Jones Ch. 5: Solow Model

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All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive.
– Robert Solow (1956 QJE)
1960

- KR: GDP per capita = $1500, population ≈ 25M, labor force participation rate ≈ 50%, college attendance in early twenties ≈ 5%
- PH: GDP per capita = $2000, population ≈ 25M, labor force participation rate ≈ 50%, college attendance in early twenties ≈ 13%

2000

- KR: GDP per capita = $16,000
- PH: GDP per capita = $3400

Why did South Korea (KR) grow so much faster than the Philippines?
Simplesst Solow Model

Model is dynamic: $K$ is changed by $I$ according to the capital accumulation equation:

\[ K_{t+1} = K_t + I_t - \bar{d}K_t \]  \hspace{1cm} (1)

\[ K_{t+1} - K_t = I_t - \bar{d}K_t \]  \hspace{1cm} (2)

\[ \Delta K_t = I_t - \bar{d}K_t \]  \hspace{1cm} (3)

Investment is determined by saving, which is a constant fraction of output

\[ I_t = \bar{s}Y_t \]  \hspace{1cm} (4)

The steady-state capital stock becomes *endogenous*. 
Simplifications

• Simplified economy: leave out $G$ and $EX - IM$:

$$Y_t = C_t + I_t$$  \hspace{1cm} (5)

• no TFP growth ($\hat{A} = 0$)

• no population growth.
Steady state of this *simplest* Solow model: gross investment equals depreciation. (I.e., no net investment.)

\[ I_{ss} = \bar{d}K_{ss} \]
\[ \bar{s}Y_{ss} = \bar{d}K_{ss} \]
\[ K_{ss}/Y_{ss} = \bar{s}/\bar{d} \]

The steady-state capital-output ratio is determined by the model parameters.
Solve for $K_{ss}$

In the simplest Solow model:

\[
\frac{K_{ss}}{Y_{ss}} = \bar{s}/\bar{d}
\]

\[
\frac{K_{ss}}{\bar{A}K_{ss}^{1/3}L^{2/3}} = \frac{\bar{s}}{\bar{d}}
\]

\[
K_{ss}^{2/3} = \bar{A}\bar{L}^{2/3}\frac{\bar{s}}{\bar{d}}
\]

\[
\left( K_{ss}^{2/3} \right)^{3/2} = \left( \frac{\bar{s}\bar{A}\bar{L}^{2/3}}{\bar{d}} \right)^{3/2}
\]

\[
K_{ss} = \left( \frac{\bar{s}\bar{A}}{\bar{d}} \right)^{3/2} \bar{L}
\] (6)
This implies the steady-state capital-labor ratio:

\[ k_{ss} = \frac{K_{ss}}{L} = \left( \frac{sA}{d} \right)^{3/2} \]  

(7)

Since \( y = Ak^{1/3} \), this implies a steady-state per capita income

\[ y_{ss} = \frac{Y_{ss}}{L} = \bar{A}^{3/2} \left( \frac{s}{d} \right)^{1/2} \]  

(8)

The corresponding level of output is

\[ Y_{ss} = \bar{A}^{3/2} \left( \frac{s}{d} \right)^{1/2} \bar{L} \]  

(9)
Application from Jones (2008, p.108, equation 5.12): let’s compare the implications for a rich country and a poor country for *relative* standard of living.

- get data on per capita real GDP and also on investment rates (data from 2000)
- assume depreciation rates are the same in both countries.
- use averages for the richest five countries and poorest five countries.
- use investment rates as a proxy for saving rates. (This is appropriate for the Solow model.)
Income differs by a factor of about 45. Investment rates differ by a factor of a bit more than 6. Deduce the relative contribution of the “technological” parameter to \textit{steady-state} differences in per capita income.

\[
\frac{y_{rss}}{y_{pss}} = \left( \frac{\bar{A}_r}{\bar{A}_p} \right)^{3/2} \left( \frac{\bar{s}_r}{\bar{s}_p} \right)^{1/2}
\]

\[
45 = \left( \frac{\bar{A}_r}{\bar{A}_p} \right)^{3/2} (6.25)^{1/2}
\]

\[
\left( \frac{\bar{A}_r}{\bar{A}_p} \right)^{3/2} = 45/(6.25)^{1/2} = 45/2.5 = 18.0
\]

Conclusion: even if we switch our emphasis to steady-state differences in per capita income, most of the difference is accounted for by the “technological” parameter.
The “technological parameter” \((A)\) is a residual that captures many things, including:

- technology
- natural resources
- human capital
- institutions
Solow Growth Model

Solow Diagram (simplified: \( \hat{L} = 0 \) and \( \hat{A} = 0 \))

**FIGURE 5.1 The Solow Diagram**
Transition Dynamics in the Simplest Solow Model

($\hat{L} = 0$ and $\hat{A} = 0$)

**FIGURE 5.2 The Solow Diagram with Output**
One-Time Destruction of Capital Stock

**FIGURE 5.11 The Solow Diagram**
One-Time Destruction of Capital Stock (Dynamics)

**FIGURE 5.12** Output over Time, 2000–2100
Prediction: output rises faster when we are farther from the steady state.
Real GDP p.c.: Level vs. Growth Rate (many countries)

**FIGURE 5.9 Growth Rates around the World, 1960–2000**
Real GDP p.c.: Level vs. Growth Rate (OECD)

FIGURE 5.8 Growth Rates in the OECD, 1960–2000
So the prediction looks a bit better if we focus on developed countries.
Increase in Saving (Simplest Solow Model)

**FIGURE 5.4** An Increase in the Investment Rate
Prediction: an increase in $s$ will lower $Y/K$ (i.e., raise $K/Y$).
$K/Y$ and $s$ (positive correlation)
Dynamic Effects of an Increase in $s$ (Solow model)

**FIGURE 5.5** The Behavior of Output Following an Increase in $s$
FIGURE 5.6 A Rise in the Depreciation Rate
$D_y$ - ve)
Dynamic Effects of an Increase in $\bar{d}$ (Solow model)

**Figure 5.7** The Behavior of Output Following an Increase in $\bar{d}$
FIGURE 5.10 Investment in South Korea and the Philippines, 1950–2000
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Solow Growth Model