When economists speak of the portfolio balance approach, they are referring to a diverse set of models. There are a few common features, however. In common with the monetary approach, portfolio balance models of flexible exchange rates focus on the role of asset stocks in the determination of exchange rates: short-run exchange-rate adjustments are determined in asset markets. There is an additional theme among these models: an attention to the links between stocks of assets and saving flows.

The link between wealth and saving can be represented as

\[
\dot{\Omega}/P = S \tag{10.1}
\]

This step is generally taken in combination with a second link between wealth and saving (see Kenen (1985, p.672) for references).

\[
S = S(\Omega/P) \quad S' < 0 \tag{10.2}
\]
To keep the model simple, we will ignore terms of trade considerations by assuming purchasing power parity.

The monetary approach to flexible exchange rates did not predict two salient events in the late 1970s.

- large deviations from purchasing power parity
- the “stylized fact” that current account surplus countries had appreciating exchange rates

We will give a separate analysis of the Dornbusch overshooting model, which can account for the large deviations from purchasing power parity. That model can also accommodate to some extent the second stylized fact. For example, starting from a zero trade balance a monetary shock generates a large depreciation that throws the trade balance into surplus, and the surplus persists as the real exchange rate appreciates toward the new equilibrium. However if we start from a current account deficit, a positive monetary shock may simply reduce the deficit in the short run without affecting it in the long run. In the simple overshooting model, asset accumulation through the current account has no cumulative effect on the economy. (The steady state of this model can therefore include a current account balance of any magnitude.)

Portfolio balance models attempt to give a more robust explanation of the observed link between the current account and exchange rate movements. We will develop a particularly simple version of the model, which focuses on this issue. As in the simple monetary approach model to flexible exchange rates, we will assume constant PPP.

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1 The role of asset accumulation through the balance of payments was central to the monetary approach to the balance of payments. The concept of saving that we will use in this chapter is similar to the concept of “hoarding” used in the monetary approach to the balance of payments. In fact, except for the tendency of the portfolio balance tradition to treat interest rates and multiple assets explicitly, there is little to separate the portfolio balance and monetary approaches to the balance of payments. However, when the monetary approach was applied to flexible exchange rates, the emphasis on stock flow links was abandoned. These were re-introduced only with the portfolio balance models of flexible exchange rates.

2 Dornbusch and Fischer (1980) and Isaac (1989) discuss a variant with endogenous terms of trade.
wealth as well as income. The resulting model predicts that full equilibrium requires current account balance. This extends the predictions of the simple monetary approach model, which makes no such prediction.

This portfolio balance model lends new emphasis to the current account by introducing the influence of asset accumulation on asset demand. (Under rational expectations, expected future current account balances should also affect $S$ because they affect expected future asset accumulation.)

### 10.1 A Partial Equilibrium Model

We begin by introducing a simple partial equilibrium approach due to Kouri (1982). Let $\alpha \Omega$ be the domestic demand for foreign assets, where $\Omega$ is nominal wealth measured in domestic currency units. Then the asset markets clear only if

\[ S_{NFA} = \alpha \Omega \]  \hfill (10.3)

Thus, given $i$, $i^*$, and $\Omega$, we can characterize exchange rate determination.

In Figure 10.1, we draw a rectangular hyperbola in ($S$, $NFA$)-space. Along this curve, the domestic currency value of net foreign asset holdings is constant ($SNFA = \alpha \Omega$). For each level of foreign assets, we can see the market clearing exchange rate. Here we see the role of the assets markets in exchange rate determination illustrated very clearly. The exchange rate adjusts so that the current portfolio is willingly held. Current net foreign assets are therefore a primary determinant of the spot exchange rate.

This may be a reasonable description of exchange rate determination in the short-run, when asset accumulation is negligible compared to the stock of existing assets. However over long periods of time, we would want to account for the influence of changes in asset stocks. We will focus on accumulation of net foreign assets through the balance of payments.

The second graph in Figure 10.1 plots the current account as a function of the exchange
The upward sloping line in \( S, CA \) space represents the level of the current account for each level of the exchange rate. Since the exchange rate is a determinant of the current account, we have a link between net foreign assets and the current account. This yields the dynamic interaction between the current account and the exchange rate. A current account deficit draws down net foreign assets, depreciating the exchange rate, and thereby eliminating the deficit.

### 10.2 A General Equilibrium Model

The very simple partial-equilibrium framework of the previous section illustrates some core considerations in a portfolio balance model of exchange rate determination. We now repair some of its most obvious shortcomings. For example, we will recognize that changes in the exchange rate affect the nominal value of wealth. More critically, we recognize that the asset market equilibrium locus of Figure 10.1 should shift over time in response to the accumulation or decumulation of net foreign assets. Our next task is therefore to incorporate

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3We are assuming the Marshall-Lerner condition is satisfied and, for the moment, ignoring changes in prices.
these changes in a more complete portfolio balance model of exchange rate determination.

In the short run, this portfolio balance model is essentially our monetary approach model. There is one change: real money demand depends on real wealth \( \Omega/P \).

\[
\frac{H}{P} = L \left( i, Y, \frac{\Omega}{P} \right) \quad L_i < 0, \ L_Y > 0, \ 1 > L_{\Omega/P} > 0
\]  

This wealth effect proves crucial: it is the core difference between this portfolio balance model and our simple monetary approach model. Aside from (10.4), we will used the standard components of a monetary approach model: full employment, interest parity (perfect capital mobility, and even perfect capital substitutability), and purchasing power parity.

We will now treat wealth in a bit more detail. Set real wealth equal to our real money balances plus the real value of our net foreign assets. We will find it convenient to explicitly characterize net foreign assets as real perpetuities that pay one unit of foreign output forever. If \( r \) is the real interest rate, then each of these perpetuities is worth \( 1/r \). We can therefore characterize real wealth as

\[
\Omega/P = H/P + a - \frac{r}{r}
\]  

where \( a \) is the number of such perpetuities owned by domestic residents. The money market equilibrium condition becomes

\[
\frac{H}{P} = L \left( i, Y, \frac{H}{P} + a - \frac{r}{r} \right)
\]  

To keep the model presentation simple, we will set foreign inflation to zero and assume absolute purchasing power parity holds and is expected to continue to hold.

\[
S = P/P^*
\]  

\[
\Delta s^e = \pi^e
\]
This allows us to characterize the nominal interest rate as follows.

\[ i = r + \pi^e \]
\[ = r + \Delta s^e \]  \hspace{1cm} (10.9)

Referring to (10.6), we see that money market equilibrium condition can be expressed as

\[ \frac{H}{P} = L \left( r + \Delta s^e, Y, \frac{H}{P} + \frac{a}{r} \right) \]  \hspace{1cm} (10.10)

Equation (10.10) implicitly defines a functional dependence of real balances on \( r, \Delta s^e, \) and \( a. \) Consider the effects of a rise in expected depreciation, starting from a situation of money market equilibrium. This unambiguously creates excess supply in the money market (i.e., lowers money demand) ceteris paribus. Is this offset by a fall in \( H/P? \) Yes, as long as an additional dollar of wealth generates less than an additional dollar of real money demand. Similarly, the excess demand created by an increase in \( a \) can be offset by an increase in \( H/P. \) Ceteris paribus, a rise in \( r \) involves both of the previous effects: a fall in money demand due to higher interest rates, and a fall in money demand due to lower real wealth. (The real value of the perpetuities is reduced by higher interest rates.) Equation (10.11) summarizes these arguments and presents the money market equilibrium condition in a simpler way. (We suppress \( Y \) since it is constant.) We will henceforth refer to \( x \) as ‘real money demand’, keeping in mind its derivation.

\[ \frac{H}{P} = x(r, \Delta s^e, Y, a) \quad x_r < 0, x_e < 0, x_Y > 0, x_a > 0 \]  \hspace{1cm} (10.11)

Here we let \( x_e \) denote the response of \( x \) to a change in expected depreciation. (So \( x_e < 0.)\)

Recall that absolute purchasing power parity can be represented as \( P = SP^*. \) So given the foreign price level, \( P^* \), the rate of change of the price level equals the rate of change of the exchange rate: \( \pi = \Delta s. \) Define the real interest rate by \( r = i - \pi^e; \) then \( r = i - \Delta s^e \) by expected PPP. (Here \( i \) is the nominal interest rate.) We will let \( r \) be the exogenously
given real interest rate. Recalling that goods market equilibrium is determined simply by purchasing power parity, \( P = S P^* \), we have a simple static determination of the exchange rate in Figure 10.2.

In Figure 10.2, the LM curve represents the money market equilibrium (10.11). It is vertical, because no matter what the current exchange rate is, there is a unique price level that can clear the money market. (Of course this would change if the current level of the exchange rate had implications for its expected future level, as under the regressive expectations hypothesis, but for now we treat \( \Delta s^e \) as exogenous.) The purchasing power parity locus is a ray with slope \( 1/P^* \).
10.3 Static Predictions

The basic predictions of our model are determined by comparative statics experiments. Graphically, our comparative statics experiments will be represented by shifts of the LM curve. Consider a one-time permanent increase in the money supply, as represented in Figure 10.3. An increase in $H$ increases the equilibrium price level proportionately. This is represented by a rightward shift in the LM curve. In the new equilibrium, the price level and the exchange rate have risen proportionately. This result is familiar to us from our work with the monetary approach.

Other comparative statics experiments involve money demand instead of money supply. Any reduction in money demand raises the equilibrium price level, and can therefore also be represented by Figure 10.3. For example, a rise in $\Delta s^e$ increases the domestic interest rate and thereby decreases money demand. The effects are a rise $S$ and $P$ proportional to the fall in money demand. Of course this is also compatible with the monetary approach.

Also compatible with our monetary approach analysis is the interest rate effect of a change in $r$: a rise in $r$ raises the domestic interest rate and reduces real money demand. However,
the effect of a change in \( r \) has been slightly complicated by its role in determining the real value of net foreign assets. The rise in \( r \) reduces the real value of net foreign assets. Since this again reduces real money demand, there is no qualitative difference from the monetary approach model.

Finally, we have a new influence on money demand: \( a \). If there is an decrease in our holding of net foreign assets, this decreases the real demand for money (by decreasing wealth). As always, the price level and exchange rate must increase in proportion to the fall in real money demand. Thus this experiment can also be represented by Figure 10.3.
We can also proceed algebraically. First combine purchasing power parity with our simplified money market equilibrium condition to get

\[
\frac{H}{SP^*} = x(r, Y, \Delta s^e, a) \tag{10.12}
\]

Then solve for the exchange rate.

\[
S = \frac{H}{x(r, Y, \Delta s^e, a) P^*} \tag{10.13}
\]

This is our basic story about exchange rate determination at each point in time. (One qualification: we will later pay more attention to expectations, so \(\Delta s^e\) will become endogenous.) Again, note the dependence of \(S\) on \(a\). This provides the dynamic link between the exchange rate and the current account.
10.4 Dynamics

This section introduces the model dynamics. As suggested above, these will be driven by savings behavior. Given our definition of wealth along with purchasing power parity, we have

\[ S \left( \frac{\Omega}{P} \right) = S \left( \frac{H}{P} + \frac{a}{r} \right) = S \left( \frac{H}{SP^*} + \frac{a}{r} \right) \] (10.14)

We are working with a target-wealth saving function (Metzler, 1951). In this framework, desired saving flows are not adjusted for any anticipated capital gains or losses on existing assets. Furthermore, the only financial assets available in our simple portfolio-balance model are domestic money and the internationally traded bond. To keep things simple, we will hold the money supply constant. It follows that desired saving can be satisfied only by the accumulation of the internationally tradable asset. Since this is a real perpetuity, we can write

\[ \frac{\dot{a}}{r} = S \] (10.15)

Equating desired to actual saving yields the dependence of asset accumulation on \( H, S, P^*, a, \) and \( r \).

\[ \frac{\dot{a}}{r} = S \left( \frac{H}{SP^*} + \frac{a}{r} \right) \] (10.16)

Given the other variables, (10.16) establishes a relationship between \( \dot{a} \) and \( a \). This relationship is a first-order differential equation: it relates asset accumulation to the current level of assets. In Figure 10.8, we explore this dynamic relationship graphically. To do so we follow the usual rule in thinking about dynamics: start by characterizing the nullclines. (A nullcline is a locus of points where dynamic adjustment stops.)

Our graphical analysis is in \((a, S)\)-space. The \( a \)-nullcline comprises the combinations of \( a \) and \( S \) such that \( \dot{a} = 0 \). Since \( \dot{a} \) is determined by saving which depends on wealth, the \( a \)-nullcline is a constant wealth locus. Increases in \( a \) increase wealth, while increases in \( S \) decrease wealth (by reducing real balances). Therefore the \( a \)-nullcline is upward sloping in
Once we have determined the nullcline, we can easily address the dynamic adjustments to its right or left. Pick any point on the nullcline, where saving is zero, and move to the right. This increases wealth, so saving must fall, leading to asset declines. (Remember that $S' < 0$!) So to the right we have $\dot{a} < 0$. Next start at the same point on the nullcline, but this time move to the left. This decreases wealth, so saving must rise, leading to asset increases. So to the left of the nullcline we find $\dot{a} > 0$.

Figure 10.8 also illustrates portfolio balance in $(a, S)$-space. Recall our comparative static argument that when $a$ is higher the exchange rate must be lower for asset market equilibrium. We summarize this consideration in the PB curve of Figure 10.8. Pay particular attention to the negative relationship between our accumulated net foreign assets, represented by $a$, and the equilibrium exchange rate.
This is because a rise in $a$ will raise wealth, and this will reduce saving. To keep saving at zero, we must have an offsetting rise in $P$, which will reduce wealth by reducing real balances. Given purchasing power parity, we therefore require increases in $S$ to offset increases in $a$ so as to maintain $\dot{a} = 0$. This relationship is represented in Figure 10.8: it is $\dot{a} = 0$ locus seen in this figure. Together these two loci summarize the dynamic behavior of the economy in this portfolio balance model.
We begin our dynamic story with the historically given level of net foreign assets. In Figure 10.8 this is labelled $a_0$. At this level of net foreign assets, there is a unique exchange rate that clears the assets markets. This is the exchange rate we determined in our LM–PPP analysis of Figure 10.2, and in Figure 10.8 we label it $S_0$. Our discussion of asset dynamics tells us that the low real wealth at point $a_0, S_0$ implies that this is a point of net asset accumulation. As we accumulate net foreign assets through current account surplusses, $a$ increases and the exchange rate appreciates. Thus the model predicts the negative correlation between the current account and exchange rate depreciation that emerged as a stylized fact in the 1970s. The adjustment continues along the PB curve until we reach the $\dot{a} = 0$ locus.

**Comment:** Note that the stability of the model dynamics can easily determined by combining (10.18) and (10.19) to get

$$\frac{\dot{a}}{r} = S\left(x(r, \Delta s^e, Y, a) + \frac{a}{r}\right) \quad (10.17)$$
Since $S' < 0$, we know $d\dot{a}/da < 0$, and the model is stable.

We can also algebraically examine the effects of changes in the foreign interest rate, the expected rate of depreciation, or the money supply. Here (10.18) of our PB curve, and we use (10.19) to determine the $\dot{a} = 0$ locus.

\[
S = \frac{H}{x(r,Y,\Delta s^e, a)P^*} \quad \text{PB curve} \quad (10.18)
\]
\[
\frac{\dot{a}}{r} = S \left( \frac{H}{SP^*} + \frac{a}{r} \right) \quad (10.19)
\]

### 10.4.1 Some Thought Experiments

Consider the following experiments (starting from long-run equilibrium):
Receive a one-time wealth transfer (increase \( a \)): Neither curve shifts, but we move to a new position on PB and then slowly adjust back to old position.

Figure 10.6: Short-Run and Long-Run Effects of Wealth Transfer
Simple flexprice portfolio balance model under static expectations.
\[ \dot{a} = S \left( \frac{H}{SP^*} + a/r \right); \] PB: \[ \frac{H}{SP^*} = x(r, \Delta s^e, Y, a). \]
Monetary expansion (increase $H$): Both curves shift up proportionately; see the PB-aa chart in Figure 10.7. There is complete monetary neutrality (no real changes).

This should look familiar from our work on the monetary approach model. Review Figure 10.3 for the effect in the PPP-LM framework. Since prices adjust flexibly, there is no further adjustment needed.
**Fiscal expansion:** We do not have an explicit fiscal variable, but we can treat this as a fall in national saving. So where we used to have $\dot{a} = 0$, we now have $\dot{a} < 0$. The aa-curve shifts left. We move along the old PB curve until we reach the new equilibrium at a reduced level of $a$.

---

**Figure 10.8:** Short-Run and Long-Run Effects of Fiscal Expansion

Simple flexprice portfolio balance model under static expectations.

$\dot{a} = S\left(\frac{H}{(SP^*)} + \frac{a}{r}\right) ;$  
PB: $\frac{H}{(SP^*)} = x(r, \Delta s^e, Y, a)$.
**Increase $\Delta s^e$:** An expected depreciation generates an actual depreciation.

Start with the PPP-LM representation in Figure 10.9. The second experiment should look familiar at the beginning: our work on the monetary approach model also showed us that an expected depreciation produces an actual depreciation.

![Figure 10.9: Increase in $\Delta s^e$ (static expectations)](image)

However, we now get a subsequent effect on the current account. We have lower real wealth due to the lower real balances caused by the rise in prices (an corresponding depreciation). We therefore save toward our target wealth. As we accumulate wealth money demand rises, driving prices back down. This continues until our original level of wealth is restored. However, the composition of wealth has changed: we now have more $a$ and somewhat less $\frac{H}{P}$. 
Note that the current account improves: the depreciation reduces wealth (via real balances), which increases saving (and thus the current account). A period of appreciation and declining current account balance follows.

One thing that may make us uncomfortable with the second experiment is the idea that expected depreciation will remain constant as the exchange rate continues to change over time. We will address this with a rational expectations analysis.
10.5 Rational Expectations

Since we are working with a deterministic model, we interpret rational expectations as exact knowledge of the path implied by the rules of motion. This means that under rational expectations we will have $\Delta s^e = \dot{S}/S$. Our dynamic system corresponding becomes

$$\frac{H}{SP*} = x \left( r, \frac{\dot{S}}{S}, Y, a \right)$$

$$\frac{\dot{a}}{r} = S \left( \frac{H}{SP*} + \frac{a}{r} \right)$$

(10.20)

The graphical representation looks just like Figure 10.8. (See problem 4.)

Figure 10.11: Rational Expectations Dynamics
To approach the algebra, we will construct linear approximation to our dynamic system around the steady state. This yields the following linear first-order differential equation system in the variables $\delta a$ and $\delta S$.

\[
\frac{H}{S^2 P^*} = x \left( r, \frac{\dot{S}}{S}, Y, a \right) \tag{10.20}
\]

\[
\frac{\dot{a}}{r} = S \left( \frac{H}{SP^*} + \frac{a}{r} \right)
\]

\[
-\frac{H}{S^2 P^*} \delta S = x_e \left[ \frac{1}{S} \delta \dot{S} - \frac{\dot{S}}{S^2} \delta S \right] + x_a \delta a \tag{10.21}
\]

\[
\frac{1}{r} \delta \dot{a} = -S_w \frac{H}{S^2 P^*} \delta S + S_w \frac{1}{r} \delta a
\]
Recall that we are linearizing around the steady state. Since the money supply is constant, we know $\dot{S} = 0$ at the steady state. This slightly simplifies our expression of the system. We will get a further simplification by using the differential operator, so that we can write $\dot{\delta a}$ as $D \delta a$ and $\dot{\delta S}$ as $D \delta S$. This allows us to write our dynamic system as

\[-\left(\frac{H}{S^2P^*} + x_e \frac{1}{S} D\right) \delta S - x_a \delta a = 0\]

\[(D - S_w) \delta a + S_w \frac{rH}{S^2P^*} \delta S = 0\]

We get a slight additional simplification by giving this system a matrix representation.

\[
\begin{bmatrix}
-\frac{H}{S^2P^*} - x_e \frac{1}{S} D & -x_a \\
S_w \frac{rH}{S^2P^*} & D - S_w
\end{bmatrix}
\begin{bmatrix}
\delta S \\
\delta a
\end{bmatrix}
= 0
\]  

(10.22)

In summary, we have

\[P(D)\begin{bmatrix}
\delta S \\
\delta a
\end{bmatrix}
= 0\]  

(10.23)

We are going to solve this system with the adjoint matrix technique. The first step is to find the characteristic roots.
In forming the characteristic equation of this system, we will slightly abuse notation: we now treat $D$ as a variable. This allows us to write the characteristic equation as

$$|P(D)| = 0 \quad (10.24)$$

In more detail, this is

$$-\frac{H}{S^2P^*}D + \frac{H}{S^2P^*}S_w - x_e \frac{1}{S} D^2 + x_e \frac{1}{S} S_w D + x_a S_w \frac{rH}{S^2P^*} = 0 \quad (10.25)$$

Rearrange to get

$$D^2 + \left( \frac{H}{x_e P^*} - S_w \right) D - S_w \frac{H}{P^*} \frac{1 + x_a r}{x_e} = 0 \quad (10.26)$$

This is a quadratic equation, so we will find two characteristic roots. Since the last term is negative, we know our two characteristic roots are real and opposite in sign. That is, we have a convergent saddle-path. Let $D_1 < 0 < D_2$ denote these roots.
Recall that we are working with the matrix operator $P(D)$ where

$$P(D) = \begin{bmatrix} -\frac{H}{S^2P_x} - x_e \frac{1}{S} D & -x_a \\ S_w \frac{r}{S^2P_x} & D - S_w \end{bmatrix}$$ (10.27)

The adjugate (or “adjoint”) matrix is therefore

$$P^\#(D) = \begin{bmatrix} D - S_w & x_a \\ -S_w \frac{r}{S^2P_x} - \frac{H}{S^2P_x} - x_e \frac{1}{S} D \end{bmatrix}$$ (10.28)

The adjoint-matrix technique states the solution to this system in terms of the characteristic roots and a column of the adjugate of $P(D)$.

$$\begin{bmatrix} \delta S \\ \delta a \end{bmatrix} = \eta_1 \begin{bmatrix} D_1 - S_w \\ -S_w \frac{r}{S^2P_x} \end{bmatrix} e^{D_1t} + \eta_2 \begin{bmatrix} D_2 - S_w \\ -S_w \frac{r}{S^2P_x} \end{bmatrix} e^{D_2t}$$ (10.29)

Recall that the values of $\eta_1$ and $\eta_2$ can be determined if we are given the initial state of the system $(\delta a_0, \delta S_0)$. But since $D_2 > 0$, this solution will diverge unless we set $\eta_2 = 0$. This is our transversality condition, and the resulting solution is

$$\begin{bmatrix} \delta S \\ \delta a \end{bmatrix} = \eta_1 \begin{bmatrix} D_1 - S_w \\ -S_w \frac{r}{S^2P_x} \end{bmatrix} e^{D_1t}$$ (10.30)

Now we have only one constant to determine, so we only need one initial condition. We use the value of $\delta a_0$ to determine the value of $\eta_1$. This is motivated by the economic interpretation of the model: only the value of $a$ is given to us as an historically predetermined variable. In contrast, $S$ is a jump variable, and the transversality condition ensures that it jumps to the convergent arm of our dynamic system.

\footnote{Since $D_1 < 0$ and $S_w < 0$, it may seem that we have failed to sign $D_1 - S_w$. However, a little algebra shows this is negative.}