brought unexpected volatility of the nominal exchange rate. Large movements of the nominal exchange rate, coupled with the relative inertia of the price level, produced large fluctuations in the real exchange rate. Purchasing power parity failed to provide a helpful short-run guide to understanding exchange rates. In some sense, the flexprice monetary approach model allows real exchange rate movements, but it treats them as exogenous. Since real-exchange-rate movements are frequent and large, we would like to develop a theory that can explain them. The present chapter explores one explanation of short-run real-exchange-rate fluctuations: sticky prices.

The most popular models of short-run real-exchange-rate movements abandon the simplifying Classical assumption of perfectly flexible prices. In contrast with the flexprice monetary approach, these models treat the price level as *sticky* in the short run.

This chapter develops a simple sticky-price exchange-rate model. Increasing the scope of our models seldom comes without costs, and this attempt to endogenize the real exchange rate will be no exception. Doing so involves discarding two key simplifying assumptions of the flexprice monetary approach model: the exogeneity of the real interest rate, and the exogeneity of the real exchange rate. The resulting sticky-price model predicts large short-run deviations from purchasing power parity, such as those described in Lecture6.

In sticky-price models, monetary policy influences real interest rates and the real exchange rate. When prices are "sticky", any change in the nominal money stock becomes a change in the real money stock, which in turn generally implies a change in the interest-rate. Similarly, with sticky prices, short-run nominal-exchange-rate fluctuations will imply corresponding real exchange rate fluctuations. In a sticky-price setting, an explanation ofshort-run nominal-exchange-rate movements becomes an explanation of short-run real-exchange-rate movements. Sticky-price models also offer new insights into the sources of exchange-rate volatility. In particular, they offer an explanation of why exchange rates may be much more volatile than monetary policy.

8.1 Overshooting: Core Considerations

Suppose an economy is disturbed by an unanticipated exogenous change, which we call a ***shock***. This might be a change in monetary or fiscal policy, but it need not be policy related. We say the endogenous variable exhibits ***overshooting*** in response to this shock if its short-run movement exceeds the change in its steady-state value.

For example, given the long-run neutrality of money, a one-time, permanent increase in the money supply will eventually lead to a proportional depreciation of the steady-state value of the currency. If the short-run depreciation is more than proportional, then the short-run depreciation of the currency is greater than the depreciation of its steady-state value. We say such an economy exhibits exchange-rate overshooting in response to money-supply shocks. Similarly, if the short-run response is less than proportional, we may say the economy exhibits undershooting.

Popular models of such exchange rate overshooting have three key ingredients: covered interest parity, regressive expectations, and a liquidity effect of money supply changes. You are already familiar with the covered interest parity relationship: it is just the requirement that domestic assets bear the same rate of return as equivalent fully hedged foreign assets. This means that $i = i^* + \text{fd}$. If we decompose the forward discount on the domestic currency into expected depreciation and a ***risk premium***, then we can write this as $i = i^* + \Delta s^e + \text{rp}$.

Figure 8.1: Rudiger Dornbush (1942-2002)

Once again, discussion of exchange-rates determination requires dealing with exchange-rate expectations. Previous chapters considered analyses under ***static*** expectations and ***rational*** expectations. This chapter additionally explores the model under ***regressive*** expectations. (In the model of this chapter, the outcomes under regressive expectations encompass those under rational expectations.) Exchange-rate expectations are ***regressive*** when the exchange rate is expected to move toward its long-run value. This captures the common-sense idea that if you think that the exchange rate is currently overvalued, then you also expect it to depreciate toward its long-run value.

The final key ingredient is that monetary policy has a ***liquidity effect*** in the short run. In particular, expansionary monetary policy lowers the real interest rate in the short run. These three ingredients—covered interest parity, regressive expectations, and short-run liquidity effects—can be combined to illustrate why the exchange rate may overshoot in response to monetary policy changes.

8.1.1 Covered Interest Parity with Regressive Expectations

The first ingredient in our exchange-rate overshooting models is covered interest parity. In Lecture2 we saw that covered interest parity is implied by perfect capital mobility in the assets markets. Recall that covered interest parity is just the requirement of equivalent returns on equivalent assets. This is the first core ingredient of overshooting models. As usual, we can rewrite this as

$$i - i^* - \text{rp} = \Delta s^e \tag{8.1}$$

Here i is the domestic nominal interest rate, i^* is the foreign interest rate, rp is the expected excess return on the domestic asset, and Δs^e is the expected rate of depreciation of the spot exchange rate. Note the presence of the expected rate of depreciation. As with the simple monetary approach model, this makes consideration of exchange rate expectations unavoidable. Holding all else constant, covered interest parity implies that the domestic interest rate must be higher when expected depreciation is higher. This implied positive correlation between interest rates and expected depreciation is summarized in Figure 8.2.

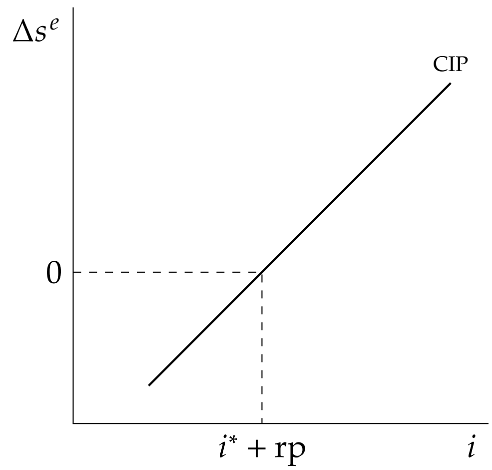


Figure 8.2: Covered Interest Parity

The interest rate varies with expected depreciation.

The second ingredient in our exchange-rate overshooting models is ***regressive*** expectations. Expectations formation is called *regressive* when a variable is expected to close any gap between its current level and its long-run equilibrium level. So if exchange-rate expectations are regressive, a spot rate that is above its long-run equilibrium level is expected to fall towards that level. The expected rate of real depreciation depends on the gap between the current exchange rate and its long-run equilibrium level.[1]

$$\Delta s^e = -\Theta[S/\bar{S}] \tag{8.2}$$

Here $\bar{S}$ is the *full-equilibrium* value of the spot rate, as elaborated below. The function Θ determines the expected speed of adjustment of the real exchange rate toward its long-run equilibrium value, based on the current gap between the spot rate and its full-equilibrium value. When S is relatively high, it is expected to fall; when it is relatively low, it is expected to rise. That, in a nutshell, is the regressive expectations hypothesis. The implied negative correlation between the current spot rate and the expected rate of depreciation is illustrated in Figure 8.3.

[1]A popular formulation of this relationship sets expected rate of depreciation to a proportion of the percentage gap between the spot rate and its long-run equilibrium level. Let δs represent the percentage deviation of s from its long-run value. We can then write the regressive expectations hypothesis as $\Delta s^e = -\Theta[\delta s]$. In terms of S, this becomes

$$\Delta s^e = -\Theta\left(\frac{S}{\bar{S}} - 1\right)$$

In an inflationary environment, expected depreciation is naturally given a slightly different formulation. See section 8.1.3.

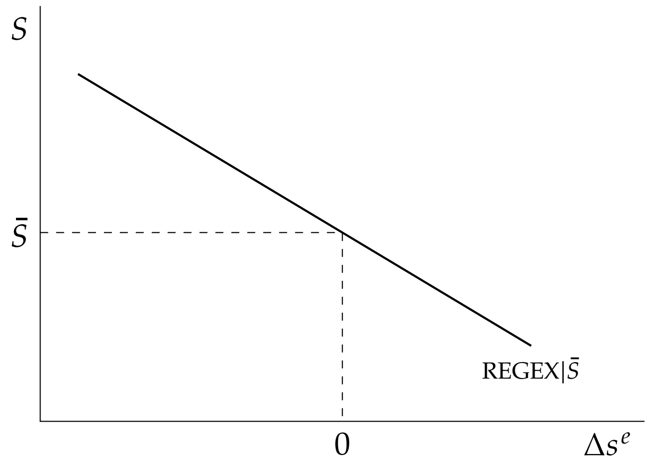


Figure 8.3: Regressive Expectations

Expected depreciation varies negatively with S.

In many ways, the regressive expectations hypothesis provides a plausible description of expectations formation. However, many economists become concerned whenever macromodels abandon the rational expectations hypothesis. At the theoretical level, it may therefore be reassuring that in many overshooting models, rational expectations proves to be a special case of regressive expectations. This will prove to be the case in the model below. In this case, regressive expectations are not only easier to model but actually encompass the behavior implied under rational expectations.

Given the central role played by regressive expectations in the model of overshooting, it is nevertheless natural to inquire as to the realism of this assumption. There are survey studies that offer some support for this the regressive expectations hypothesis. Frankel and Froot (1987) estimate expected speed of adjustment as 0.2 using such data, which offers a reasonable match to regression estimates of the speed of adjustment.

Recall that covered interest parity implies that the interest rate will be lower when the exchange rate is expected to appreciate. We also saw that under regressive expectations a currency is expected to appreciate when it is currently depreciated relative to its long-run equilibrium rate. (If it is high, it is expected to come back down.) Together these imply that the interest rate will be lower when the exchange rate is higher. That is, (8.1) and (8.2) together imply (8.3).

$$i - i^* - \text{rp} = -\Theta[S/\bar{S}] \tag{8.3}$$

For now, we treat the foreign interest rate as an exogenous constant (as in a small open economy). For now, we also treat the risk premium as exogenous.[2] Then for any given long-run equilibrium spot rate, equation (8.3) implies a negative relationship between the interest rate and the spot rate, which is represented by the CIPREG curve in Figure 8.6.

We draw a CIPREG curve for a *given* long-run equilibrium spot rate. For example, consider the CIPREG curve associated with $\bar{S}_1$ in Figure 8.5. We can deduce from the figure that the interest rate compatible with zero expected depreciation is i_0. You can see

[2]See Lecture10 for a discussion of the determination of the risk premium.

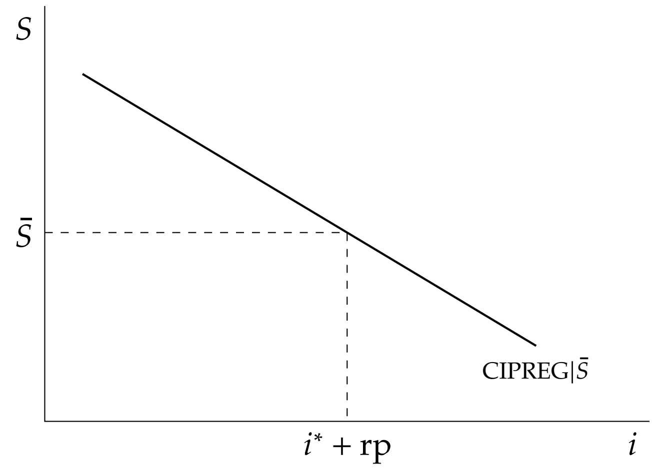


Figure 8.4: CIP with REGEX

Under covered interest parity and regressive expecations, the interest rate varies negatively with the spot rate because expected depreciation varies negatively with the spot rate.

this because the figure indicates that when the interest rate is i_0, the spot rate equals its long-run equilibrium level (from which it is not expected to move). When the interest rate is lower, say at i_{sr}, the spot rate must be higher. How high? It must be high enough that expected movement of the exchange rate, as implied by regressive expectations, offsets the interest rate differential, as required by covered interest parity.

Now consider the effect of a change in the long-run equilibrium spot rate. In Figure 8.5, the shift up of the CIPREG curve represents the effects of an increase in the long-run equilibrium spot rate from $\bar{S}_1$ to $\bar{S}_2$. The interest rate i_0 is compatible with covered interest parity only when expected depreciation is zero, but expected depreciation is zero only when the spot rate equals the long-run equilibrium exchange rate. Therefore the CIPREG curve shifts up by exactly the change in the long-run equilibrium exchange rate. If this takes place without a change in the interest rate, as in the monetary approach to flexible rates, then no overshooting is involved. However, in the presence of liquidity effects, this relationship between spot rates and interest rates provides a basis for exchange rate overshooting.

8.1.2 Liquidity Effects and Overshooting

The third ingredient in our exchange-rate overshooting models is the liquidity effects of monetary policy. In the Classical model, a one-time, permanent, unanticipated increase in the money supply has no effect on the interest rate. The interest rate is simply the fixed real rate plus expected inflation, and a one-time change in the money supply has no effect on expected inflation. A one-time, permanent increase in the growth rate of the money supply, on the other hand, has an effect on the interest rate. Expected inflation rises, and there is a corresponding rise in the interest rate. Again, the real interest rate remains unchanged. One way of describing these outcomes is to say that in the Classical model, monetary policy has no ***liquidity effects***.

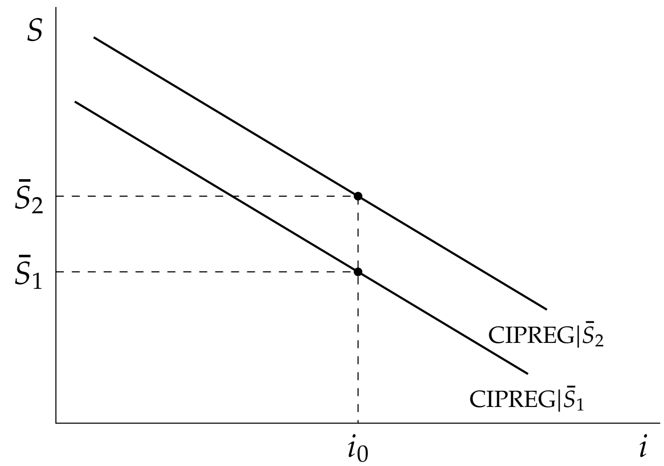


Figure 8.5: Change in $\bar{S}$

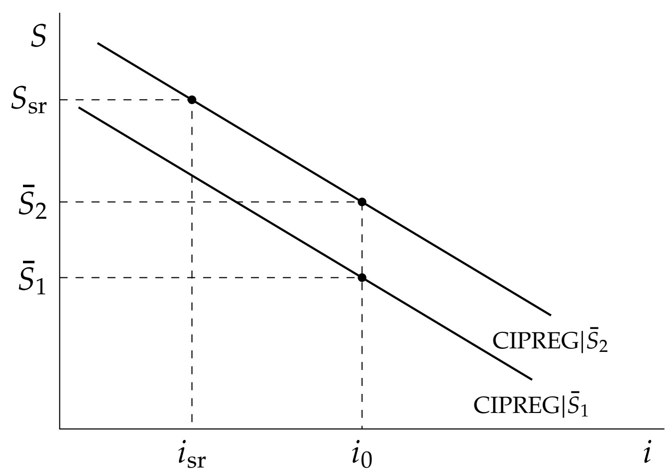


Figure 8.6: Exchange Rate Overshooting

When prices are sticky, interest rates are likely to change when the money supply changes. For example, a one-time, permanent, unanticipated increase in the money supply will increase the real money supply (since prices do not move proportionally). If the real interest rate falls in response, we say there is a liquidity effect. This can cause overshooting of the exchange rate. To see this, reconsider equation (8.3). Holding all other variables constant, a fall in the interest rate implies that $S/\bar{S}$ must rise. The increase in the spot rate must exceed the increase in the long-run equilibrium exchange rate. There is exchange rate overshooting.

These considerations are captured in Figure 8.6. The CIPREG curves represent covered interest parity under regressive expectations, at two different levels of $\bar{S}$. Let us start with an initial full equlibrium, based on $\bar{S}_1$. Starting from an initial steady state at $(i_0, \bar{S}_1)$, introduce a one-time, permanent, unanticipated change in the money supply.

By the long-run neutrality of money, we expect that the domestic currency must eventually depreciate proportionately, with no permament effect on the interest rate. We represent this as the increase from $\bar{S}_1$ to $\bar{S}_2$. The change in our beliefs about the full equlibrium exchange rate, $\bar{S}$, is the source of this shift. The upper CIPREG curve again represents covered interest parity under regressive expectations, but this curve incorporates the new steady-state spot rate $\bar{S}_2$.

In the absence of any liquidty effects, the economy could simply jump to the new full equilibrium at $(i_0, \bar{S}_2)$. However, the overshooting model acknowledges the existence of liquidity effect. The short-run liquidy effect is represented by the fall in the interest rate from i_0 to i_{sr}. As a result, we find that in the short run, the exchange rate depreciates from $\bar{S}_1$ to S_{sr}, overshooting the new steady-state spot rate.

How can (i_{sr}, S_{sr}) be compatible with equilibrium in international financial markets? After all, if we began with $i = i^*$, the liquidity effects of the monetary expansion forced $i < i^*$. As we know from covered interest parity, any risk-adjusted interest differential must be offset by expected depreciation. That is precisely what is happening at (i_{sr}, S_{sr}): the spot rate is high enough that expected appreciation of the domestic currency offsets the risk-adjusted interest differential.

This overshooting result turned on three elements: convered interest parity, regressive expectations, and a liquidity effect. If a one-time, permanent increase in the money supply has a liquidity effect, there is a fall in the interest rate. With high capital mobility, we might expect this lower interest rate to initiate massive capital outflows and thereby an exchange rate depreciation. However the interest parity condition can be satisfied if the lower interest rate is offset by an expected appreciation of the domestic currency. With regressive expectations, appreciation is expected only when the spot rate is above its steady-state value. Thus the spot rate does depreciate, but only until it is enough above the steady-state spot rate that expected appreciation offsets the interest differential. That is, the money supply increase causes a short-run depreciation of the spot rate that exceeds the depreciation of its steady-state value. There is exchange rate overshooting.

8.1.3 The Algebra of Overshooting

The algebra will again be a matter of covered interest parity, regressive expectations, and a characterization of liquidity effects. Recall that the covered interest parity condition can

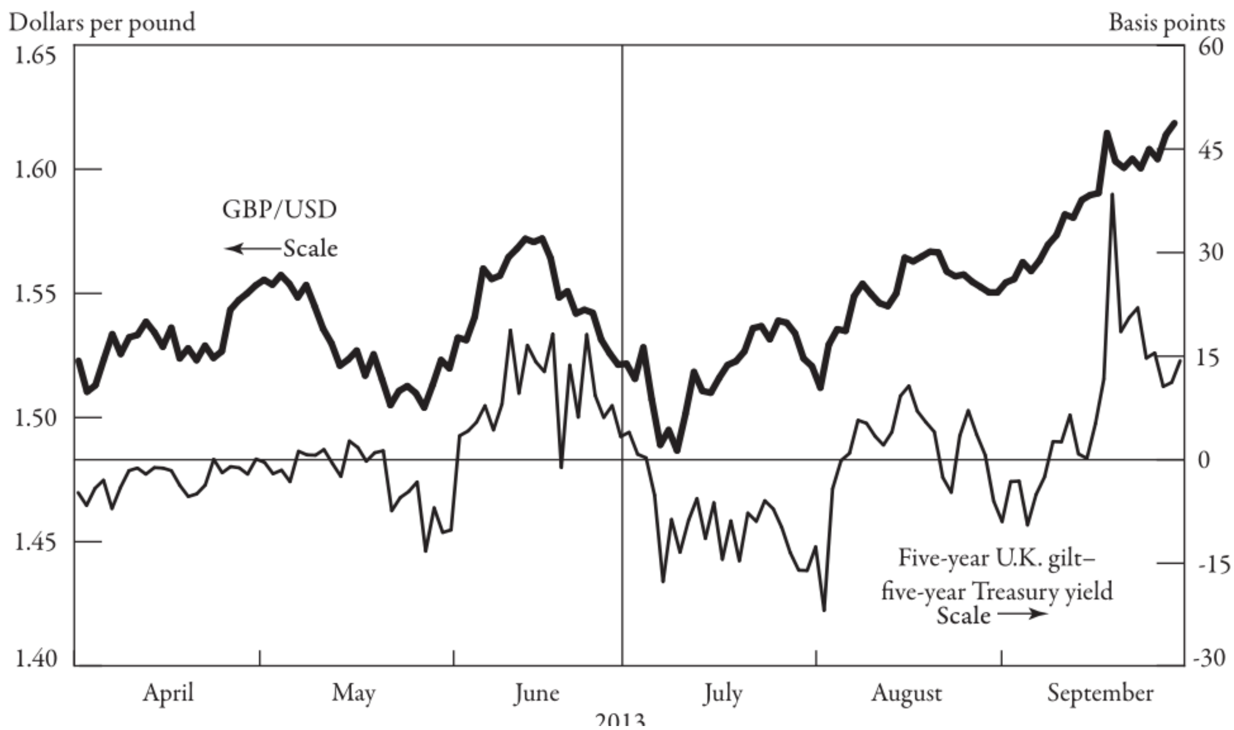


Figure 8.7: Interest-Rate Differential Versus Exchange Rate (UK vs US)
Source: `https://www.newyorkfed.org/medialibrary/media/newsevents/news/markets/2013/fxq313.pdf`, Chart 6.

be stated as in (8.1), which is restated; here for convenience.

$$i - i^* - \text{rp} = \Delta s^e \tag{8.1}$$

We will once again conjoin covered interest parity and regressive expectations, but before doing so we will slightly modify the regressive expectations hypothesis.

In an inflationary environment, expected depreciation of the nominal exchange rate is not naturally given the regressive formulation of equation (8.2). Letting $\bar{s}$ be the long-run (steady-state) value of s, we reformulate the regressive expectations hypothesis in the following natural way.[3]

$$\Delta s_t^e = \Delta \bar{s}_t^e - \theta(s_t - \bar{s}_t) \tag{8.4}$$

Again, θ is the speed at which the exchange rate is expected to move toward its long-run level, and $\Delta\bar{s}^e$ is the rate at which the full-equilibrium exchange rate is expected to change over time.[4] $\Delta\bar{s}^e$ is the rate at which the equilibrium spot exchange rate is expected to change, and it is a natural addition to expected depreciation. For example,

---

[3]In natural units, we could write this as

$$\%\Delta S^e = \%\Delta\bar{S}^e - \theta\frac{S - \bar{S}}{\bar{S}}$$

One way to look at this begins with regressive expectations for the *real* exchange rate.

$$\Delta q^e = -\theta\, \delta q$$

To move from our regressive expectations formulation for expected real exchange-rate depreciation to a characterization of expected nominal echange rate depreciation, first note that

$$\Delta s^e = \Delta q^e + \Delta p^e - \Delta p^{*e}$$

We can reasonably expect inflation even when the price level is at its "full equilibrium" level $\bar{p}$: the full equilibrium level can change over time (for example, as the money stock grows). We account for this by allowing inflation expectations to incorporate a "core-inflation" rate.

$$\begin{aligned}\Delta p^e &= -\theta\, \delta p + \Delta\bar{p}^e \\ \Delta p^{*e} &= -\theta\, \delta p^* + \Delta\bar{p}^{*e}\end{aligned}$$

Combining our characterizations of expected depreciation and expected inflation, we can characterize expected nominal exchage-rate depreciation as

$$\begin{aligned}\Delta s^e &= -\theta\, \delta q - \theta\, \delta p + \Delta\bar{p}^e \\ &= -\theta\, \delta s + \Delta\bar{p}^e - \Delta\bar{p}^{*e}\end{aligned}$$

As long as the is no long-run trend in q, this can be rewritten as

$$\Delta s^e = -\theta\, \delta s + \Delta\bar{s}^e$$

[4]A simpler formulation, lacking the term $\Delta\bar{s}^e$, of regressive expectations is often applied illustratively. See the formulation in section 8.1.1, which is essentially the illustrative treatment offered in, for example, **?** and Florentis et al. (1994). However, empirical work must allow for the long-run inflation observed in real economies. As a result, the equilibrium spot rate must be characterized as changing over time.

suppose the spot rate were at its current equilibrium value. It would still be expected to depreciate if the domestic country is a high inflation country, since such depreciation is required for the spot rate to remain at its equilibrium value over time. This implies that individuals take inflation differentials into account in forming their expectations about spot rate depreciation, and as we saw in the previous chapter, they are justified in doing so. In an inflationary environment, it is natural to allow expectations to include the underlying core-inflation differential.

Together, covered interest parity and regressive expectations imply (8.5).

$$i - i^* - \mathrm{rp} = \Delta\bar{s}^e - \theta(s - \bar{s}) \tag{8.5}$$

Solving (8.5) for s yields (8.6).

$$s = \bar{s} - \frac{1}{\theta}(i - i^* - \Delta\bar{s}^e - \mathrm{rp}) \tag{8.6}$$

Equation (8.6) has been the subject of a great deal of empirical scrutiny, which is discussed in section 8.2.1. First, however, we show that (8.6) implies overshooting by considering the effect of a one-time, permanent increase in the money supply.

$$\begin{aligned} \frac{\mathrm{d}s}{\mathrm{d}h} &= \frac{\mathrm{d}\bar{s}}{\mathrm{d}h} - \frac{1}{\theta}\frac{\mathrm{d}i}{\mathrm{d}h} \\ &> \frac{d\bar{s}}{dh} \end{aligned} \tag{8.7}$$

In the presence of liquidity effects, the increase in the money supply will immediately reduce the interest rate, so that $di/dh < 0$. We also expect that a permanent increase in the money supply will depreciate the long-run spot rate, so that $d\bar{s}/dh > 0$. So the spot rate moves move than the long-run equilibrium exchange rate. That is, the exchange rate overshoots in response to a one-time, permanent increase in the money supply.

8.2 The Long Run In Sticky-Price Models

Empirical work has focused on equation (8.6), which we repeat here for convenience.

$$s = \bar{s} - \frac{1}{\theta}(i - i^* - \Delta\bar{s}^e - \mathrm{rp}) \tag{8.6}$$

Before we can consider how well (8.6) fits the available data, we need to decide on a characterization of $\bar{s}$ and $\Delta\bar{s}^e$. Most empirical research has proceeded under the assumption of long-run PPP.

From the definition of the real exchange rate, we have

$$s_t = q_t + p_t - p_t^*$$

In our flexprice models of exchange rate determination, we treated the real exchange rate as exogenous and focused on explaining relative prices. This yielded a model of exchange rate determination. In sticky-price models, prices do not respond instantaneously to changes in economic conditions. We say that prices are *predetermined*, in the sense that their current value is determined by past economic conditions.[5] When the price level is predetermined, models of short-run nominal exchange rate movements also characterize short-run movements of the real exchange rate.

However even sticky-price models generally satisfy purchasing power parity in the long run. Although the real exchange rate fluctuates in the short run, it is assumed to have a long-run equilibrium value $\overline{q}$. We will therefore represent the *long-run equilibrium* spot exchange rate as

$$\bar{s}_t = \overline{q} + \bar{p}_t - \bar{p}^*_t \tag{8.8}$$

where $\bar{p}_t - \bar{p}^*_t$ is the *full equilibrium* value of the relative price level, as determined by the Classical model of price determination.

If we adopt the Classical model of price determination as our characterization of the full-equilibrium price level, we can proceed with various characterizations of the determination of $\bar{p} - \bar{p}^*$. Each will be associated with a characterization of the long-run equilibrium exchange rate, following our discussion in Lecture3. Table 8.1 offers a summary, where an overbar indicates a full-equilibrium value.[6]

$\bar{p} - \bar{p}^*$	Associated Model
$\bar{h} - \bar{h}^* - \phi(\bar{y} - \bar{y}^*) + \lambda(\bar{\imath} - \bar{\imath}^*)$	Crude Monetary
$\bar{h} - \bar{h}^* - \phi(\bar{y} - \bar{y}^*) + \lambda\Delta\bar{s}^e$	Standard Monetary
$\bar{h} - \bar{h}^* - \phi(\bar{y} - \bar{y}^*) + \lambda(\Delta\bar{p}^e_{t+1} - \Delta\bar{p}^{*e}_{t+1})$	Core Inflation

Table 8.1: Full-Equilibrium Relative Price Levels

How Long is the Long Run?

Duarte (2003) observes that inflation differentials have persisted among the countries of the European Monetary Union. It is even the case that inflation differentials (vis á vis Germany) have *increased* since the adoption of the common currency! One reason for this divergence may be that countries that entered the EMU with relatively low price levels, due to preceding deviations from purchasing power parity, may spend some time "catching up" under the common currency. However some countries, notably Ireland and Portugal, have continued to have much higher inflation rates than other EMU countries even after the introduction of the Euro as the medium of account in January 1999. The long-run looks to be much more than half a decade from this perspective. Another more subtle reason may be that Balassa-Samuelson effects are being observed. Region specific shocks may also contribute.

[5]Some sticky-price models allow a partial response of current prices to current economic conditions each period. See for example Papell.

[6]Note that $\overline{rp}$ has been assumed zero, so in the long run the uncovered interest parity condition is assumed. (This is easily relaxed.)

Figure 8.8: Jeffrey Frankel (1952-present, at Harvard's Kennedy School)

Indeed, price and even inflation differentials have even persisted across cities in the United States. Annual inflation rates can differ between cities by 1.6% annually for periods as long as a decade (Cecchetti et al., 2002). They estimate the "half life" of relative price convergence to be nine years. While persistence of price level differences is not surprising, the persistence of inflation differentials over such long periods is quite surprising.

8.2.1 The Real Interest Differential Model

If (8.8) is common knowledge, then

$$\Delta\bar{s}^e = \Delta\bar{p}^e - \Delta\bar{p}^{*e} \tag{8.9}$$

Here $\Delta\bar{p}^e - \Delta\bar{p^*}^e$ is the difference in the underlying long-run inflation rates. Using (8.9) to substitute for $\Delta\bar{s}^e$ in (8.6), we get an exchange rate model involving a kind of real interest differential.[7]

$$s = \bar{s} - \frac{1}{\theta}[(i - \Delta\bar{p}^e) - (i^* - \Delta\bar{p}^{*e}) - \text{rp}] \tag{8.10}$$

For this reason, this model is often referred to as the "real-interest differential" model of exchange rate determination. Keep in mind that in developing this model we used only three assumptions: perfect capital mobility, and regressive expectations, and long-run PPP (i.e., PPP for the long-run equilibrium spot rate).

Of course we can also substitute (8.8) for $\bar{s}$ to get

$$s = \overline{q} + \bar{p} - \bar{p}^* - \frac{1}{\theta}[(i - \Delta\bar{p}^e) - (i^* - \Delta\bar{p}^{*e}) - \text{rp}]$$

Frankel (1979) adopts the last of these, so that the core inflation model of exchange rate determination is our description of the long-run equilibrium spot rate, we can write (8.2.1)

[7] As Frankel (1979) notes, it is not precisely a real interest differential, as we are subtracting equilibrium inflation rates from actual short-term interest rates.

as

$$\begin{aligned} s &= \overbrace{\bar{q} + \bar{h} - \bar{h}^* - \phi(\bar{y} - \bar{y}^*) + \lambda(\Delta\bar{p}^e - \Delta\bar{p}^{*e})}^{\text{LR: monetary approach}} \overbrace{- \frac{1}{\theta}[(i - \Delta\bar{p}^e) - (i^* - \Delta\bar{p}^{*e}) - \text{rp}]}^{\text{SR: deviation}} \\ &= \bar{q} + \bar{h} - \bar{h}^* - \phi(\bar{y} - \bar{y}^*) + (\frac{1}{\theta} + \lambda)(\Delta\bar{p}^e - \Delta\bar{p}^{*e}) - \frac{1}{\theta}(i - i^*) - \frac{1}{\theta}\text{rp} \end{aligned} \tag{8.11}$$

Note the difference between (8.11) and (8.6). In (8.6) you see that the real interest differential determines the deviation from the long-run spot rate. But to implement the model empirically, we need an observable proxy for $\bar{s}$. The nominal interest differential and the expected inflation differential have different effects in (8.11) because the inflation differential is a determinant of $\bar{s}$.

We characterize Frankel's RID model in terms of four structural equations plus two simplifying auxiliary assumptions. Two of the four structural equations just give us the monetary approach predictions as a long-run outcome.

The structural equations:

- covered interest parity
- regressive expectations
- long-run purchasing power parity
- classical model of long-run price determination

The auxiliary assumptions:

- $\Delta\bar{s}^e = \Delta p^e - \Delta p^{*e}$
- observed exogenous variables are equal to their full-equilibrium levels

Empirical Implementation

Frankel (1979) considered the core inflation approach a better long-run model than short-run model of exchange rate determination. He therefore used it as only a piece of a model of short-run exchange rate determination, determining his *long run* exchange rate as in (8.11). In order to implement this model empirically, Frankel assumed that observed values of money and income equaled their full-equilibrium values. This gave him the following slightly modified version of (8.11).

$$s = \bar{q} + h - h^* - \phi(y - y^*) + (\frac{1}{\theta} + \lambda)(\Delta\bar{p}^e - \Delta\bar{p}^{*e}) - \frac{1}{\theta}(i - i^*) - \frac{1}{\theta}\text{rp} \tag{8.12}$$

Frankel contrasted this model, which he called the real interest differential model, with the crude monetary approach and core inflation approach. We make a similar comparison in table 8.2.

	Exchange Rate Model:		
	Crude	Core	Real Interest
Predicted Effect of:	Monetary	Inflation	Differential
Relative Money Supply $(h - h^*)$	1	1	1
Relative Real Income $(y - y^*)$	–	–	–
Interest Rate Differential $(i - i^*)$	+	0	–
Inflation Differential $(\pi - \pi^*)$	0	+	+

Table 8.2: Predictions of Alternative Exchange Rate Models

The first row of table 8.3 contains some coefficients estimates by Frankel (1979) for the DEM/USD exchange rate.[8] All coefficients have the correct sign and are of plausible size. In his initial (OLS) estimation, he found all estimated coefficients significant except for the interest rate coefficient. After some technical corrections (for serial correlation in the residuals and for possible regressor endogeneity) he also found a significant interest rate semi-elasticity. Frankel's results were seen as exciting initial support for the real interest rate differential model. Note that Frankel (1979) proxied expected inflation by long-term interest rates, so his empirical implementation can be seen as considering the role of long-term and short-term interest rates.

Author	Sample	$(h - h^*)$	$(y - y^*)$	$(i - i^*)$	$(\Delta\bar{p}^e - \Delta\bar{p}^{*e})$
Frankel (1979)	1974.07–1978.02	0.87	-0.72	-0.39	7.16
Frankel (1979)	1974.07–1978.02	—	-0.69	-0.44	7.53
Baillie and Selover (1987)	1973.03–1983.12	-1.404	-0.4806	0.0135	0.0080

Table 8.3: RID Model of DEM/USD Exchange Rate

The second row of table 8.3 contains additional results coefficient estimates reported by Frankel.[9] This time, Frankel imposed a coefficient of one on the relative money supplies, as implied by his theoretical model. Frankel notes that imposing this constraint addresses worries that central banks may vary money supplies in response to exchange rates, and may also improve the estimation if money demand shocks are important. However, the remaining coefficients are little changed, suggesting perhaps that these problems were not important at the time.

Later applications of the model proved less supportive. For example, extending the data set, Baillie and Selover (1987) found Frankel's earlier results no longer applied. The relative money supply and interest rate coefficients were significant but of the wrong sign.

[8]Frankel's ordinary least squares (OLS) results are reported in the table. He assumes rp = 0. His reported coefficients on interest rates and expected inflation have been divided by four to correspond to coefficients on annual rates.

[9]As in the reprint of this article in Frankel (1993), the interest rate and inflation semi-elasticities reported in Frankel (1979) have been divided by 4, so that they can be read as responses to changes in *annual* rates of return.

(Significant, perversely-signed coefficients on relative money supply were a common finding in later work—a significant problem for any "monetary" approach.) The coefficients on relative income and inflation had the right signs but are insignificant. Correcting for serial correlation in the residuals eliminated the sign difficulties, but then all these coefficients were insignificant. Baillie and Selover also found similar problems for other countries.

The real interest differential model has been widely tested. Although some initial research in the late 1970s generated promising results, extension of the sample period past 1978 typically generated insignificant, negatively signed coefficients on relative money supplies. (The theory predicts unity.) Backus (1984) is an exception in supporting this for Canadian/US data. This problem may be due to money supply endogeneity. It has been observed that there is certainly some simultaneity problem, since both i and h are regressors but at most one can be targeted by the monetary authority. Taking an example from Pentecost p.71, suppose depreciation raises i thereby reducing h. Another problem is that the residuals showed considerable serial correlation.

The real interest differential model can be modified to allow endogenous terms of trade. For example, Hooper and Morton (1982) include changes in the long run real exchange rate in their portfolio balance model. Their approach was simply to replace long run PPP with an expected long run real exchange rate, $\overline{q}_t^e$. Algebraically, the PPP equation is replaced with

$$\overline{s}_t^e = \overline{q}_t^e + p_t^e - p_t^{*e}$$

The result is very similar to the Frankel real interest differential model.

$$s_t = \overline{q}_t^e + h_t - h_t^* - \phi(y_t - y_t^*) + \lambda(\Delta p_{t+1}^e - \Delta p_{t+1}^{*e}) - \frac{1}{\theta}[(i_t - \Delta p_{t+1}^e) - (i_t^* - \Delta p_{t+1}^{*e})] - \frac{1}{\theta}\mathrm{rp}$$

Finally, there has been some attempts to endogenize the risk premium by linking it to relative asset supplies. This yields a model known as the portfolio balance model. (See Lecture5 for a discussion of this model.)

8.3 Money Market Considerations

As discussed in section 8.1.2, when prices are sticky we expect that monetary policy has liquidity effects. That is, we expect that expansionary monetary policy can lower the real interest rate in the short run. The simplest way to model these is with our standard representation of money market equilibrium.

Suppose the money market remains in constant equilibrium, which we continue to characterize as follows.

$$\frac{H}{P} = L[i, Y] \tag{8.13}$$

Let income and prices be sticky. In contrast to the Classical model, we can no longer depend on the price level to clear the money market. For the money market to clear when prices are sticky, changes in the money supply must be accommodated by changes in the interest rate.

Recall that the covered interest parity relationship and regressive expectations imply the negative relationship between the spot rate and the interest rate given by (8.3), which is repeated here for convenience.

$$i - i^{*} - \mathrm{rp} = -\theta\, \delta s$$

Solving this equation for the interest rate yields (8.14).

$$i = i^{*} + \mathrm{rp} - \theta\, \delta s \tag{8.14}$$

We can use (8.14) to substitute for the interest rate in (8.13), yielding (8.15).

$$\frac{H}{P} = L\left[i^{*} + \mathrm{rp} - \theta\ \delta s, Y\right] \tag{8.15}$$

If prices and income are "stuck" at their pre-shock values in the short run, a nominal money supply increase initially increases the real money supply. This lowers the equilibrium interest rate, which will generate disequilibrium capital outflows unless the exchange rate depreciates so far that it is expected to appreciate toward its long-run equilibrium value.

Equation (8.15) determines a unique spot rate that is compatible with any given price level. Recall that covered interest parity implies that the interest rate will be lower when the exchange rate is expected to appreciate. We also saw that, under regressive expectations, the exchange rate will be expected to appreciate when it is high (i.e., above its long-run equilibrium rate). Together these imply that the interest rate will be lower when the exchange rate is higher. We have now added a link between the price level and the interest rate. At a lower price level, there are higher real balances and the money market must therefore clear at a lower domestic interest rate. When combined with covered interest parity and regressive expectations, this yields a negative relationship between the price level and the exchange rate. Covered interest parity tells us this low interest rate must be offset by expected appreciation of the domestic currency. Capital outflows in response to the low interest rate will depreciate the currency until its expected future appreciation is just enough to offset the interest differential. So the combinations of P and S such that the assets markets clear can be represented by the LM curve in Figure 8.9.

The effects of an increase in the money supply can be represented as as shift out of the LM curve to LM′. At a given price level, such as P_0, an increase in the nominal money supply creates an increase in the real money supply. For the money market to clear, the interest rate must fall. Interest parity then requires expected appreciation of the domestic currency, which under regressive expectations implies a large depreciation of the spot rate.

We can fill in the story behind the exchange rate depreciation. With low domestic interest rates and high capital mobility, a large capital outflow will be initiated. This capital outflow drives up the spot rate. Only when the spot rate has risen far enough that it creates an adequate expectation of future appreciation does the incentive for capital outflow cease.

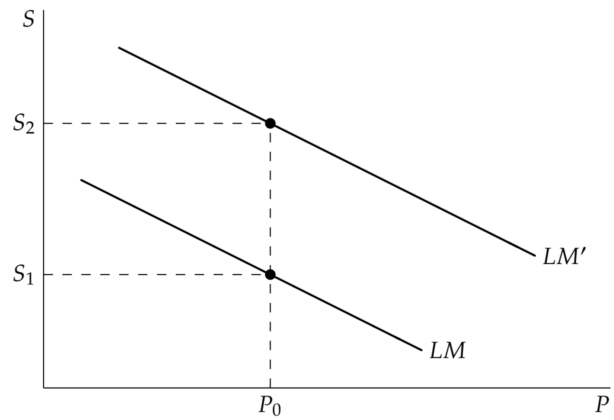


Figure 8.9: Exchange Rate Determination with Sticky Prices

8.3.1 Overshooting with Sticky Prices

We have seen how exchange rates are determined when prices are sticky. Let us review why this exchange rate movement is overshooting. First we need to characterize the full-equilibrium outcomes. We turn to the equilibrium described by the simple Classical model of Lecture7. In the Classical model, money is neutral: a one-time, permanent change in the money supply leads to a proportional change in prices and the exchange rate. We will now adopt this as our long-run description of the effect of a money supply change. Long-run neutrality of money implies that in the long run the price level and the exchange rate increase in proportion to the money supply increase, so that all real variables are unaffected. We will represent this by point A in Figure 8.10. Intuitively, if the Classical model adequately represents the long-run behavior of the exchange rate, we expect that a money supply increase should eventually lead to a proportional change in the exchange rate, a proportional change in the price level, and no change in the real exchange rate. A line from the origin through point A has a constant real exchange rate. (It is just our IS curve from the simple Classical model of Lecture7.) Point C must therefore represent the new long-run equilibrium, while point B represents the short-run response to the money supply increase. Clearly the short-run exchange-rate movement is greater than required to reach the long-run exchange rate. The excessive movement, $S_{\text{sr}} - \bar{S}_2$, is the exchange rate overshoot.

The Algebra

As in section 8.1.3, the algebra will be a matter of covered interest parity, regressive expectations, and liquidity effects. This time, however, the liquidity effects are explicitly motivated by sticky prices and income, along with continuous money market equilibrium.

Recall from section 8.1.3 that covered interest parity and regressive expectations imply (8.6), which we repeat here for convenience.

$$s = \bar{s} - \frac{1}{\theta}(i - i^* - \Delta\bar{s}^e - \text{rp}) \tag{8.6}$$

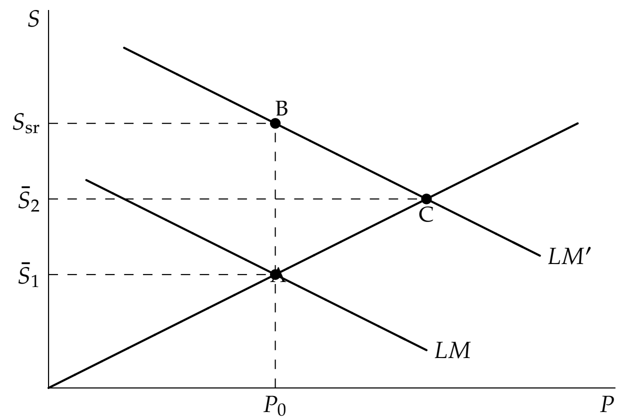


Figure 8.10: Exchange Rate Overshooting with Sticky Prices

We add money market considerations by turning to our usual representation of money market equilibrium.

$$h - p = \phi y - \lambda i \tag{8.16}$$

Solving (8.16) for i yields (8.17).

$$i = -\frac{1}{\lambda}(h - p - \phi y) \tag{8.17}$$

Together (8.6) and (8.17) imply

$$s = \bar{s} + \frac{1}{\theta\lambda}[h - p - \phi y + \lambda(\Delta\bar{s}^e + i^* + \text{rp})] \tag{8.18}$$

Equation (8.18) therefore combines three ingredients: covered interest parity, regressive expectations, and money market equilibrium.

Now consider a one-time, permanent change in the money supply, dh. Continue to treat the foreign interest rate and the risk premium as exogenous. This produces

$$\frac{\mathrm{d}s}{\mathrm{d}h} = \frac{\mathrm{d}\bar{s}}{\mathrm{d}h} + \frac{1}{\theta\lambda}(1 - \frac{\mathrm{d}p}{\mathrm{d}h} - \phi\frac{\mathrm{d}y}{\mathrm{d}h}) \tag{8.19}$$

If y and p are sticky, so that $\mathrm{d}y/\mathrm{d}h = 0$ and $\mathrm{d}p/\mathrm{d}h = 0$, then the overshooting result emerges immediately.

$$\begin{aligned}\frac{\mathrm{d}s}{\mathrm{d}h} &= \frac{\mathrm{d}\bar{s}}{\mathrm{d}h} + \frac{1}{\theta\lambda}\\ &> \frac{\mathrm{d}\bar{s}}{\mathrm{d}h}\end{aligned}$$

The long-run neutrality of money implies $\mathrm{d}\bar{s} = \mathrm{d}h$. So under long-run neutrality of money, we can write our overshooting result as

$$\frac{\mathrm{d}s}{\mathrm{d}h} = 1 + \frac{1}{\theta\lambda} \tag{8.20}$$

Here is another way to characterize the same result. Using the notation $\delta s = s - \bar{s}$, (8.18) implies

$$\delta s = -\frac{1}{\lambda\theta}(\delta p + \phi\, \delta y) \tag{8.21}$$

Starting from a steady state and increasing h (once, permanently) by one unit (at $t = 0$, say) implies in any model where prices and income are sticky and money is neutral in the long run that $\delta p_0 = -1$ and $\delta y_0 = 0$. The implication is that the exchange rate overshoots (that $\delta s_0 > 0$) since

$$\delta s_0 = \frac{1}{\lambda\theta} \tag{8.22}$$

Note that with a smaller the speed of adjustment (θ), or a less interest sensitive is the demand for money (λ), the larger is overshooting of the exchange rate.

8.4 Simple Empirical Considerations

8.4.1 Can We See the CIPREG Relationship?

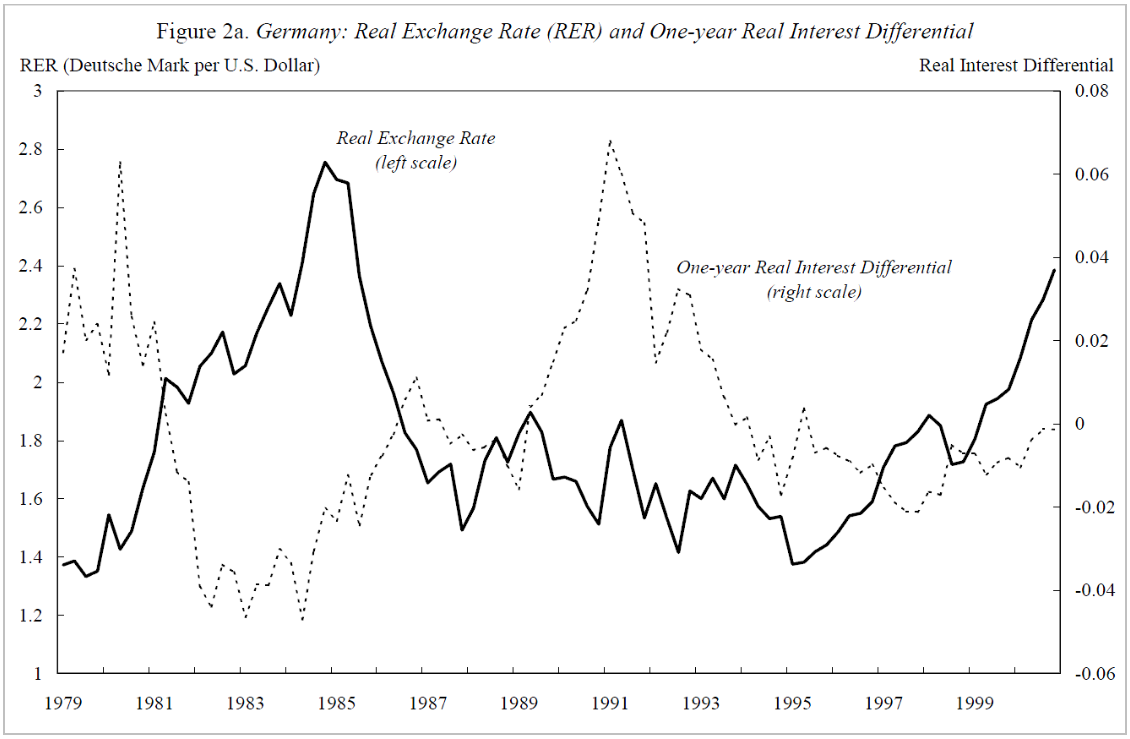
Any CIPREG relationship in the data proves empirically elusive. Meese and Rogoff (1988) fail to find a statistically significant relationship between the real exchange rate and the real interest rate. Furthermore, while time series plots appear promising for some countries, as in the German data illustrated in Figure 8.11, they do not for other countries, as in the UK data illustrated in Figure 8.12.

Taylor (1995) gives a somewhat more optimistic assessment. By considering the dollar against an aggregate of G-10 countries, he illustrates a fairly good long-term relationship between the real exchange rate and real interest rates, even though the short-run fluctuations diverge. This is illustrated in Figure 8.13.

8.4.2 Forward Rates

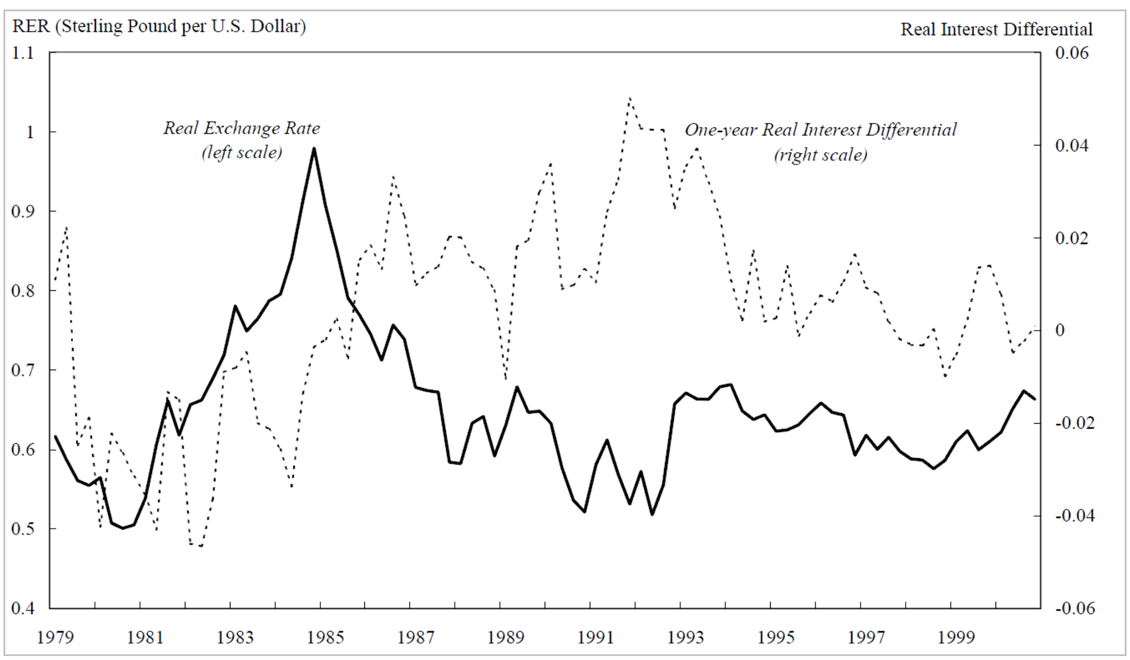
Flood (1981) points out that the overshooting model suggests that forward rates should be less responsive to monetary policy shocks than spot rates. Typically however they move very closely together, as illustrated in Figure 8.14.

> Does the empirical failure of the Mundell-Fleming-Dornbusch model mean that we have to reject it as a useful tool for policy analysis? Not at all. First, . . . the broader usefulness of the Mundell-Fleming-Dornbusch model goes well beyond the overshooting prediction. It is a generalized framework for thinking about international macroeconomic policy. Second, . . . the model does not necessarily predict overshooting when output is endogenous. Third, . . . consumption typically appears in place of output in the money demand



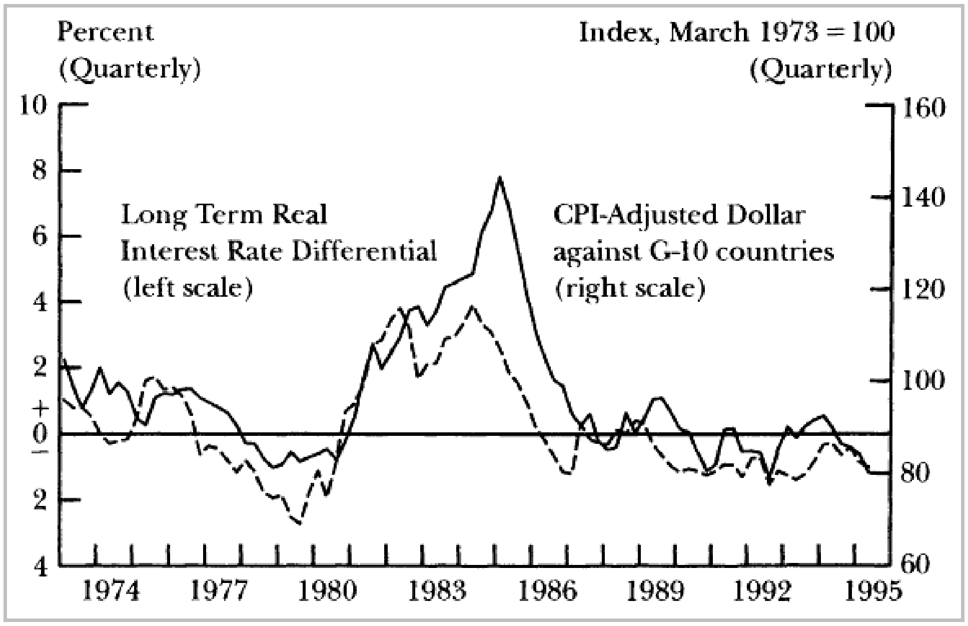
Source: Rogoff (2002)

Figure 8.11: Interest Differential ($r_{DE} - r_{US}$) and USD-DEM Exchange Rate



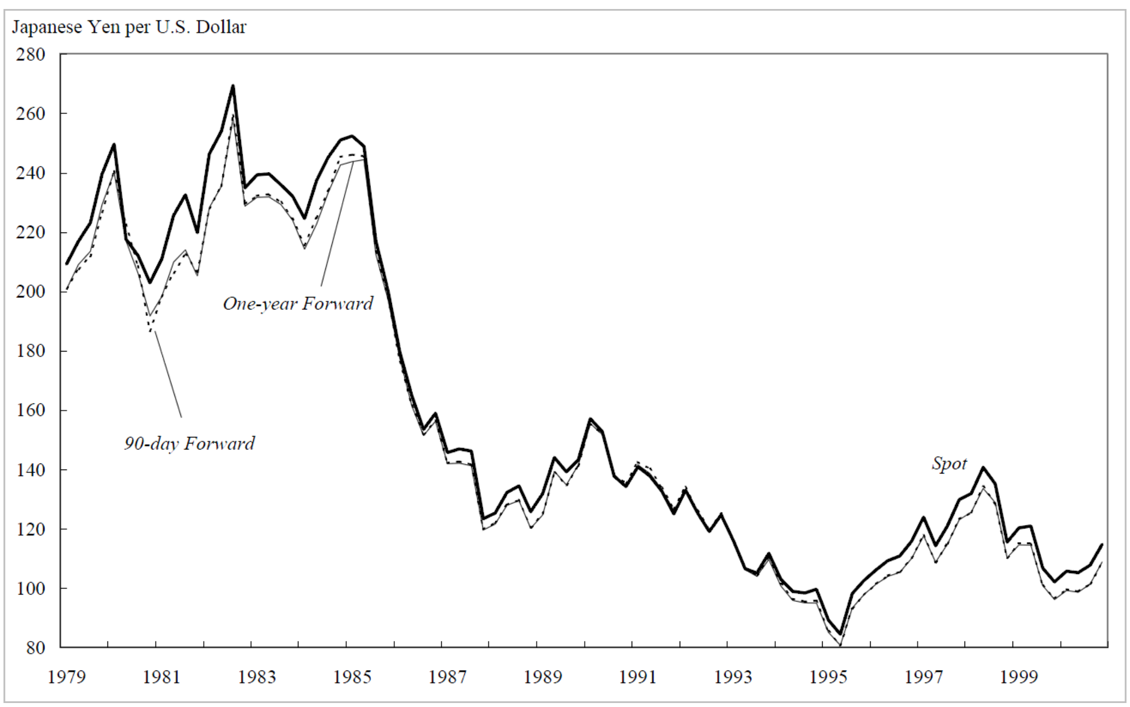
Source: Rogoff (2002)

Figure 8.12: Interest Differential ($r_{GB} - r_{US}$) and USD-GBP Exchange Rate



Source: Taylor (1995)

Figure 8.13: Interest Differential and the Dollar



Source: Rogoff (2002)

Figure 8.14: Spot and Forward Rates

> equations; this change also tilts the balance away from overshooting. . . . the apparent ability of the Dornbusch model to describe the trajectory of the exchange rate after major shifts in monetary policy is more than enough reason for us to press ahead and look more deeply at its underlying theoretical structure.
> Rogoff (2002)

8.4.3 VAR Analysis

Eichenbaum and Evans

The most famous VAR study of overshooting is probably Eichenbaum and Evans (1995). They create a variety of VAR models for a number of countries. As one measure of monetary policy they used the ratio of non-borrowed reserves to total reserves, which they call NBRX. (They also considered shocks to the federal funds rate.) Their sample is 1974.01–1990.05. In their first reported set of results, the variables are

- Y: US industrial production
- P: US CPI
- NBRX: non-borrowed reserves to total reserves
- $i^* - i$: interest differential
- S or Q: nomain or real exchange rate (two versions)

All variables were converted to logs except the interest differential. The countries considered (in separate VARs) were Japan, Germany, Italy, France, and the UK.

Results were very similar for the real and nominal exchange rate. (No surprise: we already know these are highly correlated.) A monetary contraction leads to a quick appreciation, as predicted by the overshooting model. But this appreciation continues over time before being reversed, a pattern that is sometimes called *delayed overshooting*. Monetary shocks look to be an important determinant of exchange rate dynamics. Additionally, they produce sustained deviations from covered interest parity.

The long hump in exchange rate response has been found in numerous studies. Scholl and Uhlig (2008) refer to it as the ***delayed overshooting puzzle***. As we will see, however, the overshooting puzzle is not much of a puzzle in the face of anticipated policy changes.

8.5 Conclusion

The simple overshooting approach to flexible exchange rates has two key constituents: covered interest parity and regressive expectations. The result is a very simple model of exchange rate determination that explains one of the key stylized facts of flexible exchange rate regimes: exchange volatility is high relative to the volatility of the underlying fundamentals.

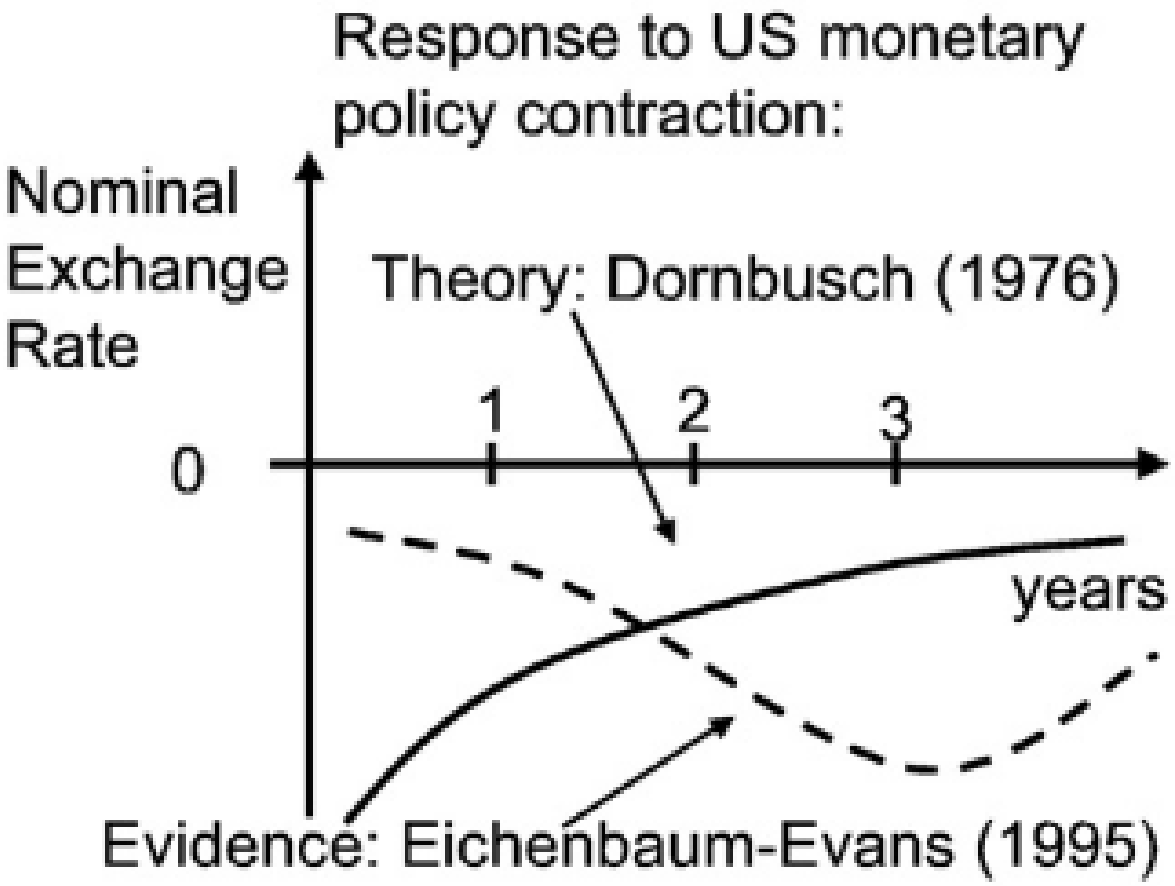


Figure 8.15: Delayed Overshooting Puzzle

Early empirical tests yielded encouraging support for the overshooting approach. Later tests proved much less satisfactory. In contrast with the monetary approach, the fact that much of the empirical work has used small samples of monthly data for countries with low average inflation rates does not raise our hopes. The overshooting approach was intended to augment the monetary approach's description of the fundamental long-run influences on the exchange rate with a story about short-run dynamics. The empirical failures of the overshooting approach are therefore extremely disappointing.

Nevertheless, the overshooting approach appears to offer some important insights. It suggests one reason why the exchange rate can be much more volatile than than the underlying fundamentals, which is clearly true with actual exchange rates. It also predicts that monetary tightening will lead to a rise in the real interest rate and an appreciation of the real exchange rate. Countries that move from high-inflation to low-inflation policy regimes often experience such effects (Rogoff, 1999).

Predictions of the overshooting approach include the following. The most important predictions is that an increase in the domestic money supply leads to a proportional depreciation of the spot exchange rate in the long run, but in the short run the depreciation is more than proportional. Furthermore, in the short run there will be a decline in the real interest rate, a depreciation of the real exchange rate, and a corresponding improvement in the trade balance. Recalling our work on the Classical model, we know that the long-run effects of an increase in domestic income are somewhat more complicated, since this will tend to appreciate the exchange rate through the money market while depreciating the real exchange rate through the goods market.

Expansionary monetary policy might be represented either as a change in the *level* or as a change in the *growth rate* of the money supply. In each case, the policy change may be a complete surprise or it may be anticipated. This leads to four possible scenarios. Once again, a primary lesson is that any effort to model exchange rates must pay careful attention to the role of expectations.

Problems for Review

1. Why do regressive expectations and covered interest parity imply a negative relationship between the interest rate and the exchange rate?

2. What is exchange rate overshooting and why is it important?

3. Explain the result in (8.22) verbally, without recourse to algebra.

4. Why does the combination of regressive expectations, covered interest parity, and liquidity effects imply overshooting in response to money supply changes?

5. How does Figure 8.3 change if we add a "core-inflation" rate to the expected rate of depreciation?

6. Consider Figure 8.6. Can an exogenous change in the risk premium cause overshooting?

7. Why does the combination of regressive expectations, covered interest parity, sticky prices, and money market equilibrium imply overshooting in response to money supply changes?

8. In the late 1970s, Argentina set its exchange rate according to a tablita: a series of small, preannounced devaluations. What does this imply for the nominal interest rate differential? At the same time the real interest rate in Argentina was high: what does this imply about expected depreciation?

9. Isard (1983) develops the link between real interest differentials and market perception of exchange rate overvaluation. In mid-1984, the real interest differential on U.S. 10 year bonds was around 3.5%/year. What does that imply for expected depreciation of the dollar. Let us say 10 years is enough time to return to PPP: what does the real interest differential say about the extent of overvaluation of the dollar?

8.6 Overshooting Price Dynamics in Continuous Time

> Dornbusch's explanation shocked and delighted researchers because he showed how overshooting did not necessarily grow out of myopia or herd behavior in markets.
> Rogoff (2002)

This section characterizes the sticky price dynamics in continuous time, which is a common theoretical treatment. The algebraic details in this section may be skipped by all MA students.

8.6.1 Regressive Expectations

Recall the basic (partially reduced) structural relationships of the model

$$\frac{H}{P} = L\,[i^* - \theta\ \delta s, Y] \tag{8.23}$$

$$\text{AD} = \text{AD}\,[i^* - \theta\ \delta s, Y, G, SP^*/P] \tag{8.24}$$

$$\dot{p} = f\left[\frac{\text{AD}\,[i^* - \theta\,\delta s, Y, G, SP^*/P]}{Y}\right] \tag{8.25}$$

For example, consider the implied movement around the long run equilibrium point $(\bar{p}, \bar{s})$. Define $\delta p = p - \bar{p}$ and $\delta s = s - \bar{s}$, the deviations of prices and the exchange rate from their long run level. Then rewrite money market equilibrium and the price adjustment equation in deviation form. This turns (8.23) and (8.25) into (8.26) and (8.27).

$$-\,\delta p = \lambda\theta\,\delta s \tag{8.26}$$

$$\text{D}\,\delta p = \pi(\rho + \sigma\theta)\,\delta s - \pi\rho\,\delta p \tag{8.27}$$

You will recognize this as a linear homogeneous first order differential equation system. One easy way to solve this system is to solve (8.26) for δs, plug this solution into (8.27), and solve the resulting differential equation in δp. The dynamics of δs can then be found by time differentiating (8.26) after substituting your solution for δp.

$$\begin{aligned} \mathrm{D}\,\delta p &= -\pi(\rho+\sigma\theta)/\lambda\theta\,\delta p - \pi\rho\,\delta p \\ &= -\pi[\rho(1+1/\lambda\theta)+\sigma/\lambda]\,\delta p \\ &= A\,\delta p \end{aligned} \tag{8.28}$$

Therefore

$$\delta p = e^{At}\,\delta p_0 \tag{8.29}$$

It makes sense to solve this in terms of δp_0, since p is a predetermined variable. Since $A < 0$, we are assured of the stability of the system.

It is also possible to attack the solution directly using the adjoint matrix technique. First, write (8.26) and (8.27) in matrix form using the differential operator.

$$\begin{bmatrix} -1 & -\lambda\theta \\ \mathrm{D}+\pi\rho & -\pi(\rho+\sigma\theta) \end{bmatrix} \begin{bmatrix} \delta s \\ \delta p \end{bmatrix} = 0 \tag{8.30}$$

The characteristic equation is

$$\lambda\theta\,\mathrm{D} + \pi(\rho\lambda\theta+\rho+\sigma\theta) = 0 \tag{8.31}$$

Solve this equation for the unique characteristic root.

$$\begin{aligned} \mathrm{D}_1 &= -\pi(\rho\lambda\theta+\rho+\sigma\theta)/\lambda\theta \\ &= -\pi\big(\rho(1+1/\lambda\theta)+\sigma/\lambda\big) \\ &< 0 \end{aligned}$$

Thus stability is assured. Recall that the adjoint matrix technique implies that (8.32) is the general solution to (8.26) and (8.27).

$$\begin{bmatrix} \delta p \\ \delta s \end{bmatrix} = k\exp\{D_1 t\} \begin{bmatrix} \lambda\theta \\ -1 \end{bmatrix} \tag{8.32}$$

Note that this involves a single arbitrary constant, so that we cannot offer arbitrary initial conditions for both δp and δs. Supply an initial condition for the predetermined variable δp_0, since prices cannot move instantaneously to clear the goods market. In contrast, the exchange rate can jump to maintain constant asset market equilibrium.

8.6.2 Rational Expectations

In this section we replace the regressive expectations hypothesis with the rational expectations hypothesis. Since there is no uncertainty in this model, the rational expectations hypothesis implies $\dot{s}^e = \dot{s}$. Given uncovered interest parity ($i = i^* + \dot{s}$), we can then use the money market equilibrium condition to express the rate of exchange rate depreciation in terms of p.

$$h - p = \phi y - \lambda(i^* + \dot{s}) \tag{8.33}$$

In the present model, $\dot{s} = 0$ in the long run. So we know

$$h - \bar{p} = \phi y - \lambda i^* \tag{8.34}$$

Comparing the short-run and long-run we see

$$p - \bar{p} = \lambda \dot{s} \tag{8.35}$$

which implies

$$\dot{s} = (p - \bar{p})/\lambda \tag{8.36}$$

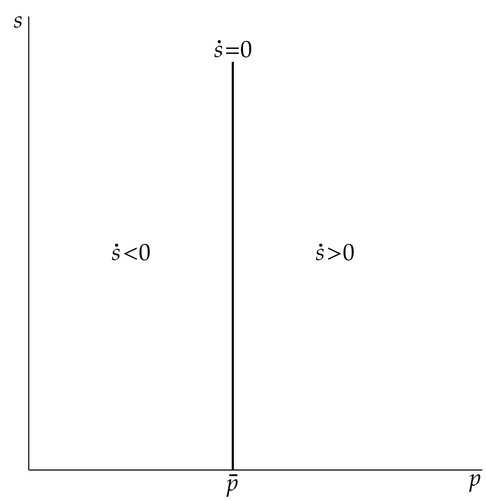


Figure 8.16: The $\dot{s} = 0$ Locus

Our basic descriptions of the goods market and of price adjustment are unchanged, so under the rational expectation hypothesis

$$\dot{p} = \pi\big(\rho(s - p) - \sigma(i^* + \dot{s}) + g - y\big) \tag{8.37}$$

Equivalently, in terms of deviations from the equilibrium values (again recalling that $\dot{s} = 0$ in the long run),

$$\dot{p} = \pi\big(\rho(s - \bar{s}) - \rho(p - \bar{p}) - \sigma\dot{s}\big) \tag{8.38}$$

Our solution for $\dot{s}$ allows us to rewrite this as[10]

$$\dot{p} = -a(p - \bar{p}) + b(s - \bar{s}) \tag{8.39}$$

making it simple to graph the $\dot{p} = 0$ locus.

[10]Here $a = \pi\rho + \pi\sigma/\lambda$ and $b = \pi\rho$, and the implied slope along the $\dot{p} = 0$ isocline is $(\mathrm{d}s/\mathrm{d}p)|_{\dot{p}=0} = a/b > 1$.

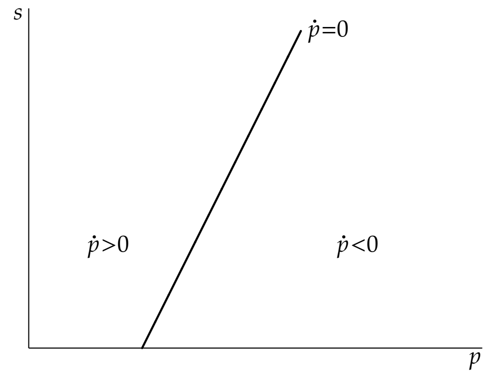


Figure 8.17: The $\dot{p} = 0$ Locus

Once again, we can solve the homogeneous system[11]

$$\begin{bmatrix} \mathrm{D}+a & -b \\ -1/\lambda & \mathrm{D} \end{bmatrix} \begin{bmatrix} p-\bar{p} \\ s-\bar{s} \end{bmatrix} = 0 \tag{8.40}$$

by the adjoint matrix technique. First solve the characteristic equation

$$D^2 + aD - b/\lambda = 0 \tag{8.41}$$

for the two roots D_1 and D_2, where $D_1 < D_2$.

$$D_1, D_2 = \frac{-a \pm \sqrt{a^2 + 4b/\lambda}}{2} \tag{8.42}$$

This clearly gives us one positive and one negative real root. Thus the following solution blows up unless $\eta_2 = 0$.

$$\begin{bmatrix} p-\bar{p} \\ s-\bar{s} \end{bmatrix} = \eta_1 \exp\{D_1 t\} \begin{bmatrix} D_1 \\ 1/\lambda \end{bmatrix} + \eta_2 \exp\{D_2 t\} \begin{bmatrix} D_2 \\ 1/\lambda \end{bmatrix} \tag{8.43}$$

It is typical for macromodels with flexible asset prices to exhibit such "saddle-point instability." Early discussions of this situation suggested that the convergent arm should be selected from all the possible dynamic paths as the only economically "reasonable" solution, and a subsequent literature provided some more sophisticated justifications for this general procedure. Selecting the covergent arm is of course the same as setting $\eta_2 = 0$ in our solution. In addition to ruling out behavior that many economists consider somewhat perverse–e.g., hyperinflation with a constant money stock–limiting our attention to the convergent arm allows us to make predictions with the model that would otherwise

[11]There is no need to substitute for $\dot{s}$ as we have done here. (Show this as a homework by working through the solution without this substitution.)

be impossible. For although the price level is predetermined, the exchange rate may potentially jump to *any* level (each corresponding to a different η_2).

8.6.3 Unanticipated Policy Shocks

If we assume the exchange rate jumps to the convergent arm, then we know that the exchange rate overshoots in response to an unanticipate monetary shock. We also know that inflation and exchange rate appreciation are correlated. So we get some strong potentially testable predictions.

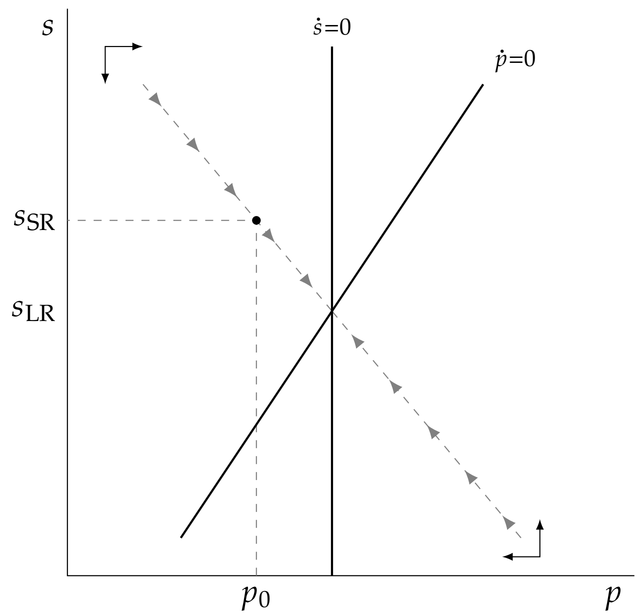


Figure 8.18: Rational Expectations and Overshooting

So with rational expectations, we will also see the same overshooting phenomenon.

The slope of the convergent arm begin apparent once we note that for any initial δp_0 we can solve $\eta_1 = \delta p_0/D_1$. This implies $\delta s_0 = \delta p_0/\lambda D_1$. Just think of the slope of the convergent arm as $\delta s_0/\delta p_0 = 1/\lambda D_1$.

Comment: Recall $\mathrm{d}\bar{p}/\mathrm{d}h = 1$. If the monetary shock disrupts an initial steady state, then $\mathrm{d}\bar{p}/\mathrm{d}h = 1$ implies $\delta p_0 = -\mathrm{d}h$. Use this to solve for $\eta_1 = -\mathrm{d}h/D_1$. This yields $\delta s_0 = -\mathrm{d}h/\lambda D_1 > 0$. The positive sign indicates overshooting: s has risen above its new long run value.

As illustrated in Figure 8.19, unanticipated fiscal shocks do not produce interesting dynamics. However, we again see the twin deficits phenomenon.

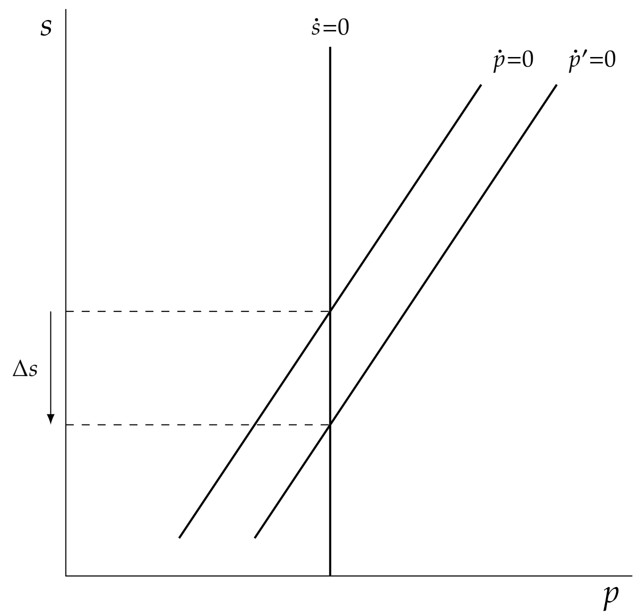


Figure 8.19: Fiscal Expansion: Overshooting Model

Lecture 9

Accounting for the Macroeconomy

International macroeconomics concerns itself with a number of variables that are often ignored in more microeconomically focused analyses of the international economy. Some of these are common to any macroeconomic analysis: aggregate output and employment, the money supply, the price level, the interest rate. Others are peculiar to the macroeconomics of the open economy: the exchange rate, the balance of payments, the relative price of traded and non-traded goods, and international capital flows.

The national income accounts and balance of payments accounts are two key data sources for macroeconomic aggregates. This chapter introduces some useful accounting identities from the national income accounts and balance of payments accounts. An identity is an algebraic relationship that is true by definition. An identity therefore has no independent economic content. However, these identities have been selected for their usefulness in categorizing interesting macroeconomic aggregates. In this sense, they can offer insights into macroeconomic relationships. (Much more interesting insights emerge elsewhere, of course, when we develop causal links between the aggregates.)

Learning Goals

After reading this chapter, you will understand:

- the balance of payments accounts and the national income accounts.
- how the current account links the balance of payments accounts to the national income accounts.
- the relationship between saving, investment, and the current account.
- how the current account affects a country's international investment position.

9.1 Balance of Payments Accounting: The Basic Concepts

The balance of payments accounts record the economic *transactions* between domestic *residents* and the rest of the world over a given period of time. As much as possible they capture international flows of goods, services, and financial claims. This data is often used by policy makers to predict the effects of international conditions on the domestic economy. Successful use of the balance of payments accounts for policy purposes depends, of course, on a sound understanding of their construction.

Residence

The concept of residence applies to individuals, firms, institutions, and government. With the exception of government employees—such as diplomats or military personnel—the balance of payments acknowledges a difference between residence and citizenship. You may reside abroad long enough to establish legal residency in another country, but this does not imply that you give up your citizenship. The balance of payments records transactions between *residents* of different nations, so once you establish residence in a new country, your transactions in that country no longer enter the balance of payments. (Government employees, however, are not considered to establish residence abroad no matter how long they actually reside abroad.)

The foreign affiliates of domestic corporations are also generally considered foreign residents, even when they are wholly owned by the parent company.[1] So if a foreign affiliate acquires materials from its parent company, this transaction enters the balance of payments accounts. (In contrast, international organizations are always considered foreign residents: they are not considered residents of any nation.)

Transactions

A balance-of-payments transaction is just a transfer of economic value from a resident of one country to a resident of another country. The economic value may be a tangible good, an economic service, or a financial asset.We consider the transaction to take place at the time of the change of ownership.[2]

The balance of payments accounts record transactions between foreign and domestic residents under a few useful categories. Roughly speaking, a balance-of-payments transaction is recorded in the Current Account if it involves goods or services (or unilateral transfers); it is recorded in the Capital and Financial Account if it involves financial assets. You can think of the Current Account as recording transactions in *current* goods and services, while the Capital and Financial Account records transactions in claims to *future* goods and services. The Financial Account is often disaggregated into the Nonreserve Financial Account (sometimes called the Private Capital Account), which records net flows of portfolio investment and direct foreign investment, and the Official Reserve Transactions Account, which records changes in the government's net holdings of international reserve assets.

9.1.1 Debits and Credits

An inflow of value is recorded as a debit. An outflow of value is recorded as a credit.

[1]We can classify foreign affiliates as subsidiaries (which are legally incorporated abroad) and branches (which are not). Although foreign branches are usually considered foreign residents for balance of payments purposes, the permanence and "center of interest" of their operations are considered to be the determinants of residency. For tax purposes, however, a foreign branch is considered a domestic resident.

[2]Ownership is normally considered to change when the transacting parties record the transaction; for details see IMF (1993).

For example, the value of imported televisions appears as a debit in the current account. If the importer paid for televisions by check, the transfer of ownership of the deposits to the foreign exporter is recorded as a credit in the Financial Account. Suppose $1M worth of televisions were imported. These transactions could be recorded as follows.

	Credit(+)	Debit(-)
Current Account		-$1M
Capital and Financial Account	+$1M	

Note how this exchange enters the balance of payments accounts twice, as a credit and a debit of the same value. This reflects the double-entry bookkeeping principles underlying the balance of payments accounts.

The balance of payments accounts use double-entry bookkeeping principles: each transaction generates a debit (value inflow) and a credit (value outflow). In principle each debit and credit should be recorded in the appropriate account when it occurs, but this is impossible in practice. Instead total debits and credits are estimated independently for various subaccounts.[3] The next section present various subaccounts and discusses how these would record a variety of international transactions.

9.2 The Balance Of Payments Accounts

9.2.1 The Current Account

We distinguish four subaccounts of the current account: merchandise trade, services, (factor) income, and unilateral transfers.

The Merchandise Trade subaccount records exports and imports of goods.[4]

The Income subaccount records the income flows generated by internationally held assets, including those held by the monetary authority. For example, this subaccount records payments by foreign residents for domestic land or capital. Interest payments, dividends, and repatriated profits appear here.[5] Here we record foreign wage payments to domestic residents.[6]

Entries in the Unilateral Transfers subaccount record official foreign aid (government grants) and private remittances.[7] Private remittances include private foreign assistance

[3]Since the debits and credits in each account are estimated independently, without making corresponding entries in other accounts, we might expect that errors in measurement—compounded by any deliberately hidden illegal activities—will generate accounts that don't "balance". This is in fact the case, and the accounts are balanced by introducing an additional category: the statistical discrepancy, formerly called errors and omissions.

[4]These are called visibles in the U.K. That choice of terminology is actually a good reminder that some trade in goods is invisible (e.g., trade in illegal drugs) and is not recorded in the balance of payments.

[5]Foreign affiliate earnings are recorded even if not remitted. Thus undistributed profits of a foreign affiliate should be measured as if received by the domestic parent.

Note that royalties, and license fees appear under other services.

[6]This follows the 5th edition of the IMF *Manual*, and is a change from earlier editions.

[7]As of the 5th edition of the IMF *Manual*, this account now only includes current transfers. Capital transfers, including unilateral debt forgiveness, are now recorded in an equivalent capital subaccount.

CURRENT ACCOUNT

1. Merchandise Trade
2. Services
   - tourism
   - transportation
   - business, professional and other services
3. Income (Factor Services)
   - Investment Income
   - Employee Compensation
4. Unilateral Current Transfers
   (incl. workers remittances)
   - government grants
   - government pensions
   - private remittances and other transfers (including taxes)

CAPITAL AND FINANCIAL ACCOUNT

1. Capital Account
   - Unilateral Capital Transfers
     (debt forgiveness, investment grants)
   - Acquisition/Disposal of IPRs
2. Financial Account (Private)
   - Direct Foreign Investment
   - Portfolio Investment (long term and short term)
3. Financial Account (Official Reserve Transactions)
   (gold, IMF credits and SDRs, foreign exchange reserves)
   - Changes in domestic assets held by foreign monetary authority
   - Changes in foreign assets held by domestic monetary authority

Table 9.1: Accounts and Subaccounts

(e.g., for famine relief), pension payments to retirees who are foreign residents, or remittances from domestic workers to their families abroad. The last of these—money and goods sent to families abroad—is the largest of the private transfers.

Services (called invisibles in the U.K.) include tourism, transportation services, and professional and other services. Transportation services include freight and insurance charges for goods as well as international travel, including tourist travel. Tourist services include *all* spending abroad by domestic residents on goods and services (food, lodging, local transportation, entertainment, and so forth), but not their expenditure on international transportation. Business and professional services encompass banking services, management consultancy, engineering services, educational services, medical services. Royalties and license fees, when paid to or received from a foreign resident, are counted in other services.

9.2.2 The Capital and Financial Account

The capital and financial account keeps track of the change of in domestic ownership of foreign assets (called "U.S. Assets Abroad") and in foreign ownership of domestic assets (called "Foreign Assets in the U.S.") The capital account keeps track of capital transfers (such as debt forgiveness or migrants transfers) and transactions involving nonproduced, nonfinancial assets (such a patents or trademarks). The financial account tracks direct investment, portfolio investment, and official transactions in reserve assets. We will disaggregate the Financial Account into the Nonreserve Financial Account (sometimes called the Private Capital Account) and the Official Reserve Transactions Account.

Nonreserve Financial Account

The Nonreserve Financial Account records the *net* changes in domestic ownership of foreign assets and in foreign ownership of domestic assets.[8] The primary distinction in the Nonreserve Financial Account is between foreign direct investment and portfolio investment. Portfolio investment flows can refer to anything from currency to long term bonds to corporate stock. Portfolio investment is sometimes broken down into short-term capital flows (original term to maturity of less than a year, including liquid funds) and long-term capital flows (of longer or undefined maturity).[9] The categorization of asset flows by contractual maturity was intended to distinguish assets by the investors' intended holding period. However it is generally agreed that contractual maturity fails to proxy this distinction.[10] It therefore makes sense to emphasize the distinction between direct foreign investment and portfolio investment, where the category of direct foreign investment is intended to indicate the acquisition of a managerial influence.[11]

Official Reserve Transactions

Reserve assets are assets that are "available to and controlled by" the monetary authorities. International reserve assets include gold, foreign exchange reserves, credit issued by the international Monetary Fund, and SDRs.[12] "Reserves" are assets under the control of the monetary authority that can be used in the implementation of balance of payments policy. Monetary authority transactions in international reserve assets are entered in the

[8] Gross flows are much larger.

While the Nonreserve Financial Account does not include transactions of central banks (monetary authorities), it does include transactions by other government agencies. For example, it would include credit extended by the ExIm Bank.

[9] I believe the IMF and the World Bank continue to report this breakdown by asset maturity, but the U.S. Department of Commerce has abandoned this distinction due to practical measurement difficulties and dubious economic relevance.

[10] Short-term assets may be held with the intent to roll them over; long-term assets can be sold before maturity; and an asset with a long-term contractual maturity may have only a few months left to maturity when traded.

[11] Definitionally, accumulating foreign equity beyond ten per cent of book value is considered DFI, as is the purchase of real estate or production facilities.

[12] SDR is an acronym for Special Drawing Right. SDRs are created by the IMF, with a value defined in terms of a basket of five currencies (dollar, yen, mark, pound, franc) .

balance of payments accounts as Official Reserve Transactions.[13] Do not be confused by the fact that a reduction in foreign exchange reserves implies Official Reserve Transactions is positive: as always, outflows of value are entered in the balance of payments accounts as credits.

An additional category—allocation of special drawing rights—is sometimes included to reflect changes in IMF allocations of SDRs to member nations. The debit of the special drawing rights account of official reserves is then offset by a credit to allocations of special drawing rights.

9.2.3 Recording Transactions

The balance of payments uses a broad measure of transactions, one that includes many activities that do not seem to involve exchanges. For example, private and official gifts, pensions, and even expropriations are included. The *Unilateral Transfers* subaccount of the Current Account allows standard double entry recording of such "transactions". In other transactions the element of exchange is conceptually more evident even if the actual transfer of goods, services, or assets is invisible. For example, the reinvestment (without remittance) of profits by a foreign subsidiary may be treated in the balance of payments as if the profits had in fact been remitted and then were reinvested abroad by the parent company. Another example is the treatment of domestic currency deposits acquired by a foreign government from a (foreign) resident bank or business: the deposit transfer is recorded in the balance of payments although total foreign claims on U.S. banks are unchanged. (Similarly, the IMF records government purchases and sales of gold to domestic residents in the balance of payments—although the Commerce Department does not.)

It is easiest to understand the balance of payments accounts by recording examples of transactions in an imaginary balance of payments account. Remember, debits record any inflow of value whether through the acquisition of goods, services, or assets or through the reduction of a liability. Similarly, a credit results from any outflow of value, whether from the disbursement of goods, services, or assets or from the increase in a liability. Consider again our example of television imports; the value of an imported television appears as a debit in the merchandise trade account. As another example, the value of banking services provided to foreign residents appears as a credit in the services account. Similarly, the acquisition of foreign stock or even a foreign factory generates a debit in the Capital and Financial Account. Each of these transactions may be thought of as one-half of an international exchange.

For example, when the U.S. exports wheat in exchange for a deposit in a foreign bank, we should record a credit in the U.S. Current Account (for the outflow of the wheat) and a debit in the U.S. Capital and Financial Account (for the acquisition of the deposit, an asset). This is a typical export transaction: it generates a Current Account credit and an equal Capital and Financial Account debit. This happens whether the export is paid for by cash, check, or trade credit.[14]

[13] The monetary authority may be an exchange stabilization fund as well as a central bank.

[14] Trade-credit is the extension, by the exporter or by a bank, of credit to the importer. It therefore appears as a debit (the acquisition of a promissory note) in the exporting country's Capital and Financial Account.

Balance of Payments Accounting Examples

Suppose that a U.S. manufacturer exports machine tools and receives payment by check. There are two international flows in this transaction, the machine tool going out of the country and the new asset coming into the country. Both of these should be recorded in the balance of payments accounts, which record international flows of goods, services, and assets. The value of a machine tool export should appear as a credit in the Merchandise Trade subaccount, while the value of the bank balances transferred in payment should appear as a debit in the Nonreserve Financial Account.

Now consider a harder example: suppose domestic residents receive additional shares as a dividend on their holdings of foreign equity (stock). Of course the additional shares acquired will generate a debit in the Capital and Financial Account, but what is the corresponding credit? It is a Current Account credit for factor services "exported," i.e., for the services of the capital represented by the original share ownership.

The oddest examples concern unilateral transfers. Consider the private donations of medical supplies to Kosovo refugees in 1999. This outflow of goods generated a credit to the Current Account, but what was the corresponding debit? We maintain balance in the balance of payments by recording a corresponding debit in the Current Account under Unilateral Transfers. As another example, in 1991 the U.S. received payments from its allies to help finance the Gulf War. The payments of course generated debits in the Capital and Financial Account; the offsetting credit was to Unilateral Transfers (in the Current Account). These credits were large enough to almost completely eliminate the U.S. current account deficit in 1991, and in fact the quarterly data register a current account surplus for the first two quarters of 1991. Occasionally you will hear an ad hoc justification of this as representing an inflow of "goodwill", but don't forget that foreign seizure of a domestic ship would look the same in the balance of payments.

As a final example, suppose an exporter decides to sell foreign exchange to the domestic central bank. There is a credit to portfolio investment in the Nonreserve Financial Account, and a debit to official reserve flows.

9.2.4 Interpreting the Data: Net Flows

Data from the balance of payments accounts is generally reported as net flows over a month, quarter, or year. Net flows for any type of transaction are calculated by adding up all the credits and debits. For example, in the U.S. one of the most widely published net flows is the monthly merchandise trade balance.[15] Exports generate credits, which are added to the merchandise trade balance. Imports generate debits, which are subtracted from the merchandise trade balance. The monthly merchandise trade balance therefore tells us by how much exports exceed imports. A deficit in the merchandise trade balance, when the balance is negative, tells us that the country is importing more goods than it is exporting.

[15]Unlike most balance of payments data, the merchandise trade balance is available monthly. This is because the data are reported by the Customs Service directly to the Commerce Department. It is likely that the availability of good quality data at such a high frequency has helped to raise the profile of this balance.

The large U.S. merchandise trade deficits have attracted considerable attention, yet the number is not particularly significant. Certainly one month movements of this volatile series tell us little about trends in the balance. But even sustained deficits are not informative, for a sustained deficit in the merchandise trade balance may simply indicate that a country has specialized in the production of services. For this reason, a better measure of the "trade balance" is the quarterly balance on goods and services. The balance on goods, services, and income is still more informative about whether a country is tending to live within its current productive capacity—including its earnings on foreign investments. Finally, the current account balance give the best idea of whether current income (including unilateral transfers) is adequate to meet current expenditures.

Statistical Discrepancy

In principle, the current account, Nonreserve Financial Account, and official reserve flows should sum to zero. In practice, they do not. Some transactions are missed, and the statistical discrepancy gives us an idea of the net international transactions not recorded in the balance of payments accounts. In the U.S., the value of the statistical discrepancy has grown tremendously in the past two decades, from a value of -$1.5B in 1969 to $32B in 1995, with a high of $54B in 1990. It is widely believed that this represents a growth in unrecorded services and especially capital flows. For this reason, the reported Capital and Financial Account balance is often the sum of the Nonreserve Financial Account balance and the Statistical Discrepancy.

It is important to note that the missing transactions are not purely random. For the world as a whole, the balance of payments accounts tend to underreport merchandise imports, foreign investment income, and the acquisition of assets abroad. It is hardly surprising that people hide their imports, their investment income, and their assets abroad from government officials interested in taxing or regulating these flows.

There is a popular impression that the Statistical Discrepancy must reflect illicit drug traffic and the earnings of undocumented immigrant workers. However note that the Statistical Discrepancy in the U.S. often shows a large credit balance, while these items would be likely to generate debits. For example, smuggled drugs should have been entered as a debit in the Merchandise Trade Balance but were not, so this debit should show up in the Statistical Discrepancy.[16] Similarly, payments to undocumented workers should have generated a debit to the Services subaccount but do not, so this debit shows up in the Statistical Discrepancy. [17]

The rise in the merchandise trade deficit may be one contributor to the statistical discrepancy in the U.S. This is due to the delayed recording of the credit arrangements used in financing international trade. A rise in the recorded trade deficit will precede the rise in the recorded Capital and Financial Account, and the missing credit will show up in the Statistical Discrepancy. Another possibility is an increase in disintermediated loan activity. For example, when a firm borrows from a foreign affiliate there should be a credit

[16] That is, it shows up in the Statistical Discrepancy to the extent that the associated credit entries appear in the Capital and Financial Account.

[17] Again, it shows up to the extent that associated credit entries appear in the Capital and Financial Account (e.g., when worker earnings are deposited in Mexico).

to the Capital and Financial Account, but such transactions are likely to be underreported. The debit side of the transaction nevertheless tends to show up when the borrowed funds are deposited in the banking system. Another possibility is unrecorded foreign purchases of U.S. real estate, especially since the U.S. does not closely monitor foreign real estate holdings. Finally, a contribution may be made by the smuggling of U.S. goods abroad.

Illegal Transactions When regulation restricts international transactions, there are incentives to evade these restrictions. Evasion may include outright smuggling of merchandise, but it may take many other forms. We now briefly consider the effects of illegal transactions on the balance-of-payments accounts. The discussion in this section draws on Gandolfo (1995).

We first consider a versatile and popular tool: misinvoicing. It is clear that misinvoicing can be used to evade import tariffs and export duties. For example, an importer may present documents claiming a lower value for imported or exported goods than the value actually paid. This may involve completely forged documents, in which case trade-data comparisons by the partner countries may disclose the illegal transactions. However if the foreign trading partner performs the misinvoicing, the illegal transaction will lead to no discrepancy. So tax evasion leads to underinvoicing, with corresponding inaccuracy in the measurement of the balance of trade.

However there can also be incentives to overinvoice imports when capital flows are restricted. For example, a foreign exporter might agree to overinvoice a shipment of goods and credit the discrepancy to a foreign account held (illegally, from the point of view of the domestic country) by the importer. Thus misinvoicing can be used to hide capital outflows. Travel, labor income, and worker's remittances can be similarly exploited (see Gandolfo (1995, p.85)).

The Overall Balance of Payments

Since the balance-of-payments accounts are kept with double entry bookkeeping, they must balance. In this sense the balance of payments is always zero. However one often encounters discussion of balance-of-payments "imbalances", or of balance-of-payments deficits. Most often this refers to what is often called the "Overall Balance": –ORT. From our discussion of the balance-of-payments accounts, this must equal the sum of the Current Account, the Capital and Financial Account, and the Statistical Discrepancy. (Since the Official Reserve Transactions balance should be accurately measured, it is natural to include the statistical discrepancy in this sum.) So when the Overall Balance is in deficit—so that we speak of a balance-of-payments deficit—the monetary authority suffers a drain of official reserve assets. This situation is not permanently sustainable, since access to reserve assets is limited, which suggests the need for a change in policy.

9.3 National Income Accounting

This section develops of some national-income-accounting identities that will be useful throughout the course. We begin with the underlying budget identities of the non-bank

```
U.S. International Transactions 2013 (billions)
===============================================

Current Account                                  -379
---------------
Exports                             3061
        Goods            1590
        Services          682
        Income Receipts   789
Imports                            -3307
        Goods           -2294
        Services         -453
        Income Payments  -560
Net Unilateral Transfers            -133

Capital Account                                    -0.4
---------------

Financial Account                                 -553
-----------------
US Assets held abroad               -553
        Reserve Assets      3
        Other Assets     -560
Foreign Assets held in US            906
        Reserve Assets    284
        Other Assets      622
Statistical Discrepancy               -2
```

Table 9.2: Balance of Payments Accounts

Source: http://www.bea.gov/scb/pdf/2014/04%20April/0414_international_transactions_tables.pdf private sector, which comprises households and firms. For both households and firms, the sources and uses of income are necessarily equal. Afterwards, we link these directly to the balance of payments.

9.3.1 Firms:

Consumption expenditure (C) by consumers, investment expenditure (I_g) by firms, and government expenditure (G), and purchases by foreign residents (X) of domestic goods and (non- factor) services all generate business income. Of course, part of consumption, investment, and government expenditure is on imports (M), and this part does not generate income for domestic firms. Also, the wages of civil servants (WN_g) constitute part of government expenditure, and this too does not generate income for domestic businesses. Total business income is therefore $C + Ig + (G - WN_g) + \mathrm{X} - \mathrm{M}$. This income is allocated to wage payments (WN_f), to direct and indirect business taxes (BTx), and to offset depreciation of equipment as a capital consumption allowance (CCA). The remainder is after tax profit (Pr).[18]

$$\begin{aligned} C + I_g + (G - WN_g) + \mathrm{X} - \mathrm{M} &= WN_f + Pr + \mathrm{BTx} + \mathrm{BTr} + \mathrm{CCA} \\ + \text{intermediate goods} &\quad + \text{intermediate goods} \end{aligned}$$

Total business income necessarily equals the allocation of business income. Don' forget that X − M includes non-factor services.

9.3.2 Households:

Household are the owners of the factors of production, including their own labor. They derive their income primarily from wages (WN) but also from the sale of factor services, both domestically (Dv) and abroad (FP).[19] In addition, they receive net transfers (Tr). The household budget identity equates total personal income to its use for consumption (C), saving ($P\mathcal{S}$), and the payment of taxes (PTx).

$$WN + \mathrm{Dv} + \mathrm{FP} + \mathrm{Tr} = C + P\mathcal{S} + \mathrm{PTx} \tag{9.1}$$

Personal income necessarily equals the uses of personal income.

Note that total transfers comprise business transfers (BTr), government transfers (GTr), and *net* unilateral transfers received from non-residents (UTr).

$$\mathrm{Tr} = \mathrm{BTr} + \mathrm{GTr} + \mathrm{UTr} \tag{9.2}$$

Wage income may be earned in the private sector (WN_f) or in the public sector (WN_g).

$$WN = W(N_f + N_g) \tag{9.3}$$

[18]Why isn't investment on the right, along with intermediate goods? Hint: think about what is done with after tax profits.

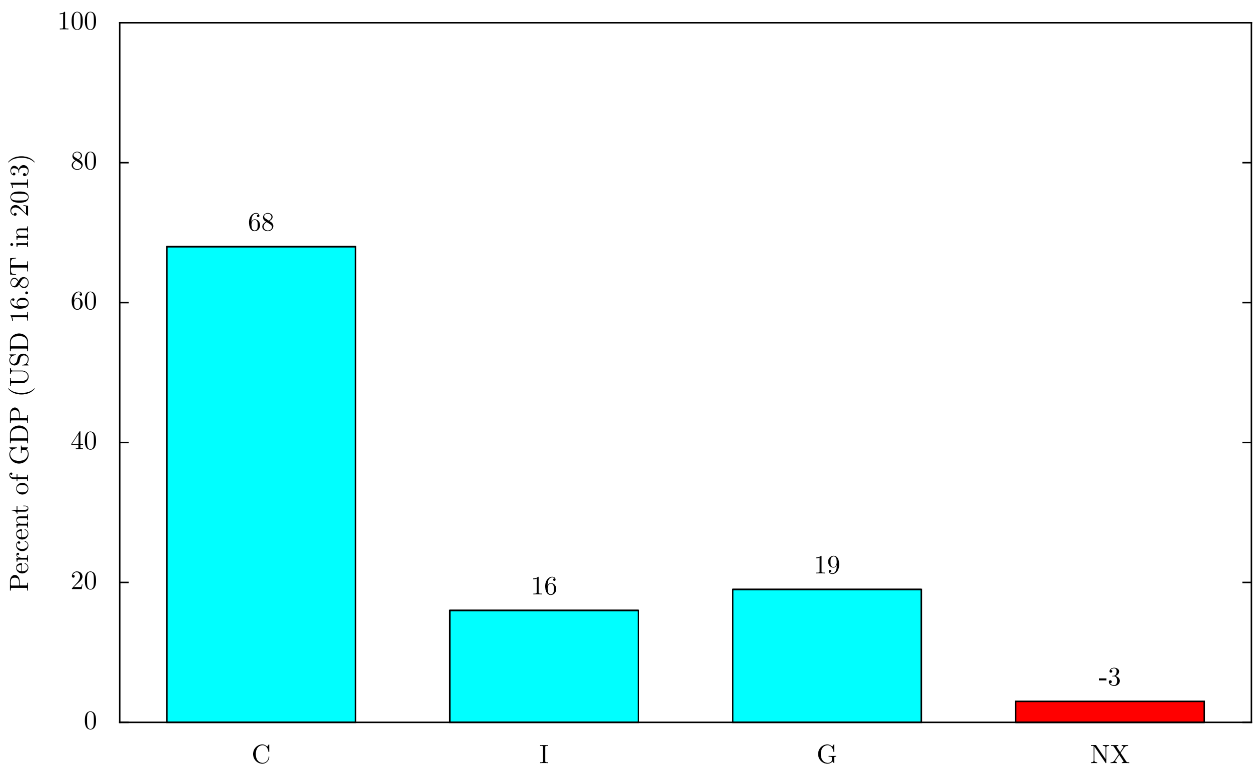
[19]These categories are interpreted extremely broadly. Thus 'wages' includes all wages, salaries, and fringe benefits, and income from factor services includes dividends, rents, and net interest.

9.3.3 Private Sector

The budget constraint for the entire private sector can be found by summing the budget constraints of households and firms. After simplification, we find our key accounting identity for total income (Y_T).[20]

$$\underbrace{\underbrace{\underbrace{\overbrace{C + I_g + G}^{A} + \mathrm{X} - \mathrm{M}}_{\mathrm{GDP}} + \mathrm{FP}}_{\mathrm{GNP}} + \mathrm{UTr}}_{Y_T} = C + \underbrace{P\mathcal{S} + Pr - \mathrm{Dv} + CCA}_{\mathcal{S}^p} + \underbrace{\mathrm{PTx} + \mathrm{BTx} - \mathrm{GTr}}_{\mathrm{Tx}}$$

Note that $Y_T = C + \mathcal{S} + \mathrm{Tx}$. Here $Y_T = C + I_g + G + \mathrm{X} - \mathrm{M} + \mathrm{FP} + \mathrm{UTr} = \mathrm{GDP} + \mathrm{FP} + \mathrm{UTr} = \mathrm{GNP} + \mathrm{UTr}$ is the total income available to the domestic economy.[21]



Source: Economic Indicators.

Figure 9.1: GDP and Its Components

[20] IMF (1993, p.13) calls Y_T gross national disposable income.

[21] Note that total income can be written directly in terms of factor incomes: $Y_T = WN + Pr + \mathrm{BTx} + \mathrm{BTr} + \mathrm{CCA} + \mathrm{FP} + \mathrm{UTr}$. This follows immediately from the budget identity of firms.

9.4 Links Between the Domestic and Foreign Sectors

Even in our national income accounting discussion above, we could not ignore the role of the foreign sector altogether. Some domestic expenditure is on imports, some foreign expenditure is on exports, and factor payments and transfers take place across national boundaries.

Our next task is to specify the links between the domestic economy and the rest of the world in more detail. That is, we will link the national income accounting concepts given above with concepts from the balance of payments accounts.[22] The first balance of payments concept we consider is the current account (CA): the net surplus on goods, services, and gifts that domestic residents run with the rest of the world.

$$\mathrm{CA} = \mathrm{X} - \mathrm{M} + \mathrm{FP} + \mathrm{UTr}$$

In order to link our national income accounting concepts with the concept of the current account from the balance of payments, it is useful to define ex post absorption (A) as the total expenditure by domestic residents, whether on domestic or foreign goods and (non-factor) services.

$$A = C + I_g + G$$

Note that an identity links the current account, total income, and absorption.

$$\mathrm{CA} = Y_T - A$$

The absorption approach to the balance of payments emphasizes this identity, noting that the current account equals the excess of income over expenditure.[23] If we desire to improve the current account, we must raise our total income or lower our absorption. The key assumption that has often been made by financial programmers who take the absorption approach to the balance of payments is that lower absorption is easier to achieve than increases in income, especially in the short run (and especially in the presence of negative profit public enterprises). As a result, severe current account imbalances have often been fought with expenditure reducing policies.[24] Note, however, that we are dealing only with identities at this point—not behavioral equations or equilibrium conditions—so we have no real clues about appropriate way to pursue our policy goals.

Similarly, since $Y_T = C + I + G + CA$ and $Y_T = C + \mathcal{S}^p + \mathrm{Tx}$, we have the identity

$$\mathrm{CA} = (\mathcal{S} - I_g) - (G - \mathrm{Tx}) \tag{9.4}$$

[22]We must simplify slightly to do this. For example, the standard for national income accounting the System of National Accounts (SNA) of the United Nations. However, as discussed in Appendix I of the IMF *Manual*, the rest-of-the-world account does not perfectly conform to the IMF balance of payments accounts. Similarly, some work is required to reconcile the U.S. National Income and Product Accounts (NIPA) with the balance-of-payments data in the Survey of Current Business. (See the July issues of the SCB.)

[23]See Alexander (1952), the originator of the “absorption” terminology, for an early treatment.

[24]Since sovereignty concerns often keep countries from approaching the IMF until time of severe balance of payments crisis, these are also common IMF stabilization policy proposals. Of course exchange regime liberalization policies are also proposed in order to improve the trade balance, but these are often argued to be more uncertain in timing and effect.

suggests that to the extent that $\mathcal{S}$ and I_g are determined in the private sector, policies to improve the current account should focus on reducing the government budget deficit. While this may be true, it is brash to treat S and I_g as exogenous and fixed when deciding on fiscal policy.

Finally, the identity

$$\mathrm{CA} = [\mathcal{S} + (\mathrm{Tx} - G)] - I_g \tag{9.5}$$

suggests that we can view the current account as being the difference between total national saving (including the budget surplus) and gross private domestic investment. This highlights the fact that a negative current account balance can reflect borrowing abroad in order to finance a high level of domestic investment. Of course, a current account deficit can also reflect a low level of national saving. The policy conclusions may be very different in these two cases, suggesting that the deficit itself is not particularly informative.

While identities cannot shed light on economic behavior—they are just identities after all—they can serve to ferret out some illogical claims made in popular discussions of international economics. For example, it is popular to argue that increased global integration now allows capital and technology to flow to low wage nations, which will then achieve the productivity levels of developed countries but have a huge competitive advantage in lower wages. High wage countries will run huge trade deficits, leading to falling wages or high unemployment.

As Krugman (1995) points out, one problem with this story is that it involves the low wage countries simultaneously running large trade surpluses and receiving large capital inflows. That is, they are financing investment in excess of their saving by borrowing abroad, while selling more goods and services abroad than they are buying there. As (9.5) makes clear, this is not possible. One cannot be simultaneously a net borrower and a net lender. Here is another way to look at this: in order to run large surpluses, the low wage countries must become net exporters of capital to the rest of the world.

9.4.1 Factor Payments

Income paid to foreign nationals often does not show up as factor payments. Consider three EU economists who work for consulting firms. Andy works for AA Consulting, a US firm, and maintains his resident in the US. Brett works for BB Consulting, a US firm, but he maintains his residence in the EU. Chris works for CC Consulting, an EU firm, and he maintains his residence in the EU.

Since Andy is a US resident, his salary is not in the balance of payments. When CC Consulting sends Chris and his team to do work in the US, the payment to the firm is counted in the balance of payments account under services, and Chris's salary does not enter the balance of payments accounts. But Brett's salary is classified as factor income, because it paid by a US firm to an EU resident. Brett's salary shows up in US GDP but EU GNP.

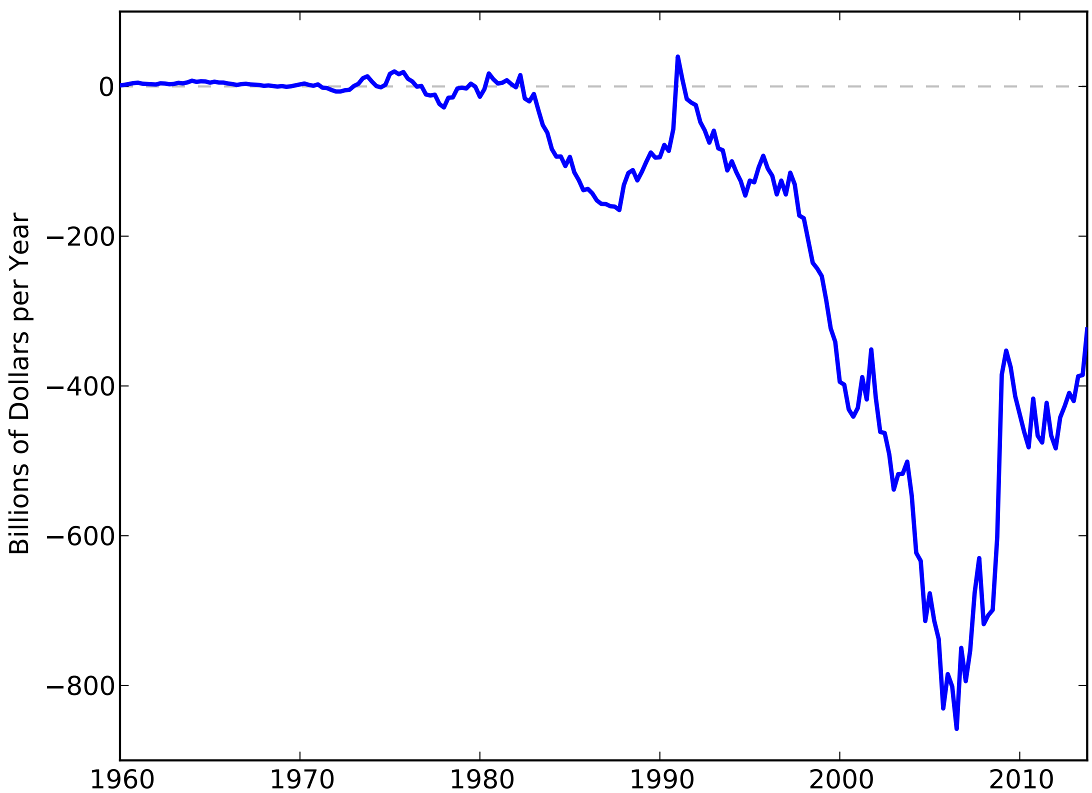


Figure 9.2: U.S. Current Account

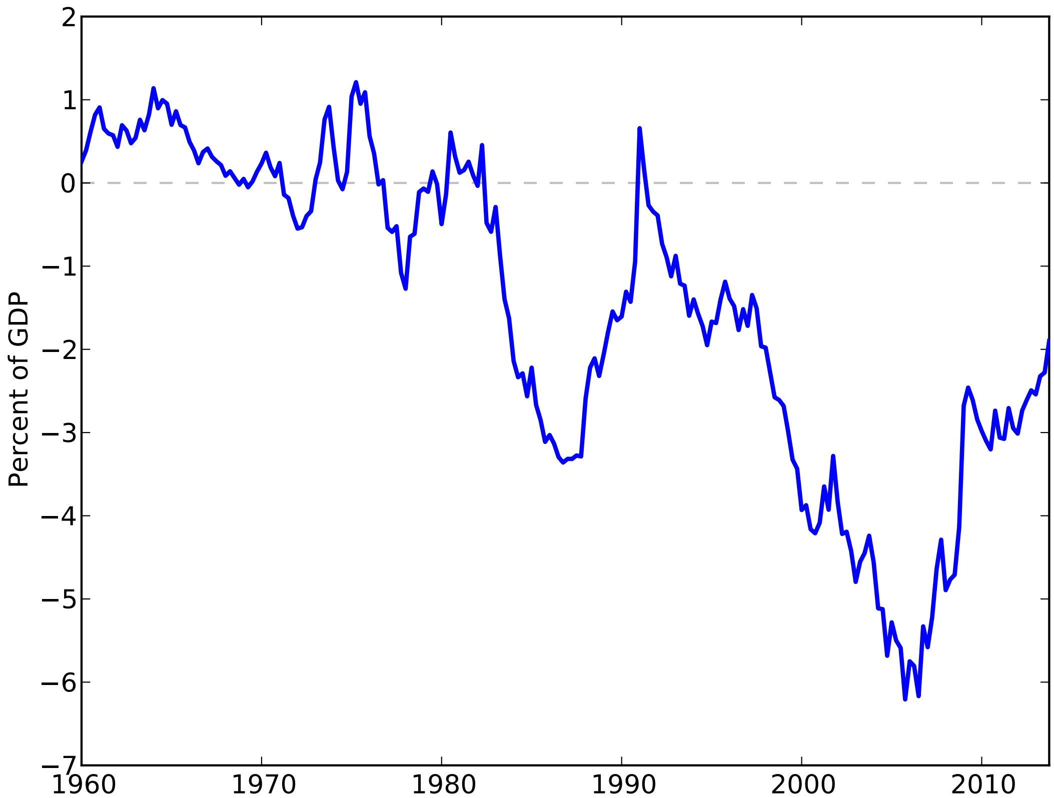


Figure 9.3: U.S. Current Account

9.5 Links to the Financial Sector

The current account, as the difference between total domestic income and total domestic expenditures, must determine the change in domestic residents' net claims on the rest of the world.[25]

$$\text{CA} = Y_T - A = \Delta\text{NFA} \tag{9.6}$$

These net claims (NFA) comprise the net foreign assets of the banking sector ($\text{NFA}^{\text{bank,ma}}$) less the foreign indebtedness of nonbank residents (FI), so we can write[26]

$$\Delta\text{NFA} = \Delta NFA^{\text{bank,ma}} - \Delta FI \tag{9.7}$$

One central observation by financial programmers—a minor elaboration of the absorption approach—is that

$$\Delta\text{NFA}^{\text{bank,ma}} = Y_T - A + \Delta FI \tag{9.8}$$

Expenditure beyond total income, which is not financed by borrowing, draws down the net foreign assets of the banking sector. Since foreign exchange reserves are finite, this is not permanently sustainable. (These days it is important to note that the reserve loss may take the form of accumulating arrears.)

9.5.1 Monetary Considerations

Traditionally, financial programmers have put primary emphasis on monetary factors in CA determination. Recall:

$$\text{CA} = \Delta\text{NFA}^{\text{bank,ma}} - \Delta FI$$

Note that the since the money supply ($M2$) is defined to be deposits (D, broadly measured) plus currency in circulation (Cu, which does not include vault cash), it must equal the domestic currency value of the net foreign assets of the banking sector ($NFA^{\text{bank,ma}}$) plus the domestic credit extended by the banking sector to the government and private sectors (DC).

$$M2/P = \text{NFA}^{\text{bank,ma}} + DC/P$$

This identity follows immediately from the simplified banking sector balance sheets below.[27]

Monetary Authority

Assets	Liabilities
$P\,\text{NFA}^{\text{ma}}$	R
$DC^{\text{ma}}_{g,p,bank}$	Cu

Financial Intermediaries

Assets	Liabilities
R	DC^{ma}_{bank}
$P\,\text{NFA}^{bank}$	D
$DC^{bank}_{g,p}$	

[25]Ignoring possible capital gains and losses on NFA.

[26]Note that

$$\text{KA} = \Delta FI - \Delta NFA^{\text{bank}}$$

[27]Current figures for the Federal Reserve can be obtained from the Federal Reserve Bulletin.

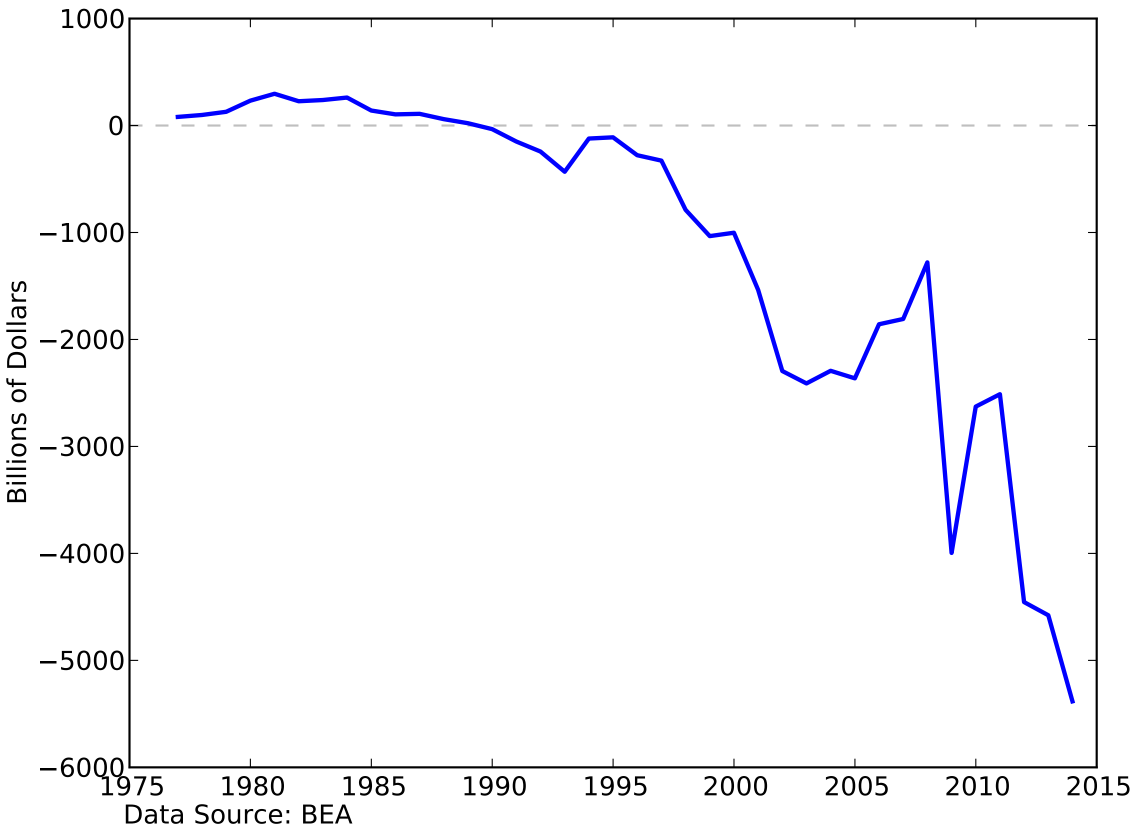


Figure 9.4: U.S. International Investment Position

9.5.2 Financial Programming

Historically, the term "financial programming" is associated with the use of a simple accounting framework to guide IMF supported adjustment programs.[28] Financial programmers attempt to draw policy implications from a theory-supplemented set of accounting identities that link the flow of funds between the non-bank private sector, the banking sector, the government sector, and the foreign sector.[29] They tend to focus on the national income accounts, the balance of payments accounts, and the balance sheets of the monetary authorities, the banking system, and the fiscal authorities. These are the basic data available to the programmer in assessing the current state of the economy, although in LDCs there may be important gaps and lags in data availability.

Financial programming developed in the IMF's Research Department in the late 1950s and early 1960s. Initially it focussed on analysis of the small open economy under fixed exchange rates, paying particular attention to the links between the monetary sector and the balance of payments (Polak, 1957; Prais, 1961). For the next thirty years the basic character of financial programming models consisted of a quantity theory model of price determination with a stablizing reciprocal link between the monetary sector and the balance of payments: current account deficits drained money from the economy, and the resulting price declines depreciated the real exchange rate and improved the balance of payments. (See Edwards (1989) for an explicit model.)

Data availability is an important consideration in modeling, since it determines the set of observable variables that can be included in a model. The fact that for many countries only monetary and balance of payments data were available on a timely and accurate basis had a strong influence on early financial programming: macroeconomic forecasts were attempted on the basis of this limited data set. (This is an important factor in the popularity of the monetary approach to the balance of payments, which we will discuss later.) Perhaps the most important role of the accounting identities emphasized by the IMF, however, has been to act as a consistency check on forecasts and policy targets independently generated for different sectors of the economy. That is, the accounting identities can be used as a tool for consistent macroeconomic planning.

From the identities above, we can conclude that

$$\mathrm{CA} = \frac{1}{P}(\Delta M2 - \Delta DC - P\,\Delta FI)$$

This identity has led many financial programmers to focus on control of ΔDC as the way to improve CA.

(i) Choose CA target CA^t (or, equivalently, $\Delta\mathrm{NFA}^{\mathrm{bank,ma}\,t} = \mathrm{CA}^t + \Delta FI$).[30] (ii) Project values $\Delta M2$, ΔFI (iii) Set $\Delta DC = \Delta M2 - P(\Delta FI - \mathrm{CA}^t) = \Delta M2 - P\,\Delta\mathrm{NFA}^{\mathrm{bank,ma}}$

Remember, we are still working with identities. (No "Model".) Nevertheless, this is the basis of the monetary approach to the balance of payments (MABOP). Just add

[28]This discussion draws heavily upon IMF (1987)).

[29]We will aggregate the monetary authorities with the banking sector, whether the relevant transactions are controlled by the treasury, a central bank, or a separate exchange equalization account.

[30]CA^t is often chosen to equal sustainable long term capital flows and unrequited transfers.

- Y Exog
- $L(i, Y)$ "Stable" and doesn't depend on DC or ΔDC (There is some support in developing countries p.14 Dornbusch)
- $M2/P = L(i, Y)$
- some way to project i and P

Here is a particularly simple case, and our first model of this chapter. We assume a constant income velocity of money ($L = k_2 Y$) and impose continuous purchasing power parity ($P = EP^*$). Given real income and net foreign indebtedness, we find that domestic credit expansion implies a current account deficit.

$$\text{CA} = k_2 \, \Delta Y - \Delta DC/P - \Delta FI \tag{9.9}$$

$$= -\Delta DC/P \qquad \text{if} \qquad \Delta Y = 0 = \Delta FI \tag{9.10}$$

The focus is on the difference between any changes in the demand for money and any attempts to change its supply.

Problems for Review

1. Consider the following table, based on July 1991 data from the IMF's *International Financial Statistics*. (The data are given as billions of US dollars.)

	US	UK	Germany	Japan
CA	-99	-22	44	57
KA	-1	14	-57	-48
Discrepancy	73	7	18	-22
ORT	27	1	-5	13

   Update this table using last July's *International Financial Statistics*.

2. For each of the following transactions, correctly record the resulting debits and credits in the U.S. balance of payments accounts (as listed in Table 9.1).
   - The U.S. exports \$1M worth of grain to Mexico.
   - Mexico pays a U.S. exporter \$1M in dollars.
   - The U.S. exports \$1M worth of grain to Mexico, and Mexico pays for this in dollars.
   - Honda builds an automobile manufacturing facility in the U.S.
   - Japanese investors buy U.S. Treasury bonds and pay by check.

3. The relationships implied by the balance of payments accounts and the national income accounts are identities. How much can they tell us about the macroeconomic links between countries? Why?

4. Locate the *International Financial Statistics Yearbook* in the library and record the U.S. current account deficits for the last decade. Do the same thing with the *Economic Report of the President*. How well do these values match?

- Bureau of Economic Analysis of the United States Department of Commerce, *Survey of Current Business* (monthly, but especially June 1978, part II for taxonomic detail).

- Clayton, Gary, and Martin Gerhard Giesbrecht, 1995, *A Guide to Everyday Economic Statistics* (New York: McGraw-Hill).

- Grayboyes, Robert F., 1991, "International Trade and Payments Data: An Introduction," *Economic Review* (Richmond: Federal Reserve Bank of Richmond) July 1975, 14–23.

- Graham, E. M., and P. R. Krugman, 1989, *Foreign Direct Investment in the United States* (Washington, D.C.: Institute for International Economics). esp. Appendix A.

- Helmers, F. Leslie C. H., 1988, "National Income Accounting Identities", in Rudiger Dornbusch and Leslie Helmers (eds), *The Open Economy: Tools for Policy Makers in Developing Countries* (Oxford: Oxford University Press, 1988) pp. 375-391.

- Kester, Anne, and Robert Baldwin, 1991, *Behind the Numbers: U.S. Trade in the World Economy* (Washington, D.C.: National Research Council, National Academy Press)

- Kindleberger, Charles P., "Measuring Equilibrium in the Balance of Payments," JPE 77(6), 1969, 873–891.

- Nicholson, R.E., 1990, "U.S. International Transactions, First Quarter 1989", *Survey of Current Business*, June.

- Stern, Robert, "The Presentation of the U.S. Balance of Payments: A Symposium," *Essays in International Finance* 123, Princeton University, August 1977.

Additional Detail

Problem: $\Delta DC = \Delta DC^{\text{ma}} + \Delta DC^{\text{bank}}$ and DC^{bank} may not be controllable.

$$P\,\text{CA} = \Delta M2 - \Delta DC - P\,\Delta FI \tag{9.11}$$

$$= \Delta M2 - \Delta DC^{\text{ma}} - \Delta DC^{\text{bank}} - P\,\Delta FI \tag{9.12}$$

So we may also need to project ΔDC^{bank} and then choose ΔDC^{ma}.. Let's reconsider the balance sheet of the monetary authorities. Recall that high powered money (H) is defined as legal reserves plus currency in circulation, which must equal the net foreign assets of the monetary authorities plus any domestic credit it extends.

$$H = R + Cu = P\,\text{NFA}^{\text{ma}} + DC^{\text{ma}}$$

Thus $P\,\Delta \text{NFA}^{\text{ma}} = \Delta H - \Delta DC^{\text{ma}}$ and, since $P\,\Delta\text{NFA}^{\text{ma}} = BOP$, deficits in the balance of payments are associated with the monetary authorities extending domestic credit at a rate that exceeds the growth in high powered money. (To understand why this happens, we need a model. Our last one should be suggestive.) So we seem to be suggesting a policy focus on the monetary authorities' extension of domestic credit.

Problem:

$$P(G - \text{Tx}) = \Delta DC_g^{\text{ma}} + \Delta DC_g^{\text{bank}} + P\,\Delta FI_g + \Delta B_p^g$$

Many developing countries find it difficult to place government bonds with the public and difficult to increase their borrowing abroad. Domestic credit then remains under pressure from large budget deficits, and this in turn pressures CA. This is one reason why so much emphasis is placed on budget restraint in adjustment programs. To see this, set $\Delta B_p^g = 0$ and note that

$$P\,\text{CA} = \Delta M2 - \Delta DC_g^{\text{ma,bank}} - \Delta DC_p - P\,\Delta FI \tag{9.13}$$

$$= \Delta M2 - P(G - \text{Tx}) - \Delta DC_p - P\,\Delta FI_p \tag{9.14}$$

A financial programmer can project ΔDC_p by relating it to GNP or investment demand and essentially proceed as before, but now current account targets imply budget deficit targets.

Comment: If we shift attention to balance of payments improvement, we can ask further whether a focus on CA or on ΔFI is more appropriate. To see this, note that

$$\text{BoP} = \Delta\text{NFA}^{\text{ma}} \tag{9.15}$$

$$= \Delta\text{NFA}^{\text{bank,ma}} - \Delta NFA^{\text{bank}} \tag{9.16}$$

$$= \text{CA} + \Delta FI - \Delta NFA^{\text{bank}} \tag{9.17}$$

$$= \text{CA} + \text{KA} \tag{9.18}$$

Financial programming remains essentially unchanged: (i) choose target BOP^t (ii) forecast your exogenous variables (iii) set policy instruments to achieve target

But hey, seriously folks, that's all impossible without a good structural model!

For now, let's set FP and UTr = 0.[31]

Then GNP = GDP = Y_T and CA = X − M(= TB). Also, note that in writing net exports as TB = X − M instead of TB = X − QM, where the real exchange rate $Q = SP^*/P$ allows us to express our imports in units of domestic output, we implicitly set $Q = 1$.

Comment:

[31]FP will be crucial later, when we consider the portfolio balance effects of foreign asset accumulation. It is also worth noting that some attention has been paid to the non-interest current account, $NICA = \text{CA} - \text{FP}$. The idea here is that high interest payments by debtor countries put their CA into deficit, disguising the "net resource transfer" o the ROW.

$$\Delta \mathrm{NFA}^{\mathrm{bank,ma}} = \Delta NFA^{\mathrm{ma}} + \Delta NFA^{\mathrm{bank}}$$
$$FI = FI_g + FI_p \quad \text{government and private}$$
$$\Delta DC = \Delta DC^{\mathrm{ma}}_{g\&p} + \Delta DC^{\mathrm{bank}}_{g\&p}$$

(Note: $DC^{\mathrm{ma}}_{\mathrm{bank}} = -DC^{\mathrm{bank}}_{ma}$)
$M2 = \mathrm{mult} * H \rightarrow \Delta M2 = \mathrm{mult} * \Delta H$
$H = P\,NFA^{\mathrm{ma}} + DC^{\mathrm{ma}} \rightarrow \Delta H = P\,\Delta NFA^{\mathrm{ma}} + \Delta DC^{\mathrm{ma}}$
H doesn't include government deposits in banks, and DC^{ma}, is net of these.

FP: net factor income received from abroad (including labor income only if non-resident, i.e., less than one year abroad.)

UTr: net transfers from abroad

M includes imported intermediate goods and raw materials

In the presence of constraints on capital flows the story is slightly more involved. The monetary effects of the balance of payments: $BOP < 0$ due to "excessive" money supply (increased M $\rightarrow$ (increases in all spending > increased spending on foreign things) $\rightarrow$ BOP deficit). Solution: monetary contraction $\rightarrow$ decreased Y, increased i $\rightarrow$ increased BOP.

Here is how the process is described by Edwards (1989). After an initial country evaluation, the IMF would

1. choose targets (for inflation, foreign reserves, etc.)
2. estimate "exogenous" components of the balance of payments (such as exports and interest payments)
3. determine the level of imports compatible with (1) and (2)
4. if the level of imports in (3) differs from the trend, decide whether devaluation is needed. If a devaluation is needed, return to step (2).
5. forecast money demand, which requires forecasting nominal income and velocity. (Velocity is often treated as exogenous.)
6. need interest rates be changed? If so, return to step (5) to revise forecasts.
7. determine the relationship between money and high powered money
8. calculate the level of DC compatible with the NFA target
9. check DC target for realism and consistency, focusing on the public sector demand for domestic credit
10. if DC target cannot be met, seek new sources of adjustment (e.g., demand reduction, supply expansion, financial policies). Return to (1) and iterate to consistency.

11. determine quantitative and non-quantitative "performance criteria" for program monitoring

12. negotiate the program with the country's authorities

The early static models used for these exercises are slowly being replaced with dynamic models that include an explicit role for expectations.

Notation

A Real absorption.[32]

$A(\cdot,\cdot,\cdot)$ A function returning the value of real absorption given the values of real government expenditure, real total income, and real wealth.

G Real government expenditure on final goods and services.

Y_T Real total income.

Ω Nominal wealth.

P Domestic price level (the domestic currency price of a unit of domestic GDP).

CA Real current account.

$\text{TB}(\cdot,\cdot)$ A function returning the real value of the trade balance given the real exchange rate and real total income.

S Nominal exchange rate (domestic currency cost of foreign currency).

P^* Foreign price level (the foreign currency price of a unit of foreign GDP).

FP Real net factor payments *received* from abroad.

UTr Real net unilateral transfers *received* from abroad.

NFA Real net foreign asset holdings.

FI Real net foreign indebtedness.

BoP The real balance of payments.

[32]Nominal means measured in domestic currency units (e.g., dollars in the U.S.). Real means measured in units of domestic GDP. You find the real value by dividing the nominal value by the GDP deflator, P.

Lecture 10

The Risk Premium

In this chapter we explore the nature and sources of currency risk, and we characterize portfolio choice behavior in the presence of currency risk. One incurs currency risk due to the currency denomination of one's portfolio. For example, if a portfolio contains unhedged foreign-currency denominated assets, then exchange rate movements can change the value of the portfolio.

With the advent of the general float, the risks of exposure to exchange rate changes were soon evident. Between June 1974 and October 1974, the Franklin National Bank of New York and the Bankhaus I.D. Herstatt of Germany failed due (at least in part) to losses from this source. At the time Franklin National was the twenty-third largest bank in the U.S.; it had an unhedged foreign exchange position of almost $2 billion and was illegally concealing losses on its foreign exchange operations. Herstatt's unhedged position was about $200 million. Exchange rate fluctuations—obviously unanticipated—pushed these positions into huge losses. On the other hand, betting against the dollar after the October 19, 1987 U.S. stock market crash generated large profits for U.S. banks.

10.1 Excess Returns

Compare the *ex post* real return from holding the domestic asset, $i_t - \pi_{t+1}$, with the (uncovered) *ex post* real return from holding the foreign asset, $i_t^* + \Delta s_{t+1} - \pi_{t+1}$. In Lecture2 we agreed to call the difference between these two *ex post* real rates of return the ***excess return*** on the domestic asset.

$$\begin{aligned} \text{er}_{t+1} &\overset{\text{def}}{=} (i_t - \pi_{t+1}) - (i_t^* + \Delta s_{t+1} - \pi_{t+1}) \\ &= i_t - i_t^* - \Delta s_{t+1} \end{aligned} \tag{10.1}$$

This is the *ex post* difference in the uncovered returns.

In Lecture2 we also developed the covered interest parity condition.

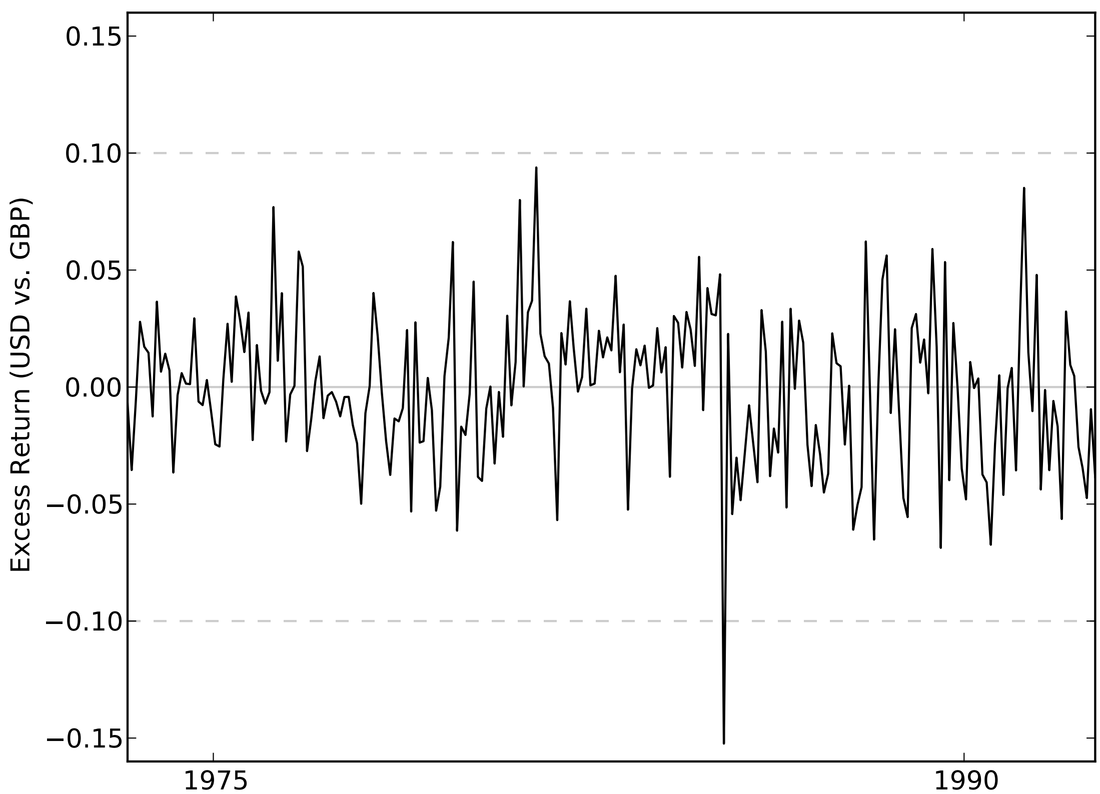
$$i = i^* + \text{fd} \tag{10.2}$$

This condition equates the rates of return on riskless assets. Currency risk is not involved in a covered interest arbitrage operation, for all currency exposure is completely hedged in the forward market.

Using covered interest parity, we can rewrite the excess return on the domestic currency as

$$\begin{aligned} \mathrm{er}_{t+1} &= \mathrm{fd}_t - \Delta s_{t+1} \\ &= (f_t - s_t) - (s_{t+1} - s_t) \\ &= f_t - s_{t+1} \end{aligned} \tag{10.3}$$

Consider Figure 10.1, which plots excess returns over time. We see these excess returns are large, variable, and mildly autocorrelated. We also see that in some subsamples, for example the early 1980s, the autocorrelation is higher than in others, for example the early 1970s. This apparent autocorrelation suggests that excess returns are somewhat predictable.

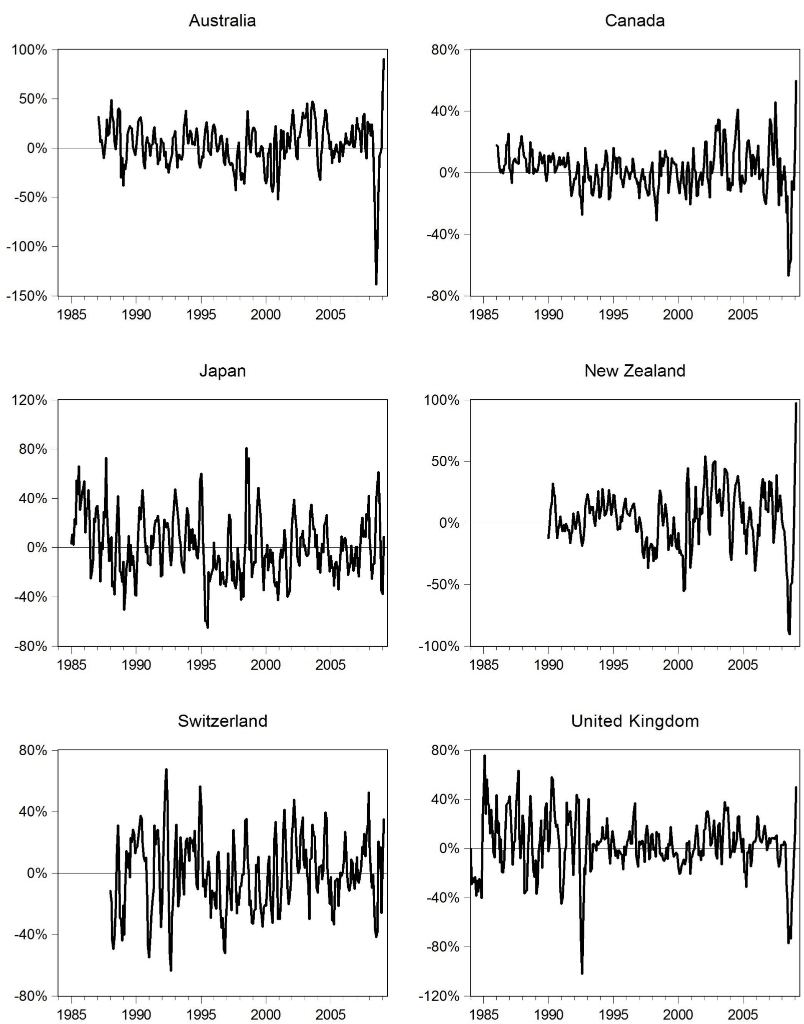


Source: Data from Hai et al. (1997)

Figure 10.1: Monthly Excess Returns: US/UK, 1973–1992

Note that covered interest arbitrage does *not* imply that the market expects zero excess return on the domestic asset. It does however imply that expected excess return shows up

Figure 2. 3-Month Excess Currency Return
(Annualized %; Home over USD return)



41

Figure 10.2: Three-Month Excess Currency Returns
Source: Chen and Tsang (2012)

as a gap between the forward rate and the expected future spot rate.

$$\begin{aligned} \text{er}^e_{t+1} &= \text{fd}_t - \Delta s^e_{t+1} \\ &= f_t - s^e_{t+1} \end{aligned} \tag{10.4}$$

We will call expected excess returns the ***risk premium***, rp_t.

$$\text{rp}_t = \text{er}^e_{t+1} \tag{10.5}$$

Let ϵ_{t+1} denote the spot-rate forecast error:

$$\epsilon_{t+1} = s_{t+1} - s^e_{t+1} \tag{10.6}$$

Then we can decompose excess returns into an expected component, and an unexpected component. The expected component is rp_t, the risk premium on the domestic currency. This is reduced by ϵ_{t+1}, the unexpected depreciation of the domestic currency.

$$\text{er}_{t+1} = \text{rp}_t - \epsilon_{t+1} \tag{10.7}$$

Economists have expended considerable effort trying to determine whether expected excess returns are zero (Hodrick, 1987). The hypothesis that expected excess returns are zero is known as the uncovered interest parity hypothesis.[1] However, as Figure 10.1 indicates, excess returns appear to be somewhat predictable, and uncovered interest parity is not supported by the data. We will explore this in more detail in the next section.

10.1.1 Uncovered Interest Parity

In Lecture2 we noted that speculative behavior ought to link the forward rate to the expected future spot rate. If the forward rate equaled the expected future spot rate, then expected excess returns would be zero. We will try to get a sense of how closely the two are tied together.

Suppose the forward rate equals the expected future spot rate. That is, assume the uncovered interest parity hypothesis.

$$f_t \overset{\text{UIP}}{=} s^e_{t+1} \tag{10.8}$$

Under UIP we can therefore write

$$f_t \overset{\text{UIP}}{=} s_{t+1} - \epsilon_{t+1} \tag{10.9}$$

where you will recall ϵ_{t+1} is the spot-rate-forecast error.

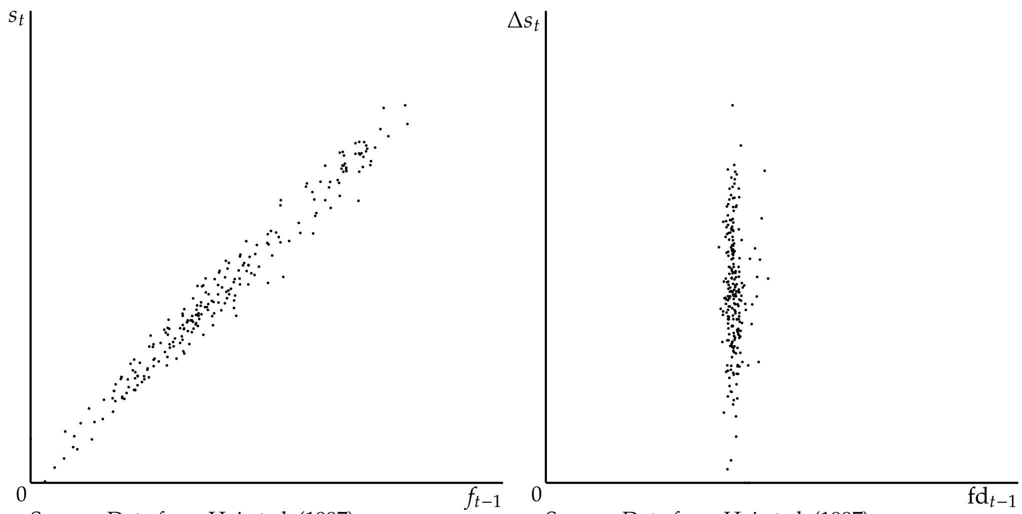
$$\epsilon_{t+1} \overset{\text{def}}{=} s_{t+1} - s^e_{t+1} \tag{10.10}$$

[1]This usage is standard, but it is not universal. For example, McCallum (1994) defined uncovered interest parity as requiring only that expected excess returns be determined exogenously. He allows, for example, exogenous risk premia, measurement errors, and aggregation effects.

If the forecast error is white noise, we then have a natural regression equation.

$$s_{t+1} = \beta_0 + \beta_1 f_t + \epsilon_{t+1} \tag{10.11}$$

We expect to find an estimate $\hat{\beta}_1 = 1$. Do we? Figure 10.3 suggests that we do.



Source: Data from Hai et al. (1997)   Source: Data from Hai et al. (1997)

Figure 10.3: Predicting Spot with Forward Rates: US/UK, 1973–1992

Note that if $\beta_1 = 1$, we can alternatively try the regression equation

$$\Delta s_{t+1} = \beta_0 + \beta_1(f_t - s_t) + \epsilon_{t+1} \tag{10.12}$$

and again expect to find an estimate $\hat{\beta}_1 = 1$. Do we? Figure 10.3 suggests that we do not.

Table 10.1 confirms that the answers to these two questions are quite different. When the spot rate is regressed on the forward rate that should "predict" it, the results look quite as anticipated. However when the apparently equivalent model (10.12) is used, quite different results emerge.[2] Apparently Tryon (1979) first noted these conflicting results, which have become known as the forward premium ***anomaly***, and they have been repeatedly confirmed (Fama, 1984; Hodrick, 1987). In table 10.1, we discover large, negative estimates for β_1 from the second model.

10.1.2 Expected Changes In The Real Exchange Rate

Combine covered interest arbitrage ($\text{fd} = i - i^*$) with the definition of the risk premium ($\text{rp} = f - s^e = \text{fd} - \Delta s^e$) to write

$$\text{rp} = i - i^* - \Delta s^e$$

[2] Huisman et al. (1998) note that the large standard errors from such regressions imply that the hypothesis that $\beta = 1$ cannot always be rejected.

Estimates of β_1 from Various Models			
Model:	USD/DEM	USD/GBP	USD/JPY
$s_{t+1} = \beta_0 + \beta_1 f_t + \epsilon_{t+1}$	0.99	0.98	0.99
$\Delta s_{t+1} = \beta_0 + \beta_1 \mathrm{fd}_t + \epsilon_{t+1}$	-4.20	-4.74	-3.33
$s_{t+1} - s_{t-1} = \beta_0 + \beta_1 (f_t - s_{t-1}) + \epsilon_{t+1}$	0.94	1.02	1.04

Sample: 1978.01–1990.07
Source: estimates from McCallum (1994).

Table 10.1: Testing UIP

Defining the real interest rates

$$r = i - \pi^e \tag{10.13}$$
$$r^* = i^* - \pi^{*e} \tag{10.14}$$

implies that the interest differential can be expressed as

$$\begin{aligned} i - i^* &= (r + \pi^e) - (r^* + \pi^{*e}) \\ &= (r - r^*) + (\pi^e - \pi^{*e}) \end{aligned}$$

We can therefore represent the risk premium as

$$\begin{aligned} \mathrm{rp} &= i - i^* - \Delta s^e \\ &= (r - r^*) + (\pi^e - \pi^{*e}) - \Delta s^e \\ &= (r - r^*) - (\Delta s^e + \pi^{*e} - \pi^e) \end{aligned} \tag{10.15}$$

The first term on the right of (10.15) is the real interest differential. The second term on the right is the expected change in the real exchange rate. Many presentations of the monetary approach to flexible exchange rates simply assume that both of these terms are zero, which implies that the risk premium is zero. Many tests for the existence of a risk premium have continued to invoke purchasing power parity, and evidence in favor of a risk premium has been interpreted as evidence of variations in $r - r^*$. However, equation (10.15) makes it clear that the presence of a risk premium may equally be due to expected changes in the real exchange rate. This possibility was emphasized by Korajczyk (1985).

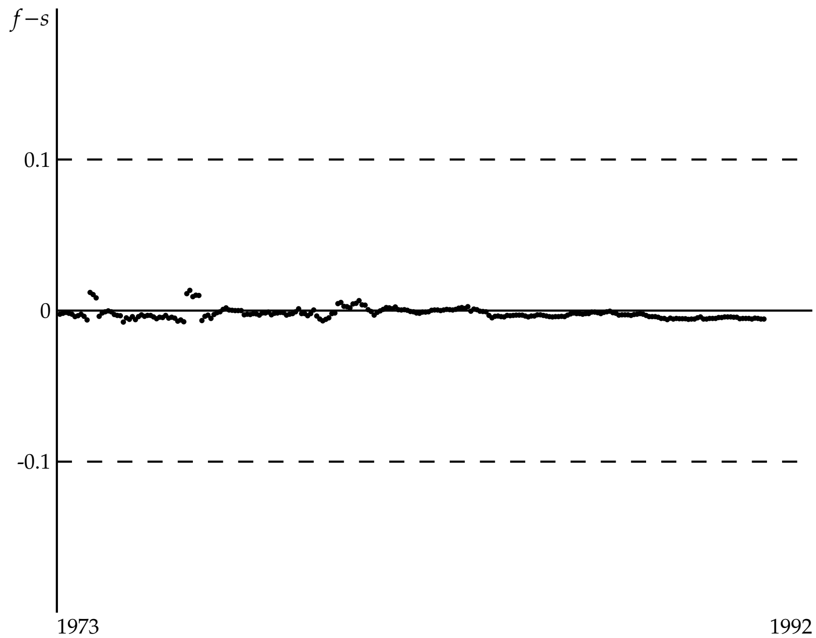
So another explanation of the forward rate bias emerges if we drop the purchasing power parity assumption, which is a cornerstone of the simplest monetary approach models. As usual, let

$$q = s + p^* - p \tag{10.16}$$

be the deviation from absolute PPP. The expected change in q is therefore

$$\Delta q^e = \Delta s^e + \pi^{*e} - \pi^e \tag{10.17}$$

Then given covered interest parity and our definitions of the real interest rates, the risk



Source: Data from Hai et al. (1997)

Figure 10.4: Contemporaneous Spot and Forward Rates: US/UK 1973–1992

premium can be written as

$$\mathrm{rp} = r - r^* - \Delta q^e$$

So we find a risk premium whenever there is a real interest rate differential *or* an expected change in the real exchange rate.[3] If real exchange rate changes are unpredictable—so that $\Delta q^e = 0$ as in the efficient markets version of PPP (Roll, 1979)—then the forward rate bias is evidence of a real interest rate differential. Real interest parity, on the other hand, implies that the forward rate bias is evidence that real exchange rate changes are anticipated. There is some evidence that real interest rates are not equal internationally (Mishkin, 1984). However Levine (JIMF, 1989) finds evidence that there is also a predictable component of real exchange rate changes. Thus it appears that both sources of forward exchange rate bias are operative. (Turning this around, (Huang, 1990) focuses on Δq^e: he finds that rp contributes more than real interest rate differentials to deviations in Δq^e.)

[3]Another common derivation works from the other end:

$$\begin{aligned} q - q^e &= (s - s^e) + (p^* - p^{*e}) - (p - p^e) \\ &= (\mathrm{rp} - \mathrm{fd}) + \pi^{*e} - \pi^e \\ &= (\mathrm{rp} + i^* - i) + \pi^{*e} - \pi^e \\ &= \mathrm{rp} + (i^* - \pi^{*e}) - (i - \pi^e) \\ &= \mathrm{rp} + r^* - r \end{aligned}$$

So we find a risk premium

$$\mathrm{rp} = r - r^* - (q^e - q)$$

10.2 Diversification of Currency Risk

When we speak of the riskiness of an asset, we are speaking of the volatility of the control over resources that is induced by holding that asset. From the perspective of a consumer, concern focuses on how holding an asset affects the consumer's purchasing power.

It might seem natural to view domestic assets as inherently less risky than foreign assets. From this perspective, domestic residents would demand a risk premium to hold a foreign asset. But clearly U.S. assets cannot pay a risk premium to Canadian residents at the same time that equivalent Canadian assets are paying a risk premium to U.S. residents. If a positive risk premium is paid in one direction, the risk premium must be negative in the other direction.

There are many possible sources of asset riskiness. For now we focus on currency risk. That is, we focus on how currency denomination alone affects riskiness. For example, we may think of debt issued in two different currency denominations by the U.S. government, so that the only clear difference in risk characteristics derives from the difference in currency denomination.

10.2.1 Sources of Currency Risk

The basic sources of risk from currency denomination are exchange rate risk and inflation risk. Exchange rate risk is the risk of unanticipated changes in the rate at which a currency trades against other currencies. Inflation risk is the risk of unanticipated changes in the rate at which a currency trades against goods priced in that currency. For example, a Canadian holding assets denominated in U.S. dollars must face uncertainty not only about the rate at which U.S. dollars can be turned into Canadian dollars but also about the price of goods in Canadian dollars. Now for most countries exchange rates are much more variable than the price level, in which case exchange rate risk deserves the most attention. However there are exceptions, especially in countries relying heavily on monetary finance of a large fiscal deficit.

If we consider the uncovered real return from holding a foreign asset, it is

$$r^{df} = i^* + \Delta s - \pi \tag{10.18}$$

So if Δs and π are highly correlated, the variance of the real return can be small—in principle, even smaller than the variance of the return on the domestic asset. Thus in countries with very unpredictable inflation rates, we can see how holding foreign assets may be less risky than holding domestic assets. This can be the basis of capital flight—capital outflows in response to increased uncertainty about domestic conditions.[4] Capital flight can simply be the search for a hedge against uncertain domestic inflation.

The notion of the riskiness of an asset is a bit tricky: it always depends on the portfolio to which that asset will be added. Similarly, the risk of currency denomination cannot be considered in isolation. That is, we cannot simply select a currency and then determine

[4] Cuddington (1986) discusses the role of inflation risk in the Latin American capital flight of the 1970s and early 1980s.

its riskiness. We need to know how the purchasing power of that currency is related to the purchasing power of the rest of the assets we are holding. The riskiness of holding a DEM denominated bond, say, cannot be determined without knowing its correlation with the rest of my portfolio.

We will use ***correlation*** as our measure of relatedness. The ***correlation coefficient*** between two variables is one way to characterize the tendency of these variables to move together. An asset return is positively correlated with my portfolio return if the asset tends to gain purchasing power along with my portfolio. An asset that has a high positive correlation with my portfolio is risky in the sense that buying it will increase the variance of my purchasing power. Such an asset must have a high expected rate of return for me to be interested in holding it.

In contrast, adding an asset that has a low correlation with my portfolio can reduce the variance of my purchasing power. For example, holding two equally variable assets that are completely uncorrelated will give me a portfolio with half the variability of holding either asset exclusively. When one asset declines in value, the other has no tendency to follow suit. In this case diversification "pays", in the sense that it reduces the riskiness of my portfolio.

From the point of view of reducing risk, an asset that is negatively correlated with my portfolio is even better. In this case there is a tendency of the asset to offset declines in the value of my portfolio. That is, when the rest of my portfolio falls in value, this asset tends to rise in value. If two assets are perfectly negatively correlated, we can construct a riskless portfolio by holding equal amounts of each asset: whenever one of the assets is falling in value, the other is rising in value by an equal amount. In order to reduce the riskiness of my portfolio, I may be willing to accept an inferior rate of return on an asset in order to get its negative correlation with my portfolio rate of return.

If we look at an asset in isolation, we can determine its expected return and the variance of that return. A high variance would seem on the face of it to be risky. However we have seen that the currency risk and inflation risk of an isolated asset are not very interesting to consider. We may be interested in holding an asset denominated in a highly variable foreign currency if doing so reduces the variance of our portfolio rate of return. To determine whether the asset can do this, we must consider its correlation with our current portfolio. A low correlation offers an opportunity for diversification, and a negative correlation allows even greater reductions in portfolio risk. We are willing to pay extra for this reduction in risk, and the risk premium is the amount extra we pay. If adding foreign assets to our portfolio reduces its riskiness, then the risk premium on domestic assets will be positive.

10.2.2 Optimal Diversification

Consider an investor who prefers higher average returns but lower risk. We will capture these preferences in a utility function, which depends positively on the average return of the investors portfolio and negatively on its variability, $U(\mathcal{E}r^p, \mathbf{var} r^p)$. We can think of portfolio choice as a two stage procedure. First we determine the portfolio with the lowest risk: the ***minimum-variance portfolio***. Second, we decide how far to deviate from the mimimum-variance portfolio based on the rewards to risk bearing.

If the domestic asset is completely safe, then of course the minimum variance portfolio does not include any risky foreign assets. For the moment, consider portfolio choice under the additional assumption that we can treat nominal interest rates as certain. In this special case, all uncertainty is exchange rate uncertainty. As shown in section 10.2.2, our optimal portfolio can then be represented by (10.19).

$$\alpha^d = -\frac{i - i^* - \Delta s^e}{\text{RRA}\,\mathbf{var}\Delta s} \tag{10.19}$$

Here α^d is the fraction of your portfolio that you want to allocate to foreign assets, and $\mathbf{var}\Delta s$ is the variance of the rate of depreciation of the domestic currency. RRA is a measure of attitude toward risk: the more you dislike taking risks, the larger is RRA, which is called the coefficient of ***relative risk aversion***.

Recall that we can write the covered interest arbitrage condition as $\text{fd} = i - i^*$, and that we defined $\text{rp} = \text{fd} - \Delta s^e$. That is, the forward discount on domestic currency can be decomposed into two parts: the expected rate of depreciation of the domestic money, and the deviation of the forward rate from the expected future spot rate. The latter we call the risk premium on domestic currency. So we can use covered interest parity to write the risk premium as

$$\text{rp} = i - i^* - \Delta s^e$$

We can therefore write our last solution for α as

$$\alpha^d = -\frac{\text{rp}}{\text{RRA}\,\mathbf{var}\Delta s} \tag{10.20}$$

Equation (10.20) says that we need a negative risk premium on the domestic currency (i.e., a positive risk premium on foreign currency) to be willing to hold any of the foreign asset. Exchange rate variance is also important of course: the higher is $\mathbf{var}\Delta s$, the lower is α, which just says that fewer foreign assets are held as they become riskier. Finally, attitude toward risk must also play a role. Here RRA is a measure of risk aversion, i.e., of how much an investor dislikes risk. An investor with a higher RRA is more risk averse and less inclined to hold the risky foreign asset.

It proves informative to turn this reasoning around. Let α be the outstanding proportion of foreign assets supplied in financial markets. In equilibrium, outstanding asset supplies must be willingly held: $\alpha = \alpha^d$. Treating α as exogenous, along with RRA and $\mathbf{var}\Delta s$, we can determine the risk premium.

$$\text{rp} = -\alpha\text{RRA}\,\mathbf{var}\Delta s \tag{10.21}$$

This is the risk premium that must be paid in order for the asset markets to be in equilibrium. Of course if investors are risk neutral (so that RRA $= 0$) then no risk premium is required. But if investors are risk averse, then a risk premium must be paid and it will vary with asset supplies. For example, as the U.S. floods international markets with dollar denominated debt, we can expect a rising risk premium to be paid on dollar assets.

This suggests that to understand variations in the risk premium we might turn to variations in asset supplies. Yet there appears to be considerable short-run variation in the risk premium, while asset accumulation is, in the short run, very small in comparison to the outstanding stocks of assets. In addition, Frankel (1986a) has suggested that a 1% increase in world wealth would imply only a 0.2% per annum increase in the risk premium. This suggests that variations in asset stocks will not prove useful in explaining short-run variations in the risk premium.

Mean Variance Optimization

Let us return to our investor who prefers higher average returns but lower risk, as represented by the utility function $U(\mathcal{E}r^p, \mathbf{var}r^p)$. Domestic assets pay $r = i - \pi$ and foreign assets pay $r^{df} = i^* + \Delta s - \pi$ as real returns to domestic residents. The total real return on the portfolio r^p will then be a weighted average of the returns on the two assets, where the weight is just α (the fraction of the portfolio allocated to foreign assets).

$$r^p = \alpha r^{df} + (1-\alpha)r \tag{10.22}$$

For example if two-thirds of your portfolio is allocated to the domestic asset, then two-thirds of the portfolio rate of return is attributable to the rate of return on that asset (and the remaining one-third to the foreign asset). Therefore the expected value of the portfolio rate of return is

$$\mathcal{E}r^p = \alpha\mathcal{E}r^{df} + (1-\alpha)\mathcal{E}r \tag{10.23}$$

There is another way to look at the same equations. The portfolio return can be thought of as the domestic rate of return adjusted for the fraction of the portfolio in the foreign asset.

$$r^p = r + \alpha(r^{df} - r) \tag{10.24}$$

$$\mathcal{E}r^p = \mathcal{E}r + \alpha(\mathcal{E}r^{df} - \mathcal{E}r) \tag{10.25}$$

Using equations (10.22) and (10.23) along with the definition of variance, we can find the variance of the portfolio rate of return.[5]

Consider how to maximize utility, which depends on the mean and variance of the portfolio rate of return. The objective is to choose α to maximize utlity.

$$\begin{aligned}\max_{\alpha} U\big(&\alpha\mathcal{E}r^{df} + (1-\alpha)\mathcal{E}r,\\ &\alpha^2\mathbf{var}r^{df} + 2\alpha(1-\alpha)\mathbf{cov}(r, r^{df}) + (1-\alpha)^2\mathbf{var}r\big)\end{aligned} \tag{10.26}$$

[5]Recall that variance measures how far you tend to be from the average: $\mathbf{var}r^p = \mathcal{E}(r^p - \mathcal{E}r^p)^2$. Covariance measures the relatedness of two variables. That is, it tells us if one of the variables tends to be larger than average whenever the other is: $\mathbf{cov}(r, r^{df}) = \mathcal{E}(r - \mathcal{E}r)(r^{df} - \mathcal{E}r^{df})$. Use these definitions along with (10.22) and (10.23) to produce

$$\mathbf{var}r^p = \alpha^2\mathbf{var}r^{df} + 2\alpha(1-\alpha)\mathbf{cov}(r, r^{df}) + (1-\alpha)^2\mathbf{var}r$$

Consider how changes in α change total utility.

$$\begin{aligned}\frac{dU}{d\alpha} =& (\mathcal{E}r^{df} - \mathcal{E}r)U_1 \\ &+ 2\left(\alpha\mathbf{var}r^{df} - (1-\alpha)\mathbf{var}r + (1-2\alpha)\mathbf{cov}(r, r^{df})\right)U_2\end{aligned} \tag{10.27}$$

As long as this derivative is positive, so that increasing α produces and increase in utility, we want to increase alpha. If this derivative is negative, we can increase utility by reducing alpha. These considerations lead to the "first-order condition": the requirement that $dU/d\alpha = 0$ at a maximum. We use the first-order necessary condition to produce a solution for α.[6]

$$\alpha = \frac{(\mathcal{E}r^{df} - \mathcal{E}r) - \frac{2U_2}{U_1}\left(\mathbf{var}r - \mathbf{cov}(r, r^{df})\right)}{-\frac{2U_2}{U_1}\left(\mathbf{var}r^{df} + \mathbf{var}r - 2\mathbf{cov}(r, r^{df})\right)} \tag{10.28}$$

Here RRA $= -2U_2/U_1$ (the coefficient of relative risk aversion) and $\sigma^2 = \mathbf{var}r^{df} + \mathbf{var}r - 2\mathbf{cov}(r, r^{df})$. Recalling that $\mathcal{E}r^{df} - \mathcal{E}r = i^* + \Delta s^e - i = -\text{rp}$ we therefore have

$$\alpha = -\frac{\text{rp}}{\text{RRA}\,\sigma^2} + \alpha_{\min\mathbf{var}} \tag{10.29}$$

Here $\alpha_{\min\mathbf{var}} = \left(\mathbf{var}r - \mathbf{cov}(r, r^{df})\right)/\sigma^2$ is the α that yields the minimum variance portfolio (Kouri 1978), so the rest can be considered the speculative portfolio share. Investors can be thought of as initially investing entirely in the minimum variance portfolio and then exchanging some of the lower return asset for some of the higher return asset. They accept some increase in risk for a higher average return. If the assets have the same expected return, they will simply hold the minimum variance portfolio.

For a moment, let us focus on the case where the covariance of the returns is zero. Diversification still reduces risk. (Compare $\alpha = 0$ to $\alpha = 1/2$ with equal variances of the two assets.) Thus there is an incentive to diversify even when one asset has a lower average return. Of course relative variance matters: ceteris paribus you wish to hold less of a more variable asset. In the extreme case when one asset has zero variance—say the domestic asset is viewed as completely safe—the other asset will be held only if it has a higher return.[7]

Let us consider this case in a little detail. If the domestic asset is completely safe, then of course the minimum variance portfolio does not include any of the foreign asset. Suppose in addition we can treat nominal returns as certain, so that all uncertainty is exchange rate uncertainty. Our optimal portfolio can then be represented by (10.30).

$$\alpha = \frac{i^* + \Delta s^e - i}{\text{RRA}\,\mathbf{var}\Delta s} \tag{10.30}$$

[6]See problem 5.

[7]Zero variability of the real return implies the domestic security is perfectly indexed to inflation or, equivalently, that inflation is perfectly predicted. However, corporations might be induced to act as if the domestic return were certain by accounting rules that emphasize nominal profits measured in the domestic currency.

Here RRA is a measure of attitude toward risk: the more you resist taking risks, the larger is RRA.

Recall that we can write the covered interest arbitrage condition as $\text{fd} = i - i^*$, and that $\text{rp} = \text{fd} - \Delta s^e$. That is, the forward discount on domestic currency can be decomposed into the expected rate of depreciation of the domestic money and the deviation of the forward rate from the expected future spot rate, which we call the risk premium on domestic currency. So we can use covered interest parity to write the risk premium as

$$\text{rp} = i - i^* - \Delta s^e$$

We can therefore write our last solution for α as

$$\alpha = \frac{-\text{rp}}{\text{RRA}\,\mathbf{var}\Delta s} \tag{10.31}$$

10.3 An Empirical Puzzle

Section 10.1.1 provided evidence that if foreign exchange markets are efficient then part of the forward discount on domestic currency is a risk premium. Risk averse investors will insist on a higher return before taking a position in a risky currency. Such behavior offers a potential explanation of deviations of the forward rate from the expected future spot rate.

For example, suppose we find $0 < \beta < 1$ when we estimate (10.12). Then when the Canadian dollar is selling at a forward discount, we will interpret only a fraction β of that discount as expected depreciation of the Canadian dollar. We attribute the remaining fraction $(1 - \beta)$ of the discount to a risk premium reflecting the perceived risk of holding Canadian dollars. This may be clearer if we recall the covered interest parity condition, which says that when foreign exchange sells at a forward discount then the interest rate paid on foreign currency must be above the domestic interest rate. Some of this higher rate of return just offsets expected depreciation, on average, but the rest of it offsets the perceived risk of having a position in foreign exchange.

$$i_t - i_t^* = \Delta s_t^e + \text{rp}_t$$

Probably the most famous study of the properties of spot and forward rates is that of Fama (1984). Using monthly data he showed that $f - s_{+1}$ has a larger standard deviation than Δs_{+1}. In this sense we can say that the current spot rate is a better predictor of the future spot rate than is the current forward rate. Further, the autocorrelations in Δs are close to zero, and those in $f - s_{+1}$ are also small, but the autocorrelations in $f - s$ are large and show a slow decay. Since $f - \Delta s$ is the risk premium plus expected depreciation, at least one of these appears to be autocorrelated.

The forward rate appears more closely pegged to the current spot rate than the future spot rate: the standard deviation of $f_t - s_t$ is much smaller than that of $f_t - s_{t+1}$. This may suggest that most of the innovation in both is due to news.

10.3.1 Algebra

To simplify notation, we will drop time subscripts as long as this will not generate confusion. Define the forward discount $\text{fd} = f - s$, the risk premium $\text{rp} = f - s^e = \text{fd} - \Delta s^e$ where $\Delta s_t^e = s_{t+1}^e - s_t$ is the expected rate of depreciation of the spot rate, and the forecast error $\epsilon_{t+1} = s_{t+1} - s_{t+1}^e = \Delta s - \Delta s^e$. Then if we project (regress) Δs on fd, we will find[8]

$$\Delta s = \hat{\beta}_0 + \hat{\beta}_1 \,\text{fd} + \text{residual}$$

where

$$\hat{\beta}_1 = \frac{\mathbf{cov}(\text{fd}, \Delta s)}{\mathbf{var}\text{fd}}$$
$$\hat{\beta}_1 = \frac{\mathbf{cov}(\text{fd}, \Delta s^e) + \mathbf{cov}(\text{fd}, \epsilon)}{\mathbf{var}\text{fd}}$$

The second equality follows from $\Delta s = \Delta s^e + \epsilon$. Now note that we can write $\Delta s^e = \text{fd} - \text{rp}$, so that $\mathbf{cov}(\text{fd}, \Delta s^e) = \mathbf{cov}(\text{fd}, \text{fd} - \text{rp}) = \mathbf{var}(\text{fd}) - \mathbf{cov}(\text{fd}, \text{rp})$. It follows that

$$\begin{aligned}\hat{\beta}_1 &= \frac{\mathbf{var}(\text{fd}) - \mathbf{cov}(\text{fd}, \text{rp}) + \mathbf{cov}(\text{fd}, \epsilon)}{\mathbf{var}(\text{fd})} \\ &= 1 - b_{\text{rp}} - b_{\text{re}}\end{aligned}$$

where we define[9]

$$\begin{aligned} b_{\text{re}} &= -\mathbf{cov}(\text{fd}, \epsilon)/\mathbf{var}(\text{fd}) \\ b_{\text{rp}} &= \mathbf{cov}(\text{fd}, \text{rp})/\mathbf{var}(\text{fd}) \end{aligned} \tag{10.32}$$

Note that $b_{\text{rp}} = 0$ if the risk premium is constant (or otherwise uncorrelated with fd). Similarly, $b_{\text{re}} = 0$ if there are no systematic prediction errors in the model. Most empirical work has assumed that there are no systematic prediction errors in the foreign exchange markets, that ϵ is a white noise error term uncorrelated with anything else.[10] In this case, $b_{\text{re}} = 0$ and $\hat{\beta}_1 = 1 - b_{\text{rp}}$.

$$\hat{\beta}_1 = 1 - \frac{\mathbf{cov}(\text{fd}, \text{rp})}{\mathbf{var}(\text{fd})} \tag{10.33}$$

In this case, if movement in fd are not reflecting movements in rp then $\hat{\beta}_1 = 1$. In addition, since

$$\begin{aligned} \mathbf{cov}(\text{fd}, \text{rp}) &= \mathbf{cov}(\text{fd}, \text{fd} - \Delta s^e) \\ &= \mathbf{var}(\text{fd}) - \mathbf{cov}(\text{fd}, \Delta s^e) \end{aligned} \tag{10.34}$$

we also know that if Δs^e is constant then $\hat{\beta}_1 = 0$. For example, if the spot rate is believed to follow a random walk, then $\Delta s^e = 0$. In this case, all of the forward discount is due to a

[8]Some empirical studies replace fd with $i - i^*$, which should be equivalent by covered interest parity.
[9]Froot and Frankel (1989) make use of $\mathbf{cov}(\text{fd}, \text{rp}) = \mathbf{var}(\text{rp}) + \mathbf{cov}(\text{rp}, \Delta s^e)$ in their definition of b_{rp}.
[10]White noise is zero mean, finite variance, and uncorrelated.

risk premium, and we should find an estimated regression coefficient of zero.

So if rp is constant, $\hat{\beta}_1 = 1$; but if Δs^e is constant, $\hat{\beta}_1 = 0$.[11] The empirical results don't support either: in fact, we generally find $\hat{\beta}_1 < 0$.[12] Now this result, that $0 > \mathbf{cov}(\text{fd}, \Delta s)$, implies $0 > \mathbf{cov}(\text{rp}, \Delta s^e)$ since

$$0 > \mathbf{cov}(\text{fd}, \Delta s) = \mathbf{var}(\Delta s^e) + \mathbf{cov}(\text{rp}, \Delta s^e) \geq \mathbf{cov}(\text{rp}, \Delta s^e)$$

Further, we must have $\mathbf{var}\text{rp} > \mathbf{var}\Delta s^e$ since

$$\begin{aligned} \mathbf{var}\text{fd} &= \mathbf{var}\Delta s^e + 2\mathbf{cov}(\text{rp}, \Delta s^e) + \mathbf{var}\text{rp} \\ &= [\mathbf{var}\Delta s^e + \mathbf{cov}(\text{rp}, \Delta s^e)] + [\mathbf{cov}(\text{rp}, \Delta s^e) + \mathbf{var}\text{rp}] \\ > 0 \text{ and} &\qquad\qquad < 0 \qquad\qquad \Leftrightarrow \qquad\qquad > 0 \end{aligned}$$

10.4 Explanations Of The Puzzle

Fama (1984) first noted the implications that $\mathbf{cov}(\text{rp}, \Delta s^e) < 0$ and that most of the variance in the forward discount is due to variance in the risk premium, and he conjectured that the maintained rationality assumption is too strong. Another possibility noted by Isard (1988) is that central banks peg i and i^* so that $f - s$ is relatively constant. Exogenous increments in rp would then be reflected in decrements to Δs^e. More generally, as pointed out by Boyer and Adams (1988), interest elastic money supply or demand suffices for this result. This is discussed in the next section.

10.4.1 Peso Problem

Evidence of systematic forecast errors is puzzling: it suggests that financial market participants repeatedly make the same mistakes. The "peso problem" shows how even in the presence of rational expectations we may turn up such evidence. The idea is that low frequency expected events may lead to long periods of ex post excess returns (Krasker, 1980; Kaminsky, 1993). We can motivate this as agents having imperfect information about their economic environment.

During most of the 20th century, the MXP was a model currency.[13] The oil crisis hit

---

[11] This can also be seen from a common alternative derivation that begins with the assumption that ϵ is white noise to argue

$$\begin{aligned} \hat{\beta}_1 &= \mathbf{cov}(\text{fd}, \Delta s)/\mathbf{var}\text{fd} \\ &= \mathbf{cov}(\Delta s^e + \text{rp}, \Delta s^e + \epsilon)/\mathbf{var}(\Delta s^e + \text{rp}) \\ &= [\mathbf{var}\Delta s^e + \mathbf{cov}(\text{rp}, \Delta s^e)]/[\mathbf{var}\Delta s^e + 2\mathbf{cov}(\text{rp}, \Delta s^e) + \mathbf{var}\text{rp}] \end{aligned}$$

[12] So if you move your money with the interest differential, that will tend to pay! (Of course, it will be a risky strategy.) Note that the focus here is on short run correlations: $i - i^*$ tends to correctly forecast long run depreciation since higher inflation generates both higher interest rates and depreciation.

[13] The nuevo peso, introduced January 1, 1993, has ISO code MXN. One MXN was worth 1,000 MXP at the time of the conversion. The word 'nuevo' was removed from the currency on January 1, 1996.

Mexico hard, however. In the late 1970s, (Krasker, 1980) observed a persistent interest rate differential in favor of the MXP, despite a fixed exchange rate. From a US perspective, we observed

$$i - i^* = f - s < 0 \tag{10.35}$$

We also observed

$$s_{+1} = s \tag{10.36}$$

(because of the successfully fixed exchange rate), so we persistently have

$$f < s_{+1} \tag{10.37}$$

In this sense, we have systematic forward rate forecast errors.

if agents know that the exchange rate is fixed so that

$$s^e_{+1} = s_{+1} \tag{10.38}$$

it seems we persistently have

$$f < s^e_{+1} \tag{10.39}$$

However, following Mark, let us tell a different story. Suppose that each period there is a probability p of devaluation from $\bar{s}_0$ to $\bar{s}_1$. Then at each t prior to any devaluation we have

$$s_{t+1} = \begin{cases} \bar{s}_1 & \text{with probability } p \\ \bar{s}_0 & \text{with probability } 1-p \end{cases} \tag{10.40}$$

This gives us a one period ahead expected value of

$$\mathcal{E}[s_{t+1}] = p\bar{s}_1 + (1-p)\bar{s}_0 \tag{10.41}$$

The implied forecast error as long as the peg is maintained is

$$\bar{s}_0 - \mathcal{E}_t[s_{t+1}] = p(s_0 - s_1) < 0 \tag{10.42}$$

In this sense, we get a rational expectations explanation of the systematic forward rate forecast errors. Furthermore, our forecast errors are serially correlated, but they contain no information that would help us better predict the future.

Learning

Lewis (1989) pursues the peso problem logic one step further by adding learning. In this case, agents are not immediately certain whether a regime shift has taken place. (Consider for example the changes in Fed operating procedures in 1979 or 1982.) Lewis introduces these considerations into our basic monetary approach model

$$s_t = \frac{1}{1+\lambda}\tilde{m}_t + \frac{\lambda}{1+\lambda}\mathcal{E}_t s_{t+1} \tag{10.43}$$

Assume that fundamentals follow a random walk with drift:

$$\tilde{m}_t = \mu_0 + \tilde{m}_{t-1} + v_t \tag{10.44}$$

where $v_t \sim N(0, \sigma_v^2)$ is white noise. Recall that (using the method of undetermined coefficients, for example) you can show that in our basic monetary model this has the solution

$$s_t = \lambda\mu_0 + \tilde{m}_t \tag{10.45}$$

Now introduce the possibility of a regime shift to $\mu_0' > \mu_0$. This changes the expected future fundamentals:

$$\mathcal{E}_t\tilde{m}_{t+1} = p\mu_0 + (1-p)\mu_0' + \tilde{m}_t \tag{10.46}$$

Solving again (e.g., using the method of undetermined coefficents), we get

$$s_t = \tilde{m}_t + \lambda[p_t\mu_0 + (1-p_t)\mu_0'] \tag{10.47}$$

After some manipulation, this implies

$$\mathcal{E}_t s_{t+} = \tilde{m}_t + (1+\lambda)[p_t\mu_0 + (1-p_t)\mu_0'] \tag{10.48}$$

This gives us a forecast error of

$$s_{t+1} - \mathcal{E}_t s_{t+1} = \lambda[(\mu_0' - \mu_0)(p_{t+1} - p_t)] + \mu_0' + v_{t+1} - [\mu_0 + (\mu_0' - \mu_0)(1 - p_t)] \tag{10.49}$$

10.4.2 Exogenous Risk Premia

In order to relax the standard monetary approach assumption of risk neutrality, we will now consider the following model due to Boyer and Adams (1988).

$$s = q + p - p^* \tag{10.50}$$
$$i - i^* = f - s \tag{10.51}$$
$$f = s^e + \text{rp} \tag{10.52}$$
$$h - h^* - (p - p^*) = \phi(y - y^*) - \lambda(i - i^*) \tag{10.53}$$

The model is a simple version of the monetary approach model developed in Lecture3: (10.50) is absolute purchasing power parity, (10.51) is covered interest parity, (10.52) defined the risk premium (rp), and (10.53) is money market equilibrium. The variables s, $p - p^*$, $i - i^*$, and f are endogenous. This is a fairly standard monetary approach model. However, we allow $f \neq s^e$ so that there can be an *exogenous* risk premium: $\text{rp} \stackrel{\text{def}}{=} f - s^e$.[14]

The solution procedure is largely unchanged from Lecture3. The only real change is that we will distinguish two data generating processes: one for the risk premium, and one

[14]Note that we call rp the risk premium; it is the risk premium paid by domestic assets. In the literature $\text{rp}^* \equiv -\text{rp}$ is often called the risk premium as well; it is the risk premium paid by foreign assets. To avoid confusion, just keep in mind that the risk premium is a real return differential and ask yourself: who is being paid to bear what risk? Here, rp is the expected extra cost (the premium) of buying future foreign exchange without risk. Note that—like the real interest rate—the risk premium is an ex ante concept.

for the remaining exogenous determinants of the exchange rate. Recall that $\text{rp} \stackrel{\text{def}}{=} f - s^e$, so that $f - s \equiv \text{rp} + s^e - s$. Thus relative prices can be written as

$$\begin{aligned} p - p^* &= h - h^* - \phi(y - y^*) + \lambda(i - i^*) \\ &= h - h^* - \phi(y - y^*) + \lambda(f - s) \\ &= h - h^* - \phi(y - y^*) + \lambda(\text{rp} + s^e - s) \end{aligned} \tag{10.54}$$

Recalling that $s = q + p - p^*$, we therefore have

$$s = q + h - h^* - \phi(y - y^*) + \lambda(\text{rp} + s^e - s) \tag{10.55}$$

So, if we combine terms in s (and let $\tilde{m} \equiv h - h^* - \phi(y - y^*)$ to reduce notation),

$$s = \frac{1}{1+\lambda}(\tilde{m} + \lambda\text{rp}) + \frac{\lambda}{1+\lambda}s^e \tag{10.56}$$

Equation (10.56) can be solved using the recursive substitution procedure developed in Lecture4. Adding time subscripts for clarity, we find

$$s_t = \frac{1}{1+\lambda}\sum_{i=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^i \left(\mathcal{E}_t\tilde{m}_{t+i} + \lambda\mathcal{E}_t\text{rp}_{t+i}\right) \tag{10.57}$$

To explore this, suppose $\tilde{m} = \tilde{m}_{t-1} + u_t$ where u_t and rp_t ~White Noise. Note that we are treating rp as *exogenous*! Then

$$\begin{aligned} s_t &= \tilde{m}_t + \frac{\lambda}{1+\lambda}\text{rp}_t \\ \Longrightarrow \quad s_{t+1} &= \tilde{m}_{t+1} + \frac{\lambda}{1+\lambda}\text{rp}_{t+1} \\ \Longrightarrow \quad \Delta s_{t+1} &= \Delta\tilde{m}_{t+1} + \frac{\lambda}{1+\lambda}\Delta\text{rp}_{t+1} \\ \Longrightarrow \quad \Delta s^e_{t+1} &= -\frac{\lambda}{1+\lambda}\text{rp}_t \end{aligned}$$

From these relations we can derive

$$\mathbf{cov}(\Delta s^e, \text{rp}) = -\frac{\lambda}{1+\lambda}\mathbf{var}\text{rp} < 0 \tag{10.58}$$

$$\mathbf{var}\Delta s^e = \left(\frac{\lambda}{1+\lambda}\right)^2 \mathbf{var}\text{rp} < \mathbf{var}\text{rp} \tag{10.59}$$

Keeping in mind the rational expectations hypothesis under which this analysis is conducted, so that the exchange rate forecast error is uncorrelated with any current information, we can also derive

$$\begin{aligned}\mathbf{cov}(\Delta s, \mathrm{rp}) &= \mathbf{cov}(\Delta s^e + \epsilon, \mathrm{rp}) \\ &= -\frac{\lambda}{1+\lambda}\mathbf{var}\mathrm{rp} < 0\end{aligned} \tag{10.60}$$

$$\begin{aligned}\mathbf{cov}(\Delta s, \mathrm{fd}) &= \mathbf{cov}(\Delta s^e + \epsilon, \Delta s^e + \mathrm{rp}) \\ &= \mathbf{var}\Delta s^e + \mathbf{cov}(\Delta s^e, \mathrm{rp}) \\ &= -\frac{\lambda}{(1+\lambda)^2}\mathbf{var}\mathrm{rp} < 0\end{aligned} \tag{10.61}$$

Note that equation (10.61) explains the negative coefficient Fama (1984) and others found in the unbiasedness regressions discussed in section 10.3. In addition, Fama (1984) regressed $\mathrm{fd} - \Delta s$ on fd. This supplies no additional information since

$$\mathbf{cov}(\mathrm{fd} - \Delta s, \mathrm{fd}) = \mathbf{var}\mathrm{fd} - \mathbf{cov}(\Delta s, \mathrm{fd})$$

However, since $\mathrm{fd}(= f - s = \mathrm{rp} + \Delta s^e) = \mathrm{rp}/(1+\lambda)$, treating rp as *exogenous* suggests that the regression

$$\mathrm{fd} = a + b\mathrm{rp} + v$$

should yield $0 < b < 1$, and Boyer and Adams find $\hat{\beta}_1 = .033$.

They also note that we observe $\mathrm{rp}^x = \mathrm{fd} - \Delta s = \mathrm{rp} - \epsilon$ instead of the true risk premium, creating an errors in variables problem.[15] Fama (1984) ran $\mathrm{rp}^x = a' + b'\mathrm{fd}$. Boyer and Adams suggest 1) this reverses the exogeneity and 2) this ignores errors in variables. Correcting for both problems,[16] Boyer and Adams find $\hat{\beta}_1 = .129$, which implies $\lambda \approx 6.8$ and a resulting interest rate elasticity between .14 + .68 (see their paper for the details). This estimate is consistent with other work on money demand.

10.4.3 Irrational Expectations

Survey data on exchange rate expectations uses the reported forecasts of participants in the foreign exchange market. Recently, such data have been increasingly exploited to address questions of the source of the rejection of unbiasedness. Surveys allow us to actually collect data on Δs^e_{t+1} and rp. For example, Cavaglia et al. (1994) estimate the regression

$$\Delta s^e_{t+1} = \alpha + \beta \mathrm{fd}_t + \epsilon_{t+1}$$

[15] That is, the ex post measure of the risk premium most often used in empirical work is

$$\mathrm{rp}^x = f - s_{+1} = f - (s^e_{+1} + \epsilon) = \mathrm{rp} - \epsilon$$

However, survey data may offer direct measurements of rp.

[16] However, if we take their model seriously Fama's procedure in fact gives consistent and more efficient estimates of λ than their errors in variables procedure. This is because under the rational expectations hypothesis the forecast error is not correlated with the forward discount.

using survey data. Perfect substitutability between domestic and foreign assets would imply $\alpha = 0$ and $\beta = 1$. If there is a risk premium but it is zero on average, then $\alpha = 0$. They generally find evidence against perfect substitutability, but they cannot reject the risk premium being zero on average. They also find the risk premium tends to be autocorrelated, in line with some theoretical predictions.[17]

Survey data allows us to consider the possibility that the problem lies in the rational expectations assumptions. Early discussions of this possibility include Bilson (1981), Longworth (1981), Cumby and Obstfeld (1984), and Fama (1984). However, economists were reluctant to accept this interpretation. It seems to imply that speculators are overlooking ready profit opportunities, and economists have been inclined to view financial markets as particularly likely to efficiently exploit all available information. However, alternatives to the rational expectations hypothesis have drawn increasing attention since the dollar's 65% real appreciation in the mid 1980s and its subsequent offsetting depreciation sent economists scrambling for possible explanatory fundamentals.[18] Ito (1990) has found considerable evidence in the survey data that expectations are not rational—individuals appear to have "wishful" expectations, and their short run and long run expectations are not consistent. We will examine Froot and Frankel (1989), who also address this question using survey data on the expectations of foreign exchange traders; they find the forecast error to be correlated with the forward discount. While this may reflect learning or "peso problems", it remains a provocative problem for the REH.[19] In particular, it suggests that *ex post* data do not provide a good econometric proxy for the expectations of market participants.

Recall our regression results

$$\Delta s = \hat{\beta}_0 + \hat{\beta}_1 \text{fd} + \text{residual}$$

[17]Nijman, Palm, and Wolff (1993) show that a certain class of models predicts that the risk premium follows a first order autoregressive process. Cavaglia et al. (1994) offer some support for this prediction.

[18]Froot and Thaler (1990, p.185): "From late 1980 until early 1985, dollar interest rates were above foreign rates so the dollar sold at a forward discount, implying that the value of the dollar should fall. However, the dollar appreciated (more or less steadily) at a rate of about 13 percent per year. Under the risk-premium scenario, these facts would suggest that investors' (rational) expectation of dollar appreciation was strongly positive (perhaps even the full 13 percent), but that the risk premium was also positive. Therefore, according to this view, dollar-denominated assets were perceived to be much riskier than assets denominated in other currencies, exactly the opposite of the "safe-haven" hypothesis which was frequently offered at that time as an explanation for the dollar's strength.

The subsequent rapid fall in the value of the dollar would conversely imply a reversal in the risk premium's sign, as investors in 1985 switched to thinking of the dollar as relatively safe. Something very dramatic must have happened to the underlying determinants of currency risk to yield such enormous swings in the dollar's value: during the appreciation investors must have been willing to give up around 16 percent per year (13 percent from dollar appreciations plus 3 percent from an interest differential in favor of the dollar) in order to hold the "safer" foreign currency, whereas during the later depreciation phase they must have been willing to forego about 6 percent in additional annual returns (8 percent average annual depreciations minus the 2 percent average interest differential) in order to hold dollars. These premia are very large. It is hard to see how one could rely on the risk-premium interpretation alone to explain the dollar of the 1980s.

[19]Lewis (1989) suggests slow learning about monetary policy can account for some of the US forward rate anomaly, but she notes that the errors seem to persist. Another problem for learning and peso problem explanations, as noted by Froot and Thaler (1990) is cross country consistency of the anomalous results.

where

$$\hat{\beta}_1 = 1 - b_{re} - b_{rp}$$

Here we define $b_{re} = -\mathbf{cov}(\text{fd}, \epsilon)/\mathbf{var}(\text{fd})$ and $b_{rp} = \mathbf{cov}(\text{fd}, \text{rp})/\mathbf{var}\text{fd}$.[20] Recall also that $b_{rp} = 0$ if the risk premium is constant (or otherwise uncorrelated with fd). Similarly, $b_{re} = 0$ if there are no systematic prediction errors in the model. With survey data, both b_{re} and b_{rp} can be estimated.

You can estimate b_{re} by regressing the forecast error on the forward discount. The rational expectations hypothesis implies that $b_{re} = 0$, since the forecast error should not be predictable based on current information. However survey data tend to reject this implication (Frankel and Froot, 1987; Cavaglia et al., 1994). In fact Froot and Frankel not only find support for systematic prediction errors but also find that these prediction errors are the primary source of bias in the forward discount: they could not reject $b_{rp} = 0$. These are the opposite conclusions of the earlier work that assumed rational expectations![21] More recently, Cavaglia et al. (1994) found a significant contribution of both irrationality and the risk premium to forward discount variability.[22]

Recall

$$\begin{aligned} b_{rp} &= \frac{\mathbf{cov}(\text{fd}, \text{rp})}{\mathbf{var}\text{fd}} \\ &= \frac{\mathbf{cov}(\Delta s^e + \text{rp}, \text{rp})}{\mathbf{var}(\Delta s^e + \text{rp})} \\ &= \frac{\mathbf{cov}(\Delta s^e, \text{rp}) + \mathbf{var}\text{rp}}{\mathbf{var}\Delta s^e + \mathbf{var}\text{rp} + 2\mathbf{cov}(\Delta s^e, \text{rp})} \end{aligned}$$

So $b_{rp} > 0.5$ iff $\mathbf{var}\text{rp} > \mathbf{var}\Delta s^e$. Cavaglia et al. (1994) use this direct test of the relative variance of the risk premium and expected depreciation and find at best modest support. Note that unlike the test offered by Fama (1984), this test does not invoke rational expectations.

Problems for Review

1. What do figures 10.3 and 10.3 tell us, and what do we learn from the difference between them?

2. Use your own words to define in economic terms the risk premium in the international assets markets (rp).

3. If the forward rate exceeds the expected future spot rate

[20]Froot and Frankel (1989) make use of $\mathbf{cov}(\text{fd}, \text{rp}) = \mathbf{var}(\text{rp}) + \mathbf{cov}(\text{rp}, \Delta s^e)$ in their definition of b_{rp}.

[21]Recall that under rational expectations $b_{re} = 0$. This implies $b_{rp} > 1$ since $\hat{\beta}_1 < 0$ and also $\mathbf{cov}(\text{fd}, \Delta s^e) < 0$ since $1 - b_{rp} = \mathbf{cov}(\text{fd}, \Delta s^e)/\mathbf{var}\text{fd}$.

[22]We should allow that concluding irrationality from the predictability of the forecast error is problematic. For example, a rational allowance for a small probability of a large exchange rate movement could manifest *ex post* as biased forecasts. (This is the "peso problem" of Krasker (1980).)

(a) domestic assets pay a risk premium.

(b) foreign assets pay a risk premium.

(c) covered interest parity is violated.

(d) domestic and foreign assets are perfect substitutes.

(e) none of the above.

4. Which are true statements about currency risk?

(a) A foreign investor will always consider the dollar risky if its exchange rate vis-a-vis the foreign currency is uncertain.

(b) The currency risk of an asset depends on the correlation of the real return on the asset with the return on the investor's portfolio.

(c) In a country with unstable monetary policy and a highly variable price level, the domestic assets may be riskier than foreign currency denominated assets.

(d) b. and c.

(e) all of the above

5. Check the second-order sufficient condition for the mean-variance optimization problem above. (Remember that the partial derivatives of U are still functions of α.)

Appendix

10.5 Quadratic Programming

Mean-variance optimization is an application of quadratic programming with equality constraints. In this section, we focus on a specialized problem: minimize $x'Ax$ subject to $Cx = b$ where A is symmetric and positive definite. We form the Lagrangian

$$\mathcal{L}(x, \lambda) = x'Ax - 2\lambda'(Cx - b) \tag{10.62}$$

Differentiating yields

$$\partial\mathcal{L}/\partial x = (A + A')x - 2C'\lambda \tag{10.63}$$
$$\partial\mathcal{L}/\partial\lambda = -2(C'x - b) \tag{10.64}$$

When A is symmetric (as in mean-variance optimization), our first-order conditions become

$$Ax - C'\lambda = 0 \tag{10.65}$$
$$Cx = b \tag{10.66}$$

or

$$\begin{bmatrix} A & C' \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ -\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} \tag{10.67}$$

Recalling that in this application A is symmetric positive definite, assume the leftmost matrix is invertible. (I.e., assume that C has full rank: $C_{M \times K}$ has rank M.) Then we can produce an inverse and solve

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} A^{-1}[I - C'(CA^{-1}C')^{-1}CA^{-1}] & A^{-1}C'(CA^{-1}C')^{-1} \\ (CA^{-1}C')^{-1}CA^{-1} & -(CA^{-1}C')^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ b \end{bmatrix} \tag{10.68}$$

This gives us our solution for x as

$$x = A^{-1}C'(CA^{-1}C')^{-1}b \tag{10.69}$$

Consider the resulting value of our objective function. (Recall that A is symmetric, and thus so is A^{-1}.)

$$\begin{aligned} x'Ax &= b'(CA^{-1}C')^{-1}CA^{-1}AA^{-1}C'(CA^{-1}C')^{-1}b \\ &= b'(CA^{-1}C')^{-1}b \end{aligned} \tag{10.70}$$

Consider any other value, $x + \mathrm{d}x$, that also satisfies the constraint. This means

$$C(x + \mathrm{d}x) = b \tag{10.71}$$

$$Cx + C\,\mathrm{d}x = b \tag{10.72}$$

$$C\,\mathrm{d}x = 0 \tag{10.73}$$

Note that

$$(x + \mathrm{d}x)'A(x + \mathrm{d}x) = x'Ax + x'A\,\mathrm{d}x + \mathrm{d}x'Ax + \mathrm{d}x'A\,\mathrm{d}x \tag{10.74}$$

By the symmetry of A

$$\begin{aligned} x'A\,\mathrm{d}x + \mathrm{d}x'Ax &= 2x'A\,\mathrm{d}x \\ &= b'(CA^{-1}C')^{-1}CA^{-1}A\,\mathrm{d}x \\ &= b'(CA^{-1}C')^{-1}C\,\mathrm{d}x \\ &= 0 \end{aligned} \tag{10.75}$$

So

$$(x + \mathrm{d}x)'A(x + \mathrm{d}x) = x'Ax + \mathrm{d}x'A\,\mathrm{d}x > x'Ax \tag{10.76}$$

since A is positive definite. Thus we have found a minimizer.

10.5.1 Mean-Variance Optimization

An investor can hold any linear combination of n assets, with random returns $\boldsymbol{R}$. Here $R \sim (\mu, \Sigma)$ where $\mu = \mathcal{E}R$ and $\Sigma = \mathbf{cov}R = \mathcal{E}(R-\mu)(R-\mu)'$. Note that $\mathbf{cov}R$ is nonnegative definite; if it is also positive definite then the investor cannot eliminate all risk. The portfolio weights are ω, and the weights must sum to 1. The portfolio return is ωR. The mean and variance of the portfolio return are therefore and the variance is

$$\mathcal{E}(\omega' R) = \omega'\mu \tag{10.77}$$

$$\begin{aligned} \mathbf{var}(\omega' R) &= \mathcal{E}[\omega'(R-\mu)(R-\mu)'\omega] \\ &= \omega'\Sigma\omega \end{aligned} \tag{10.78}$$

Assume Σ is positive definite. Then from our work in section 10.5, we know the minimum variance portfolio is

$$\omega_0 = \Sigma^{-1}\mathbf{1}\left(\mathbf{1}'\Sigma^{-1}\mathbf{1}\right)^{-1} \tag{10.79}$$

To explore the efficient frontier we add a second constraint:

$$\mathcal{E}\omega R = \mu_0 \tag{10.80}$$

Let us stack the constraints as

$$\begin{bmatrix} \mathbf{1}' \\ \mu' \end{bmatrix} \omega = \begin{bmatrix} 1 \\ \mu_0 \end{bmatrix} \tag{10.81}$$

or $W\omega = [1\mu_0]'$. Assuming not all mean returns are equal, we find the minimum variance portfolio to be

$$\omega(\mu_0) = \Sigma^{-1}W(W\Sigma^{-1}W)^{-1}\begin{bmatrix} 1 \\ \mu_0 \end{bmatrix} \tag{10.82}$$

Evidently this satisfies our two linear constraints. The portfolio variance is

$$\begin{aligned} \sigma(\mu_0) &= \omega'\Sigma^{-1}\omega \\ &= \begin{bmatrix} 1 & \mu \end{bmatrix}\left(W'\Sigma^{-1}W\right)^{-1}\begin{bmatrix} 1 \\ \mu \end{bmatrix} \end{aligned} \tag{10.83}$$

To show this is the minimum variance portfolio for this level of return, we again consider $d\omega$ such the $Wd\omega = \mathbf{0}$. (The proof is almost identical.) Note that we end up with a variance that is quadratic in μ. Naturally the unique minimum is at the mean return of the minimum variance portfolio.

The efficient frontier minimizes portfolio variance subject to an average return constraint.

10.5.2 Characterizing the Data

Gather together T observations on the return on K assets into a $T \times K$ matrix $\tilde{X}$. Store the mean returns for the assets in $\mu = \text{rowsum}(\tilde{X})/T$. Create the matrix X of deviations from the mean for each variable. (So $X_{tk} = \tilde{X}_{tk} - \mu_k$.) Store the covariance matrix for the assets in

$$[\sigma_{rk}] = \Sigma = \frac{1}{T} X^\top X \tag{10.84}$$

(This is also called the variance-covariance matrix.) It is common to standardize the covariance matrix by creating the correlation matrix

$$\rho_{rk} = \sigma_{rk} / \sqrt{\sigma_{rr}\sigma_{kk}} \tag{10.85}$$

Naturally the correlation matrix has ones along its diagonal. Additionally every element is in the interval $[-1, 1]$.

The following is based on a example in Halliwell (1995). We begin with the means and covariances of three asset classes:

$$\mu = \begin{bmatrix} 0.129 \\ 0.053 \\ 0.043 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 0.042025 & 0.00466375 & -0.0002296 \\ 0.00466375 & 0.004225 & 0.0002912 \\ -0.0002296 & 0.0002912 & 0.000784 \end{bmatrix} \tag{10.86}$$

Based on this data, we compute the minimum variance portfolio weights: $[0.011, 0.098, 0.891]$, which produces and expected return of 0.045 (i.e., 4.5% annually). The variance of this return is 0.0007, implying a standard deviation of about 4.5%. The resulting efficient frontier is shown in Figure 10.5.

- Branson, W. H., and D. Henderson, 1985 "The Specification and the Influence of Asset Markets", in R.W. Jones and P.B. Kennn (eds), *Handbook of Internationational Economics* vol. 2 (Amsterdam: North Holland Publishing Co.)

- Cumby, Robert, and M. Obstfeld, 1984, "International Interest Rate and Price Level Changes under Flexible Exchange Rates: A Review of the Recent Evidence", in J. F. O. Bilson and R. C. Marston (eds) *Exchange Rate Theory and Practice* (Chicago: University of Chicago Press), 121-51.

- Frankel, Jeffrey, and Kenneth Froot, "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations," AER 77(1), March 1987, 133–53.

- Fama, Eugene, "Forward and Spot Exchange Rates," JME 14, 1984, 319-33.

- Frankel, Jeffrey, 1980, SEJ

- Isard, Peter, 1988

- Lucas, Robert, 1982, "Interest Rates and Currency Prices in a Two-Country World", JME 10, 335-59.

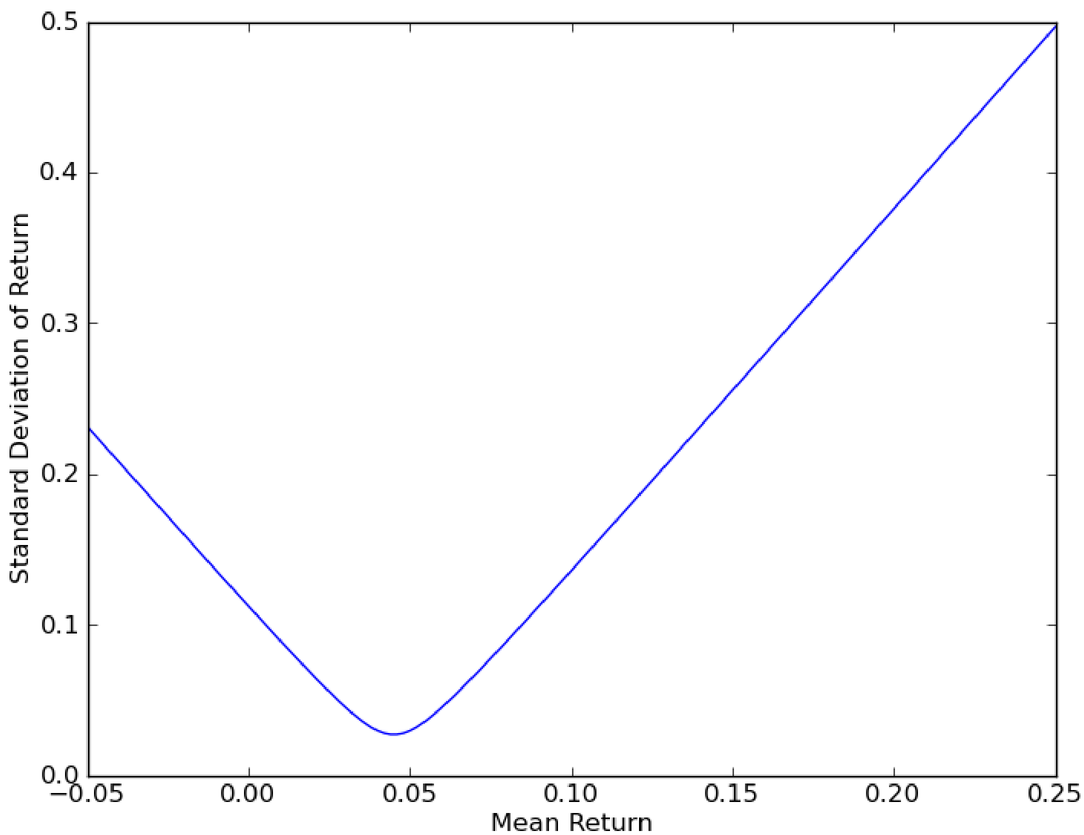


Figure 10.5: Efficient Frontier

On the failure of rational expectations:

- Bilson, John, "The Speculative Efficiency Hypothesis," JBus 54 (1981): 435-51. (Excitable speculators.)
- Cavaglia, Stefano, Willem Werschor, and Christian Wolff, 1994, "On the Biasedness of Forward Foreign Exchange Rates: Irrationality or Risk Premia?", JBus 67(3), 321-343.
- Dominguez, Kathryn, "Are Foreign Exchange Forecasts Rational? New Evidence from Survey Data," EL 21 (1986): 277-282.
- Frankel, Jeffrey, and K. Froot, 1987, "Using Survey Data to Test Some Standard Propositions Regarding Exchange Rate Expectations," AER 77, 133-53.
- MacDonald, R. and T. Torrance, 1989, "Some Survey Based Tests of Uncovered Interest Parity", in R. MacDonald and M. P. Taylor (eds), *Exchange Rates and Open Macroeconomics* (Cambridge, MA: Basil Blackwell).
- Takagi, S., 1991, "Exchange Rate Expectations: A Survey of Survey Studies", IMF , March.

Optimal Diversification

- Lessard, Donald R., 1983, "Principles of International Portfolio Selection", in Abraham George and Ian H. Giddy (eds) *International Finance Handbook* (New York: John Wiley & Sons).
- Dornbusch
- Niehans 7, 8, 10
- Aliber, R. Z., "The Interest Rate Parity Theorem: A Reinterpretation", JPE Nov./Dec., 1977. (Examines the role of political risk.)
- Bilson, J., "The 'Speculative Efficiency' Hypothesis", JBus 1981, 54, 435–51. (evidence of bias in forward rate).
- Cumby, R. E., and M. Obstfeld, "International Interest Rate and Price Level Linkages Under Flexible Exchange Rates: A Review of Recent Evidence", in Bilson, J. F. O. and R. Marston (eds.), *Exchange Rates: Theory and Practice*. Chicago: University of Chicago Press, 1984. (presents evidence that real exchange rate is not a martingale, finds bias in forward rate).
- Cumby, R., "Is It Risk? Explaining Deviations From Uncovered Interest Parity", JME 22(2), Sep. 1988. (Empirical evidence that the consumption based asset pricing model is not an adequate model of the returns to forward speculation.)
- Dooley, Michael, and Peter Isard, "Capital Controls, Political Risk, and Deviations from Interest Rate Parity", JPE 88(2), 1980, 370–354.

- *Fama, E. F., 1984, “Forward and Spot Exchange Rates”, JME 14, 319–38. (finds time varying risk premia cause most forward rate variation).

- Frankel, J., “In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization”, JIMF December, 1982.

- Hansen, L. P., and R.J. Hodrick, “Forward Exchange Rates as Optimal Predictions of Future Spot Rates”, JPE 88, Oct. 1980, 829–53.

- Hodrick, R. J. and S. Srivastava, “The Covariation of Risk Premiums and Expected Future Spot Exchange Rates”, JIMF 5, 1986, 5–21. (Provides additional empirical evidence for Fama’s results, with theoretical backing from a simple version of Lucas’ model.)

- Hodrick, R. J., “The Covariation of Risk Premiums and Expected Future Spot Exchange Rates”, JIMF, 1988. (A confirmation of Fama’s results).

- Hodrick, Robert J., *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets* (Chur, Switzerland: Harwood Academic Publishers, 1987).

- Korajczyk, R. A., 1985, “The Pricing of Forward Contracts for Foreign Exchange”, JPE 93 (April), 346–68. (relates risk premia and ex ante real interest differentials under re if real exchange rate is a martingale).

- Levich, R. M. “On the Efficiency of Markets for Foreign Exchange”, in R. Dornbusch and J. Frenkel (eds.), *International Economic Policy: Theory and Evidence*, Johns Hopkins, Baltimore, Md. (1979) (early evidence of bias in forward rate.)

- Levich, R. M., “Empirical Studies of Exchange Rates: Price Behavior, Rate Determination, and Market Efficiency“ in *Handbook of International Economics* vol. 2, ed. P. Kenen and R. McKinnon (Amsterdam: North–Holland, 1984)

- Mussa, M., “Empirical Regularities in the Behavior of Exchange Rates”, in CRCS 11, 1979.

- Roll, R. “A Critique of the Asset Pricing Theory’s Tests”, JFin, March 1977, 129–78.

- Meese, Richard A. and Kenneth Rogoff. “Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?”, JIE, 1985. (They don’t.)

- Schinasi, G., and P.A.V.B. Swamy, “The Out of Sample Performance of Exchange Rate Models when Coefficients are Allowed to Change”, I.F. Discussion paper #301, January 1987. (Improves on Meese and Rogoff.)

- Stulz, R., “The Forward Exchange Rate and Macroeconomics”, JIE 12, 1982, 285–299.

The Relationship between Spot and Forward Rates: Is There a Risk Premium?

Tests Based on Rational Expectations

We will discuss two approaches to testing the assumption of risk neutrality in efficient markets. We call a market efficient to the extent that (i) current market prices reflect market participants' beliefs about future prices *and* (ii) these beliefs incorporate relevant information.[23] The first condition requires the absence of significant market imperfections or high transactions costs. The second condition, that beliefs incorporate relevant information, is known as the rational expectations hypothesis.

In the absence of market imperfections and transactions costs, risk neutrality will ensure that the forward rate for the purchase of foreign exchange next period f_t will equal the expected value of next period's spot rate s^e_{t+1}. This is because risk neutrality implies that investors care only about their average rate of return, and market equilibrium must then reflect this in an equal average rate of return for all investments.

$$f_t = s^e_{t+1} \tag{10.87}$$

This is the assumption of a zero risk premium. We can also write this as

$$\mathrm{fd}_t = \Delta s^e_{t+1} \tag{10.88}$$

Let the information set used by market participants at time t be $\mathcal{I}_t$. We will represent the rational expectations hypothesis by the following assumption:

$$s^e_{t+1} = \mathcal{E}\{s_{t+1} \mid \mathcal{I}_t\} \tag{10.89}$$

where $\mathcal{I}_t$ is the publicly available information at time t and $\mathcal{E}$ is the mathematical expectation operator. This is the rational expectations assumption. Let us define the forecast error by

$$\epsilon_{t+1} = s_{t+1} - s^e_{t+1} \tag{10.90}$$

and note that the rational expectations assumption implies that the expected forecast error is zero. We can also write

$$\epsilon_{t+1} = \Delta s_{t+1} - \Delta s^e_{t+1} \tag{10.91}$$

which implies that on average depreciation is what it is expected to be.

[23]In the literature, the "efficient markets hypothesis" is sometimes defined to include a third element: a zero risk premium. Such a definition is not concerned with market efficiency per se. We can distinguish weak, semi-strong, and strong efficiency as $\mathcal{I}_t$ comprises past prices, all publicly known information, or all relevant information (Fama). Since the foreign exchange market is huge, with trade volumes around forty times world trade volumes in goods and services and twenty time US GNP, we expect that it is highly liquid and at least semi-strong efficient.

Implication 1: Unbiased Prediction

We have seen that in the absence of a risk premium, the forward rate is an unbiased predictor of the future spot rate. Equivalently, the forward discount on domestic money is an unbiased predictor of the rate of depreciation of the spot rate. By "unbiased" we simply mean that the forecast error is zero on average. The forecast error is ϵ_{t+1}, which is zero on average, in the following equations.

$$s_{t+1} = f_t + \epsilon_{t+1} \tag{10.92}$$
$$\Delta s_{t+1} = \text{fd}_t + \epsilon_{t+1} \tag{10.93}$$

This unbiasedness lends itself readily to empirical tests. Econometric tests are often based on the following regression equations.

$$s_{t+1} = \alpha + \beta f_t + \epsilon_{t+1} \tag{10.94}$$
$$\Delta s_{t+1} = \alpha + \beta \text{fd}_t + \epsilon_{t+1} \tag{10.95}$$

Equations (10.94) and (10.95) allow for a more general empirical relationship between the spot and forward rate than equations (10.92) and (10.93). Comparing the two equations, we see that in the absence of a risk premium, we should find that $\alpha = 0$ and $\beta = 1$. Most attention has been paid to β: estimates are generally less than one, often as low as 0.5, and it is not uncommon to find estimates near or even below zero.[24] Estimates near zero imply that the forward rate contains no information about the future spot rate: rather than base a prediction on the forward rate, it is better to guess the exchange rate will not change. That is, the current spot rate is then the best guess for the future spot rate; the exchange rate follows a martingale.

So given rational expectations, we reject the absence of a risk premium. Of course it may be the rationality hypothesis that is the basis of this rejection. However until recently most economists have been willing to maintain the assumption of rationality and view this as evidence of a risk premium.

Consider the implications of finding $\beta = 0.5$ in (10.93). Then if the U.S. dollar sells at a forward premium of 2%/year against the Canadian dollar, our best guess is that the dollar will appreicate against the Canadian dollar by 1%/year. On the other hand if $\beta = 0$, then the forward premium would contain no information about expected future movement of the exchange rate.

Implication 2: Uncorrelated Forecast Errors

If $\mathcal{E}\{x_{t+1} \mid \mathcal{I}_t\} = x_t$ for $t \geq 0$, then we say x_t is a martingale with respect to $\mathcal{I}_t$. Define $\epsilon_{t+1} = x_{t+1} - x_t$.[25] Then ϵ_t has a zero *unconditional* mean and is serially uncorrelated:

---

[24]For example, Froot and Thaler (1990) report an average coefficient across several dozen studies of -0.88. Note however that survey evidence generally suggests that currencies at a forward discount are also expected to depreciate (Frankel and Froot 1987; Cavaglia et al. 1994).

[25]So we can write $x_{t+n} = x_t + \sum_{j=1}^{n} \epsilon_{t+j}$, and the n-period forecast error is $x_{t+n} - \mathcal{E}\{x_{t+n} \mid \mathcal{I}_t\} = \epsilon_{t+j}$.

($\mathcal{E}\epsilon_t = 0$) and ($\mathcal{E}\epsilon_{t+n}\epsilon_t = 0$).[26]

Suppose your period t forecast of the period τ spot rate is $\mathcal{E}\{s_\tau \mid \mathfrak{I}_t\}$. If information sets only increase, i.e., $\mathfrak{I}_t \subset \mathfrak{I}_{t+1} \subset \mathfrak{I}_{t+2} \ldots$, then your forecasts of the period τ spot rate will follow a martingale. ($\mathcal{E}\{s_\tau \mid \mathfrak{I}_t\}$ will be a martingale with respect to $\mathfrak{I}_t$ since your best guess of your next period's best guess of a future spot rate is just your best guess of that future spot rate.) Let

$$\mathcal{E}\{s_\tau \mid \mathfrak{I}_\tau\} = s_\tau \tag{10.96}$$

so that the current spot rate is in the current informaton set. Then it is easy to see that non-overlapping spot rate forecast errors should be serially uncorrelated. For example, if the time t one period ahead forward rate is equal to the market participants' expectation of next period's spot rate (so that $f_t = \mathcal{E}\{s_{t+1} \mid \mathfrak{I}_t\}$), we should find that $\epsilon_{t+2} = s_{t+2} - f_{t+1}$ is uncorrelated with $\epsilon_{t+1} = s_{t+1} - f_t$.

In the early 1980s, economists began to document serial correlation in $s_{t+1} - f_t$ (Frankel, 1980; Hansen and Hodrick, 1980). The bias in the forward rate prediction of the future spot rate attracted considerable attention for two major reasons. First, it was evidence against the *joint* hypothesis of no risk premium ($f_t = s^e_{t+1}$ due to perfect substitutability of domestic and foreign assets) and rational expectations ($s^e_{t+1} = \mathcal{E}\{s_{t+1} \mid \mathfrak{I}_t\}$). This joint hypothesis had been used in much of the research on the popular monetary approach to exchange rate determination. The implication was that either market participants were making persistent forecast errors of there was a risk premium in the market for foreign exchange. Suspicion fell on the former, since economists are naturally reluctant to give up the rationality hypothesis. The serial correlation is taken to be evidence of imperfect substitutability of domestic and foreign assets. Second, imperfect substitutability is of policy interest since it suggests that sterilized intervention can be effective and that even a small open economy under flexible exchange rates can effectively use fiscal policy. Empirically, the risk premium seems to be about 6-8% annually.[27]

Technical Analysis of Exchange Rates

Most academic work on exchange rate forecasting has focussed on structural models rather than technical analysis, and this book places a corresponding emphasis on structural modeling. In practice, however, forecasters use many approaches to predicting exchange rates.

Many forecasters focus on trends in the recent behavior of exchange rates, basically ignoring the kinds of models we emphasize in this book. This approach is often called "technical analysis", although for "chartists" the extrapolation techniques may literally involve no more than hand drawn charts. Moving average models often recommend

[26] A widely used special case is the random walk. We say that x_t follows a random walk with respect to $\mathfrak{I}_t$ if $x_{t+1} - x_t = \epsilon_{t+1}$ and $\epsilon_t \sim$i.i.d.$(0, \sigma^2)$. If $x_{t+1} - x_t = c + \epsilon_{t+1}$ where $c \neq 0$, we say that x_t follows a random walk with drift. The variance of the forecast error is then $n\sigma^2$ (which obviously increases without bound with the forecast horizon). Note that while the forecast errors ϵ_t must be uncorrelated for a martingale, they are independent for a random walk. If $\mathcal{E}\{x_{t+1} - x_t \mid \mathfrak{I}_t\} > 0 (< 0)$ we say x_t is a sub (super) martingale with respect to $\mathfrak{I}_t$.

[27] This conflicts with the much smaller predictions of many models: see Engel (1990).

buying a currency when its short run moving average rises above its long run moving average. Momentum models recommend buying a currency on sustained increases. This tendency to extrapolate recent trends can add instability to the foreign exchange markets.

Exchange rate predictions vary widely. For example, Frankel and Froot (1990) show that at a 6-month horizon, forecasts vary over a range averaging more than 15%. Whether traders rely on technical or fundamental analysis, they lose money on many trades.[28] Yet to profit, traders need only a slight edge over chance in predicting spot rate movements.

However Schulmeister and Goldberg (1989) and Goodman (1979) offer some evidence that moving average and momentum models can in fact be profitable. This may explain the shift in the early 1980s among foreign exchange forecasting firms from fundamental to technical analysis. However, fundamentals are generally given a role for forecasts over longer horizons (say, 6 months or more).

- Driskill, Robert A., Nelson C. Mark, and Steven M. Sheffrin, 1992, "Some Evidence in Favor of a Monetary Rational Expectations Exchange Rate Model with Imperfect Capital Substitutability", *International Economic Review* 33(1), Feb., 223–37.
- Frankel, Jeffrey, and Kenneth Froot, 1990, "Chartists, Fundamentalists, and Trading in the Foreign Exchange Market," AER 80(2), May, 181-5.

  Woo, W. T., 1985, "The Monetary Approach to Exchange Rate Determination
- under Rational Expectations," *Journal of International Economics* 18, 1–16.
- Somanath, V. S., 1986, "Efficient Exchange Rate Forecasts: Lagged Models Better than the Random Walk," JIMF 5, 195-220.
- McDonald, Ronald, and Mark Taylor, "The Monetary Model of the Exchange Rate: Long-Run Relationships, Short-Run Dynamics, and How to Beat a Random Walk," JIMF 13(3), June 1994, 276–290.

On chartists and noise trading:

- Goodman, S.H., "Foreign Exchange Forecasting Techniques: Implications for Business and Policy," JFin 34 (May 1979): 415-27.
- Schulmeister, Stephan and Michael Goldberg, "Noise Trading and the Efficiency of Financial Markets," in G. Luciani (ed) *Structural Change in the American Financial System* (Rome: Centro di Studi Americani, 1989) pp. 117-164.
- Frankel, Jeffrey and Kenneth Froot, "Chartists, Fundamentalists and the Demand for Dollars," in A. Courakis and M. Taylor (eds) *Policy Issues for Interdependent Economies* (London: Macmillan, 1990).

[28] Even when most traders lose money we may observe only profitable traders in the market if losers leave the market.

- Goodhard, Charles, “The Foreign Exchange market: a Random Walk with a Dragging Anchor,” Economica 55 (November 1988, 437-460.

- Taylor, Mark, and Helen Allen, “The Use of Technical Analysis in the Foreign Exchange Market,” JIMF 11(3), June 1992, 304–14.

- Schulmeister, Stephan, and Michael Goldberg, “Noise Trading and the Efficiency of Financial Markets,” in G. Luciani (ed), *Structural Change in the American Financial System* (Rome: Centro di Studi Americani 1989), pp.117–164.

- Goodman, S. H., “Foreign Exchange Forecasting Techniques: Implications for Business and Policy,” JFin 34, May 1979, 415–427.

Efficient Markets

- Bilson, John, “The Speculative Efficiency Hypothesis,” JBus 54, 1981, 435–451. (Suggests overexcitability.)

- Engel, Charles, “Why is the Forward Exchange Rate Forecast Biased? A Survey of Recent Evidence,” *Journal of Empirical Finance*, forthcoming 1996?

- Froot, Kenneth, and Jeffrey Frankel, “Forward Discount Bias: Is it an Exchange Risk Premium?” QJE , February, 1989, 139–161.

- Hansen, Lars Peter, and Robert J. Hodrick, 1980, “Forward Exchange Rates as Optimal Predictors of Future spot Rates: An Econometric Analysis”, JPE 88(5), 829-853.

- Hodrick, Robert, *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets* (Chur, Switzerland: Harwood Academic Publishers, 1988).

- Hodrick, Robert, and Sanjay Srivastava, 1984, “An Investigation of Risk and Return in forward Foreign Exchange,” JIMF 3 (1984): 5-30.

- Hsieh, David, “Tests of Rational Expectations and No Risk Premium in Forward Exchange Markets,” JIE 17 (1984): 173-184.

- Isard, 1988, “The Empirical Modeling of Exchange Rates,” in *Empirical Macro for Interdependent Economies*, Brookings, R. Bryant et al., eds.

  Ito, Takatoshi, “Foreign Exchange Rate Expectations: Micro Survey Data,” AER 80(3), June 1990, 434-449.

- Korajczyk, R. A., 1985, “The Pricing of Forward Contracts for *Foreign* Exchange”, JPE 93, 346-68.

- Kouri, P., 1978

We will describe the excess demand of hedgers in the spot market by

$$T_t = \beta(S_t - S^*) \tag{10.97}$$

Comment: if the foreign exchange market only contains hedgers, then we will have

$$F = S(1 + i_{\text{us}})/(1 + i_{uk}) \tag{10.98}$$

and the forward exchange market will be redundant.

Bibliography

Abauf, Niso and Philippe Jorion (1990, March). "Purchasing Power Parity in the Long Run." *Journal of Finance* 45(1), 157–174.

Adler, M. and B. Lehmann (1983, December). "Deviations from Purchasing Power Parity in the Long-run." *Journal of Finance* 38(5), 1471–87.

Alexander, Sidney S. (1952). "Effects of a Devaluation on the Trade Balance." *International Monetary Fund Staff Papers* 2, 263–78.

Argy, Victor (1994). *International Macroeconomics*. New York: Routledge.

Artus, J.R. (1975, November). "The 1967 Devaluation of the Pound Sterling." *International Monetary Fund Staff Papers* 22, 595–640.

Asea, Patrick K. and Max Corden (1994, October). "The Balassa-Samuelson Model: An Overview." *Review of International Economics* 2(3), 191–200.

Backus, D. (1984, November). "Empirical Models of the Exchange Rate: Separating the Wheat from the Chaff." *Canadian Journal of Economics* 17(4), 824–46.

Baillie, R.T. and D.D. Selover (1987). "Cointegration and Models of Exchange Rate Determination." *International Journal of Forecasting* 3(1), 43–52.

Beaugrand, Philippe (1997). "Zaire's Hyperinflation, 1990–96." IMF Working Paper WP/97/50, International Monetary Fund, Washington, DC.

Bilson, J.F.O. and R.C. Marston, eds. (1984). *Exchange Rate Theory and Practice*. Chicago: University of Chicago Press.

Bilson, John F. O. (1978a, March). "The Monetary Approach to The Exchange Rate—Some Empirical Evidence." *International Monetary Fund Staff Papers* 25, 48–75.

Bilson, John F. O. (1978b). "Rational Expectations and the Exchange Rate." In Jacob A. Frenkel and Harry G. Johnson, eds., *The Economics of Exchange Rates: Selected Studies*, Chapter 5, pp. 75–96. Reading, MA: Addison-Wesley.

Bilson, John F. O. (1981). "The Speculative Efficiency Hypothesis." *Journal of Business* 54, 435–51.

BIS (2000). *70th Annual Report*. Basle: Bank for International Settlements.

BIS (2002, March). *Triennial Central Bank Survey: Foreign Exchange and Derivatives Market Activity in 2001*. Basle: Bank for International Settlements.

Blanchard, Olivier, Francesco Giavazzi, and Filipa Sa (2005, February). "International Investors, the U.S. Current Account and the Dollar." *Brookings Papers on Economic Activity* 2005(1), 1–49.

Boughton, James M. (1987). "Tests of the Performance of Reduced-Form Exchange Rate Models." *Journal of International Economics* 23, 41–56.

Boughton, James M. (1988). "The Monetary Approach to Exchange Rates: What Now Remains?" *Essays in International Finance* 171.

Boyer, Russell S. and F. Charles Adams (1988, November). "Forward Premia and Risk Premia in a Simple Model of Exchange Rate Determination." *Journal of Money, Credit, and Banking* 20(4), 633–44.

Brasse, V. (1983). "The Inaccuracy of Exchange Rate Forecasting Devices in the UK." *CUBS Economic Review* 1, 1–15.

Burstein, Ariel and Gita Gopinath (2014). "International Prices and Exchange Rates." In Elhanan Helpman, Kenneth Rogoff, and Gita Gopinath, eds., *Handbook of International Economics*, Volume 4 of *Handbooks in Economics*, Chapter 7, pp. 391–452. Elsevier.

Cagan, Phillip (1956). "The Monetary Dynamics of Hyperinflation." In Milton Friedman, ed., *Studies in the Quantity Theory of Money*, Chapter 2, pp. 25–117. Chicago: University of Chicago Press.

Cassel, Gustav (1918). "Abnormal Deviations in International Exchanges." *Economic Journal* 28, 413–5.

Cavaglia, Stefano, Willem Werschor, and Christian Wolff (1994). "On the Biasedness of Forward Foreign Exchange Rates: Irrationality or Risk Premia?" *Journal of Business* 67(3), 321–343.

Cecchetti, Stephen G., Nelson C. Mark, and Robert J. Sonora (2002, November). "Price Index Convergence Among United States Cities." *International Economic Review* 43(4), 1081–99.

Cerra, V. and S. Saxena (2010, July). "The Monetary Model Strikes Back: Evidence from the World." *Journal of International Economics* 81(2), 184–196.

Choi, Chi-Young, Nelson C. Mark, and Donggyu Sul (2006, June). "Unbiased Estimation of the Half-Life to PPP Convergence in Panel Data." *Journal of Money, Credit and Banking* 38(4), pp. 921–938.

Council of Economic Advisors (1999, February). *Economic Report of the President*. Washington, DC: United States Government Printing Office.

Cuddington, John (1986, December). "Capital Flight: Estimates, Issues and Explanations." *Princeton Studies in International Finance* 58.

Cumby, Robert and Maurice Obstfeld (1984). "International Interest Rate and Price Level Linkages under Flexible Exchange Rates: A Review of the Recent Evidence." See Bilson and Marston (1984), pp. 121–51.

De Long, J. B., A. Shleifer, L. Summers, and R. Waldmann (1987, October). "The Economic Consequences of Noise Traders." NBER Working Paper 2395, National Bureau of Economic Research.

Deardorff, A. V. (1979, April). "One-way Arbitrage and Its Implications for the Foreign Exchange Markets." *Journal of Political Economy* 87(2), 351–64.

Dornbusch, Rudiger (1976, December). "Expectations and Exchange Rate Dynamics." *Journal of Political Economy* 84(6), 1161–76.

Dornbusch, Rudiger and Stanley Fischer (1980, December). "Exchange Rates and the Current Account." *American Economic Review* 70(5), 960–971.

Dornbusch, Rudiger and Jeffrey A. Frankel (1988). "The Flexible Exchange Rate System: Experience and Alternatives." In Silvio Borner, ed., *International Finance and Trade*. London: International Economics Association, in association with Macmillan Press.

Driskill, Robert A., Nelson C. Mark, and Steven M. Sheffrin (1992, February). "Some Evidence in Favor of a Monetary Rational Expectations Exchange Rate Model with Imperfect Capital Substitutability." *International Economic Review* 33(1), 223–37.

Driskill, Robert A. and Steven S. Sheffrin (1981, December). "On the Mark: Comment." *American Economic Review* 71(5), 1068–74.

Duarte, Margarida (2003, Summer). "The Euro and Inflation Divergence in Europe." Federal Reserve Bank of Richmond *Economic Quarterly* 89(3), 53–70.

Edison, Hali J. (1987, August). "Purchasing Power Parity in the Long Run: A Test of the Dollar/Pound Exchange Rate (1890–1978)." *Journal of Money, Credit, and Banking* 19(3), 376–87.

Edwards, Sebastian (1989). "The International Monetary Fund and the Developing Countries: A Critical Evaluation." *Carnegie-Rochester Conference Series on Public Policy* 31, 7–68.

Edwards, Sebastian (1998). "Capital Flows, Real Exchange Rates, and Capital Controls: Some Latin American Experiences." NBER Working Paper 6800, National Bureau of Economic Research.

Eichenbaum, Martin and Charles Evans (1995, November). "Some Empirical Evidence of the Effects of Shocks to Monetary Policy on Exchange Rates." *Quarterly Journal of Economics* 110(4), 975–1009.

Einzig, Paul (1970). *The History of Foreign Exchange*. London: Macmillan.

Engel, Charles M. and John H. Rogers (2001). "Deviations fromm Purchasing Power Parity: Causes and Welfare Costs." *Journal of International Economics* 55, 29–57.

Fama, Eugene F. (1970). "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance* 25, 383–417.

Fama, Eugene F. (1984, November). "Forward and Spot Exchange Rates." *Journal of Monetary Economics* 14(3), 319–38.

Feenstra, Robert C. and Jon D. Kendall (1994, August). "Pass-Through of Exchange Rates and Purchasing Power." NBER Working Paper 4842, National Bureau of Economic Research.

Feldstein, Martin and Charles Horioka (1980, June). "Domestic Saving and International Capital Flows." *Economic Journal* 90(358), 314–29.

Felmingham, B.S. (1988, January). "Where is the Australian J-Curve?" *Bulletin of Economic Research* 40, 43–65.

Flood, Robert P. (1981, Autumn). "Explanations of Exchange-Rate Volatility and Other Empirical Regularities in Some Popular Models of the Foreign Exchange Market." *Carnegie-Rochester Conference Series on Public Policy* 15, 219–249. Available as NBER Working Paper 625, February 1981.

Flood, Robert P. and Andrew K. Rose (1999, November). "Understanding Exchange Rate Volatility without the Contrivance of Macroeconomics." *Economic Journal* 109(459), F660–72.

Florentis, G., D.V. Gordon, and P. Huber (1994, Autumn). "Monetary Models of the Canadian-U.S. Exchange Rate: A Reexamination of the Empirical Evidence, 1971–1986." *Quarterly Journal of Business and Economics* 33(4), 27–43.

Frankel, Jeffrey (1990). "Zen and the Art of Modern Macroeconomics: The Search for Perfect Nothingness." In W. Haraf and T. Willett, eds., *Monetary Policy for a Volativle Global Economy*. Washington, DC: American Enterprise Institute. Reprinted in (Frankel, 1993, ch.7).

Frankel, Jeffrey and Kenneth Froot (1987, March). "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations." *American Economic Review* 77(1), 133–53.

Frankel, Jeffrey A. (1979, September). "On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Rate Differentials." *American Economic Review* 69(4), 610–22.

Frankel, Jeffery A. (1986a). "The Implications of Mean-Variance Optimization for Four Questions in International Macroeconomics." *Journal of International Money and Finance* 5(1, Supplement), 533–575.

Frankel, Jeffery A. (1986b). "International Capital Mobility and Crowding Out in the U.S. Economy: Imperfect Integration of Financial Markets or Goods Markets?" In R. Hafer, ed., *How Open Is The U.S. Economy?*, Chapter 2, pp. 33–67. Lexington, MA: Lexington Books.

Frankel, Jeffery A. (1991). "Quantifying International Capital Mobility in the 1980s." In D. Bernheim and J. Shoven, eds., *National Saving and Economic Performance*, pp. 227–60. Chicago: University of Chicago Press. As reprinted in **?**.

Frankel, Jeffery A. (1993). *On Exchange Rates*. Cambridge, Massachusetts: The MIT Press.

Frankel, Jeffery A. and Andrew Rose (1996). "Mean Reversion Within and Between Countries: A Panel Project on Purchasing Power Parity." *Journal of International Economics* 40, 209–24.

Frenkel, Jacob A. (1976, May). "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empricial Evidence." *Scandinavian Journal of Economics* 78(2), 200–224. Reprinted in Frenkel and Johnson (1978).

Frenkel, Jacob A. (1978). "Purchasing Power Parity: Doctrinal Perspectives and Evidence from the 1920s." *Journal of International Economics* 8, 161–91.

Frenkel, Jacob A. (1981). "Flexible Exchange Rates, Prices and the Role of 'News': Lessons from the 1970s." *Journal of Political Economy* 89, 665–705.

Frenkel, Jacob A. and Harry G. Johnson, eds. (1978). *The Economics of Exchange Rates: Selected Studies*. Reading, MA: Addison-Wesley.

Frenkel, Jacob A. and Michael L. Mussa (1985). "Asset Markets, Exchange Rates, and the Balance of Payments: The Reformulation of Doctrine." See Jones and Kenen (1985), pp. 679–747.

Friedman, Milton (1953a). "The Case for Flexible Exchange Rates." See Friedman (1953b), pp. 157–203.

Friedman, Milton (1953b). *Essays in Positive Economics*. Chicago, IL: University of Chicago Press.

Friedman, Milton and Rose D. Friedman (1998). *Two Lucky People: Memoirs*. Chicago, IL: University of Chicago Press.

Froot, Kenneth and Jeffrey Frankel (1989, February). "Forward Discount Bias: Is It an Exchange Risk Premium?" *Quarterly Journal of Economics* 104(1), 139–161.

Froot, Kenneth A. and Kenneth Rogoff (1995). "Perspectives on PPP and the Long-Run Real Exchange Rate." In G.M. Grossman and K. Rogoff, eds., *Handbook of International Economics*, Volume 3. Amsterdam: North-Holland.

Froot, Kenneth A. and Richard H. Thaler (1990, Summer). "Foreign Exchange." *Journal of Economic Perspectives* 4(3), 179–192.

Gandolfo, Giancarlo (1995). *International Economics II: International Monetary Theory and Open-Economy Macroeconomics* (Second, Revised ed.). New York: Springer-Verlag.

Genberg, H. (1978). "Purchasing Power Parity under Fixed and Flexible Exchange Rates." *Journal of International Economics* 8, 247–76.

Gerlach, Stefan (1989, November). "Intertemporal Speculation, Devaluation, and the 'J-Curve'." *Journal of International Economics* 27, 335–345.

Goldfeld, S. (1973). "The Demand for Money Revisited." *Brookings Papers on Economic Activity* 3, 577–638.

Goldfeld, Stephen M. and Daniel E. Sichel (1990). "The Demand for Money." In Benjamin M. Friedman and Frank H. Hahn, eds., *Handbook of Monetary Economics*, Volume 1, pp. 299–356. New York: North-Holland.

Goodhart, Charles A. E. (1988, November). "The Foreign Exchange Market: A Random Walk with a Dragging Anchor." *Economica* 55(220), 437–460.

Grilli, Vittorio and Gian Maria Milesi-Ferretti (1995, September). "Economic Effects and Structural Determinants of Capital Flows." *International Monetary Fund Staff Papers* 42(3), 517–51.

Hack, G.N., ed. (1970). *Approaches to Greater Flexibility of Exchange Rates*. Princeton, NJ: Princeton University Press.

Hai, Weike, Nelson C. Mark, and Yangru Wu (1997, Nov.–Dec.). "Understanding Spot and Forward Exchange Rate Regressions." *Journal of Applied Econometrics* 12(6), 715–734.

Hakkio, Craig S. (1985). "A Reexamination of Purchasing Power Parity: A Multi-Country and Multi-Period Study." *Journal of International Economics* 17, 265–77.

Halliwell, Leigh Joseph (1995, May). "Mean-Variance Analysis and the Diversification of Risk." Discussion Paper 1-22, Casualty Actuarial Society.

Harberger, Arnold (1978). "Perspectives on Capital and Technology in Less Developed Countries." In M. Artis and A. Nobay, eds., *Contemporary Economic Analysis*, pp. 151–69. Lodon: Croom Helm.

Haughton, Jonathan (1998). "Reconstruction of a War-torn Economy: The Next Steps in the Democratic Republic of Congo." CAER II Discussion Paper 24, Consulting Assistance on Economic Reform II, Cambridge, MA.

Helkie, William L. and Peter Hooper (1988). "An Empirical Analysis of the External Deficit 1980–86." In Ralph C. Bryant, Gerald Holtham, and Peter Hooper, eds., *External Deficits and the Dollar: The Pit and the Pendulum*, pp. 10–56. Washington, D.C.: The Brookings Institution.

Heston, Alan and Robert Summers (1996, May). "International Price and Quantity Comparisons: Potentials and Pitfalls." *American Economic Review* 86(2), 20–24.

Hodrick, Robert J. (1978). "An Empirical Analysis of the Monetary Approach to the Determination of the Exchange Rate." See Frenkel and Johnson (1978), Chapter 6, pp. 97–116.

Hodrick, Robert J. (1987). *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets*. Chur, Switzerland: Harwood Academic Publishers.

Hoffman, Dennis L. and Don E. Schlagenhauf (1983, March). "Rational Expectations and Monetary Models of Exchange Rate Determination: An Empirical Examination." *Journal of Monetary Economics* 11, 247–260. check.

Hooper, Peter and Jaime Márquez (1995). "Exchange Rates, Prices, and External Adjustment." In Peter B. Kenen, ed., *Understanding Interdependence: The Macroeconomics of the Open Economy*. Princeton: Princeton University Press.

Hooper, Peter and J. Morton (1982, April). "Fluctuations in the Dollar: A Model of Nominal and Real Exchange Rate Determination." *Journal of International Money and Finance* 1, 39–56.

Houthakker, Hendrik S. (1962). *Factors Affecting the United States Balance of Payments*. U.S. Government Printing Office.

Houthakker, Hendrik S. and Stephen P. Magee (1969, May). "Income and Price Elasticities in World Trade." *Review of Economics and Statistics* 51(2), 111–25.

Huang, Roger (1984). "Some Alternative Tests of Forward Exhange Rates as Predictors of Future Spot Rates." *Journal of International Money and Finance* 3, 157–67.

Huang, Roger D. (1990, August). "Risk and Parity in Purchasing Power." *Journal of Money, Credit, and Banking* 22(3), 338–356.

Huisman, Ronald, Kees Koedijk, Clemens Kool, and François Nissen (1998). "Extreme Support for Uncovered Interest Parity." *Journal of International Money and Finance* 17, 211–28.

Huizinga, John H. (1987, Autumn). "An Empirical Investigation of the Long-run Behaviour of Real Exchange Rates." *Carnegie-Rochester Conference Series on Public Policy* 27, 149–214.

IMF (1987). "Theoretical Aspects of the Design of Fund-Supported Adjustment Programs." IMF Occasional Paper 55, International Monetary Fund, Washington, DC.

IMF (1993). *Balance of Payments Manual* (5th ed.). Washington, DC.

Isaac, Alan G. (1989). "Wealth Effects and the Current Account with Endogenous Terms of Trade." *Journal of Macroeconomics* 11(4), 609–612.

Isard, Peter (1977). "How Far Can We Push the Law of One Price?" *American Economic Review* 67, 942–8.

Isard, Peter (1983). "An Accounting Framework and Some Issues for Modeling How Exchange Rates Respond to the News." In Jacob A. Frenkel, ed., *Exchange Rates and International Macroeconomics*, Chapter 2, pp. 19–56. Chicago: University of Chicago Press.

Isard, Peter (1988). "Exchange Rate Modeling: An Assessment of Alternative Approaches." In Ralph C. Bryant, Dale W. Henderson, Gerald Holtham, Peter Hooper, and Steven A. Symansky, eds., *Empirical Macroeconomics for Interdependent Economies*, Chapter 8, pp. 183–201. Washington, D.C.: The Brookings Institution.

Isard, Peter (1995). *Exchange Rate Economics*. Cambridge: Cambridge University Press.

Ito, Takatoshi (1990, June). "Foreign Exchange Rate Expectations: Micro Survey Data." *American Economic Review* 80(3), 434–449.

Johnson, Harry G. (1958). *International Trade and Economic Growth*. Cambridge, MA: Harvard University Press.

Johnson, Harry G. (1969, June). "The Case for Flexible Exchange Rates, 1969." *Federal Reserve Bank of St. Louis Review* 51, 12–24. Reprinted in Hack (1970).

Johnston, R. Barry and Natalia T. Tamirisa (1998, December). "Why Do Countries Use Capital Controls?" IMF Working Paper 98/181, International Monetary Fund, Washington, DC.

Jones, Ronald W. and Peter B. Kenen, eds. (1985). *Handbook of International Economics*, Volume 2. Amsterdam: North Holland Publishing Co.

Junz, Helen B. and Rudlof R. Rhomberg (1965, July). "Prices and Export Performance of Industrial Countries." *International Monetary Fund Staff Papers* 12, 224–271.

Kaminsky, G. (1993). "Is There a Peso Problem? Evidence from the Dollar/Pound." *American Economic Review* 83(3), need these.

Kappler, Marcus, Helmut Reisen, Moritz Schularick, and Edouard Turkisch (2013). "The Macroeconomic Effects of Large Exchange Rate Appreciations." *Open Economies Review* 24(3), 471–494.

Kenen, Peter (1985). "Macroeconomic Theory and Policy: How the Closed Economy was Opened." See Jones and Kenen (1985), Chapter 13, pp. 625–677.

Keynes, John Maynard (1925). *The Economic Consequences of Mr. Churchill*. London: Hogarth Press.

Koch, Paul D. and Jeffrey A. Rosensweig (1990, July). "The Dynamic Relationship between the Dollar and Components of U.S. Trade." *Journal of Business and Economic Statistics* 8, 355–364.

Korajczyk, R.A. (1985, April). "The Pricing of Forward Contracts for Foreign Exchange." *Journal of Political Economy* 93, 346–68.

Kouri, Pentti J.K. (1976, May). "The Exchange Rate and the Balance of Payments in the Short Run and the Long Run: A Monetary Approach." *Scandinavian Journal of Economics* 78(2), 280–304.

Kouri, Pentti J.K. (1982). "The Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model." In Jagdeep S. Bhandari and Bluford H. Putnam, eds., *Economic Interdependence and Flexible Exchange Rates*, pp. 116–56. Cambridge, MA: MIT Press. Also available as NBER Working Paper number 644.

Krasker, W.S. (1980). "The Peso Problem in Testing the Efficiency of Forward Exchange Markets." *Journal of Monetary Economics* 6, 269–76.

Kravis, Irving B., Alan W. Heston, and Robert Summers (1978a, June). "Real GDP Per Capita for more than One Hundred Countries." *Economic Journal* 88(350), 215–42.

Kravis, Irving B., Alan W. Heston, and Robert Summers (1978b). *United Nations International Comparison Project: Phase II; International Comparisons of Real Product and Purchasing Power*. Baltimore, MD: Johns Hopkins University Press.

Kravis, Irving B., Alan W. Heston, and Robert Summers (1982). *World Product and Income*. Baltimore, MD: Johns Hopkins University Press.

Kravis, Irving B. and Robert E. Lipsey (1983). *Toward an Explanation of National Price Levels*. Number 52 in Princeton Studies in International Finance. Princeton: Princeton University Press.

Krugman, Paul (1978). "Purchasing Power Parity and Exchange Rates: Another Look at the Evidence." *Journal of International Economics* 8, 397–407.

Krugman, Paul (1989, May). "The J-Curve, the Fire Sale, and the Hard Landing." *American Economic Review* 79, 31–35.

Krugman, Paul (1990). "Equilibrium Exchange Rates." In William Branson, Jacob A. Frenkel, and Morris Goldstein, eds., *International Policy Coordinations and Exchange Rate Fluctuations*, pp. 159–87. Chicago: University of Chicago Press.

Krugman, Paul (1995, Winter). "The Illusion of Conflict in International Trade." *Peace Economics, Peace Science, and Public Policy* xxx, 9–18.

Lastrapes, William D. (1992, August). "Sources of Fluctuations in Real and Nominal Exchange Rates." *Review of Economics and Statistics* 74(3), 530–9.

Levich, Richard M. (1981). "Exchange Rates and Currency Exposure." In E. Altman, ed., *Financial Handbook*, Chapter 12, pp. 98–111. New York: John Wiley.

Levich, Richard M. (1982, August). "How the Rise of the Dollar Took Forecasters by Surprise." *Euromoney*, 98–111.

Lewis, Karen K. (1989). "Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange." *American Economic Review* 79, 621–36.

Llewellyn, David T. (1980). *International Financial Integration: The Limits of Sovereignty*. London: The Macmillan Press Ltd.

Longworth, David (1981). "Testing the Efficiency of the Canadian-U.S. Exchange Market Under the Assumption of No Risk Premium." *Journal of Finance* 36, 43–49.

Lothian, James R. and Mark P. Taylor (1996, June). "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries." *Journal of Political Economy* 104(3), 488–509.

Lumsdaine, Robin L. and Eswar S. Prasad (2003, January). "Identifying the Common Component of International Economic Fluctuations: A New Approach." *Economic Journal* 113, 101–127.

Macdonald, Ronald (1983, November). "Some Tests Of The Rational Expectations Hypothesis In The Foreign Exchange Market." *Scottish Journal of Political Economy* 30(3), 235–250.

MacDonald, Ronald (1993, November). "Long-Run Purchasing Power Parity: Is It For Real?" *Review of Economics and Statistics* 75(4), 690–695.

MacDonald, Ronald and Mark P. Taylor (1994, June). "The Monetary Model of the Exchange Rate: Long-Run Relationships, Short-Run Dynamics, and How to Beat a Random Walk." *Journal of International Money and Finance* 13(3), 276–290.

Magee, Stephen P. (1973). "Currency Contracts, Pass Through, and Devaluation." *Brookings Papers on Economic Activity* 1973(1), 303–325.

Maor, Eli (1994). *e: The Story of a Number*. Princeton, NJ: Princeton University Press.

Marquez, Jaime (1991, February). "The Dynamics of Uncertainty or the Uncertainty of Dynamics: Stochastic J-Curves." *Review of Economics and Statistics* 73, 125–133.

McCallum, Bennett T. (1989). *Monetary Economics: Theory and Policy*. New York: Macmillan.

McCallum, Bennett T. (1994, February). "A Reconsideration of the Uncovered Interest Parity Relationship." *Journal of Monetary Economics* 33(1), 105–132.

McKinnon, Ronald I. (1976). "Floating Exchange Rates 1973–74: The Emperor's New Clothes." *Carnegie-Rochester Conference Series on Public Policy* 3, 79–114. Reprinted as McKinnon (1996, pp.271-90).

McKinnon, Ronald I. (1996). *The rules of the Game: International Money and Exchange Rates*. Cambridge, MA: MIT Press.

Meese, Richard A. and Kenneth Rogoff (1983). "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" *Journal of International Economics* 14, 3–24.

Meese, Richard A. and Kenneth Rogoff (1988). "Was It Real? The Exchange Rate-Interest Differential Relation over the Modern Floating-Rate Period." *Journal of Finance* 43(4), 933–948.

Metzler, L. (1951). "Wealth, Savings, and the Rate of Interest." *Journal of Political Economy* 59(2), 93–116.

Mishkin, Frederic S. (1984). "Are Real Interest Rates Equal Across Countries? An Empirical Investigation of International Parity Conditions." *Journal of Finance* 39, 1345–58.

Moffett, Michael H. (1989, September). "The J-Curve Revisited: An Empirical Examination for the United States." *Journal of International Money and Finance* 8, 425–44.

Moggridge, Donald E. (1972). *British Monetary Policy 1924–1931: The Norman Conquest of 4.86*. Cambridge: Cambridge University Press.

Montiel, Peter J. (1994, September). "Capital Mobility in Developing Countries: Some Measurement Issues and Empirical Estimates." *The World Bank Economic Review* 8(3), 311–50.

Mussa, Michael L. (1984). "The Theory of Exchange Rate Determination." See Bilson and Marston (1984), Chapter 1, pp. 13–78.

Neely, Christopher J. (1999, Nov./Dec.). "An Introduction to Capital Controls." Federal Reserve Bank of St. Louis *Review* 81(6), 13–30.

Noland, Marcus (1989, February). "Japanese Trade Elasticities and the J-Curve." *Review of Economics and Statistics* 71, 175–79.

O'Connell, Paul G.J. and Shang-Jin Wei (2002, February). "'The Bigger They Are the Harder They Fall': Retail Price Differences across U.S. Cities." *Journal of International Economics* 56(1), 21–53.

O'Connell, Paul G. J. (1998, February). "The Overvaluation of Purchasing Power Parity." *Journal of International Economics* 44(1), 1–19.

Officer, Lawrence H. (1976, March). "The Purchasing Power Parity Theory of Exchange Rates: A Review Article." *International Monetary Fund Staff Papers* 23, 1–60.

Officer, Lawrence H. (1980, May). "Effective Exchange Rates and Price Ratios over the Long Run: A Test of the Purchasing-Power-Parity Theory." *Canadian Journal of Economics* 13(2), 206–230.

Oh, K.Y. (1996). "Purchasing Power Parity and Unit Root Tests Using Panel Data." *Journal of International Money and Finance* 15, 405–18.

Pakko, Michael R. and Patricia S. Pollard (1996, January/February). "For Here or To Go? Purchasing Power Parity and the Big Mac." Federal Reserve Bank of St. Louis *Review* 78(1), 3–21.

Papell, David H. (1997). "Searching for Stationarity: Purchasing Power Parity under the Current Float." *Journal of International Economics* 43(3–4), 313–332.

Patel, J. (1990, Oct.-Dec.). "Purchasing Power Parity as a Long-Run Relation." *Journal of Applied Econometrics* 5(4), 367–79.

Pigott, C. and R.J. Sweeney (1985). "Testing the Exchange Rate Implications of Two Popular Monetary Models." In S.W. Arnott, R.J. Sweeney, and T.D. Willet, eds., *Exchange Rates, Trade and the US Economy*. Ballinger.

Polak, J. J. (1957). "Monetary Analysis of Income Formation and Payments Problems." *International Monetary Fund Staff Papers* 4, 1–50.

Prais, S. J. (1961). "Some Mathematical Notes on the Quantity Theory of Money in an Open Economy." *International Monetary Fund Staff Papers* 8(2), 212–26.

Prais, S. J. (1962). "Econometric Research in International Trade: A Review." *Kyklos* 15, 560–77.

Putnam, Bluford H. and John R. Woodbury (1979–80, Winter). "Exchange Rate Stability and Monetary Policy." *Review of Economic and Business Research* 15(2), 1–10.

Ricardo, David (1817). *The Principles of Political Economy and Taxation*. As reprinted 1987 by J.M. Dent & Sons, Ltd.

Rogoff, Kenneth (1996, June). "The Purchasing Power Parity Puzzle." *Journal of Economic Literature* 34, 647–668.

Rogoff, Kenneth (1999, November). "Monetary Models of Dollar/Yen/Euro Nominal Exchange Rates: Dead or Undead." *Economic Journal* 109(459), F655–9.

Rogoff, Kenneth (2002, February). "Dornbusch's Overshooting Model After Twenty-Five Years." Working Paper 02/39, International Monetary Fund, Washington, DC. Also see `http://www.imf.org/external/np/speeches/2001/112901.htm`.

Roll, Richard (1979). "Violations of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets." In Marshall Sarnat and Giorgio P. Szego, eds., *International Finance and Trade*, Volume 1, pp. 133–76. Cambridge, MA: Ballinger.

Rose, Andrew K. (1990). "Exchange Rates and the Trade Balance." *Economics Letters* 34, 271–75.

Rose, Andrew K. and Janet L. Yellen (1989, July). "Is There a J-Curve?" *Journal of Monetary Economics* 24(1), 53–68.

Samuelson, Paul A. (1964, May). "Theoretical Notes on Trade Problems." *Review of Economics and Statistics* 46(2), 145–54.

Sargent, Thomas J. (1977, February). "The Demand for Money During Hyperinflations under Rational Expectations." *International Economic Review* 48, 49–73.

Schinasi, Garry J. and P.A.V.B. Swamy (1987). "The Out-of-Sample Forecasting Performance of Exchange Rate Models when Coefficients are Allowed to Change." Special Studies Paper 22, Federal Reserve Board, Washington,D.C. check.

Scholl, Almuth and Harald Uhlig (2008). "New Evidence on the Puzzles: Results from Agnostic Identification on Monetary Policy and Exchange Rates." *Journal of International Economics* 76, 1–13.

Sohmen, Egon (1961). *Flexible Exchange Rates*. Chicago, IL: University of Chicago Press.

Somanath, V.S. (1986). "Efficient Exchange Rate Forecasts: Lagged Models Better than the Random Walk." *Journal of International Money and Finance* 5, 195–220.

Summers, Robert and Alan Heston (1991, May). "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950–1988." *Quarterly Journal of Economics* 106(2), 327–68.

Taylor, John B. (1995, Fall). "The Monetary Transmission Mechanism: An Empirical Framework." *Journal of Economic Perspectives* 9(4), 11–26.

Taylor, Mark P. (1988). "An Empirical Examination of Long-Run Purchasing Power Parity Using Cointegration Techniques." *Applied Economics* 20, 1369–81.

Tryon, R. (1979). "Testing for Rational Expectations in Foreign Exchange Markets." International Finance Discussion Paper 139, Board of Governors of the Federal Reserve System, Washington, DC.

Vanek, Jaroslav (1962). *International Trade: Theory and Economic Policy*. Homewood, IL: Richard D. Irwin, Inc.

Williamson, John (1965). *The Crawling Peg*. Essays in International Finance. Princeton: Princeton University Press.

Woo, W.T. (1985). "The Monetary Approach to Exchange Rate Determination under Rational Expectations: The Dollar-Deutschmark Rate." *Journal of International Economics* 18, 1–16.

Wu, Y. (1996). "Are Real Exchange Rates Nonstationary? Evidence from a Panel-Data Test." *Journal of Money, Credit, and Banking* 28, 54–63.

Young, J.P. (1925). "European Currency and Finance." Technical Report Serial 9, Commission of Gold and Silver Enquiry.

Index