The Risk Premium

11 The Risk Premium

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In this chapter we explore the nature and sources of currency risk, and we characterize portfolio choice behavior in the presence of currency risk. Currency risk is the risk one incurs due to the currency denomination of one’s portfolio. For example, if your portfolio contains unhedged foreign-currency denominated assets, then exchange rate movements can changes the value of your portfolio.

With the advent of the general float, the risks of exposure to exchange rate changes were soon evident. Between June 1974 and October 1974, the Franklin National Bank of New York and the Bankhaus I.D. Herstatt of Germany failed due (at least in part) to losses from this source. At the time Franklin National was the twenty-third largest bank in the U.S.; it had an unhedged foreign exchange position of almost $2 billion and was illegally concealing losses on its foreign exchange operations. Herstatt’s unhedged position was about $200 million. Exchange rate fluctuations—obviously unanticipated—pushed these positions into huge losses. On the other hand, betting against the dollar after the October 19, 1987 U.S. stock market crash generated large profits for U.S. banks.

### 11.1 Excess Returns

Compare the *ex post* real return from holding the domestic asset, $i_t - \pi_{t+1}$, with the (uncovered) *ex post* real return from holding the foreign asset, $i^*_t + \Delta s_{t+1} - \pi_{t+1}$. In chapter 2 we agreed to call the difference between these two *ex post* real rates of return the **excess return** on the domestic asset.

$$
er_{t+1} \overset{\text{def}}{=} (i_t - \pi_{t+1}) - (i^*_t + \Delta s_{t+1} - \pi_{t+1})$$

(11.1)

This is the *ex post* difference in the uncovered returns.
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In chapter 2 we also developed the covered interest parity condition.

\[ i = i^* + fd \]  

(11.2)

This condition equates the returns on riskless assets. Currency risk is not involved in a covered interest arbitrage operation, for all currency exposure is completely hedged in the forward market.

Using covered interest parity, we can rewrite the excess return on the domestic currency as

\[ er_{t+1} = fd_t - \Delta s_{t+1} \]

\[ = (f_t - s_t) - (s_{t+1} - s_t) \]  

(11.3)

\[ = f_t - s_{t+1} \]

Consider Figure 11.1, which plots excess returns over time. We see these excess returns are large, variable, and mildly autocorrelated. We also see that in some subsamples, for example the early 1980s, the autocorrelation is higher than in others, for example the early 1970s. This apparent autocorrelation suggests that excess returns are somewhat predictable.

Note that covered interest arbitrage does not imply that the market expects zero excess return on the domestic asset. It does however imply that expected excess return shows up as a gap between the forward rate and the expected future spot rate.

\[ er^e_{t+1} = fd_t - \Delta s^e_{t+1} \]  

(11.4)

\[ = f_t - s^e_{t+1} \]

We will call expected excess returns the risk premium, \( rp_t \).

\[ rp_t = er^e_{t+1} \]  

(11.5)
Let $\epsilon_{t+1}$ denote the spot-rate forecast error:

$$\epsilon_{t+1} = s_{t+1} - s_{t+1}^e$$

(11.6)

Then we can decompose excess returns into an expected component, and an unexpected component. The expected component is $r_{p_t}$, the risk premium on the domestic currency. This is reduced by $\epsilon_{t+1}$, the unexpected depreciation of the domestic currency.

$$er_{t+1} = r_{p_t} - \epsilon_{t+1}$$

(11.7)

Economists have expended considerable effort trying to determine whether expected ex-
Figure 2. 3-Month Excess Currency Return
(Annualized %; Home over USD return)

Figure 11.2: Three-Month Excess Currency Returns
cess returns are zero (Hodrick, 1987). The hypothesis that expected excess returns are zero is known as the uncovered interest parity hypothesis.\footnote{This usage is standard, but it is not universal. For example, McCallum (1994) defined uncovered interest parity as requiring only that expected excess returns be determined exogenously. He allows, for example, exogenous risk premia, measurement errors, and aggregation effects.} However, as figure 11.1 indicates, excess returns appear to be somewhat predictable, and uncovered interest parity is not supported by the data. We will explore this in more detail in the next section.

### 11.1.1 Uncovered Interest Parity

In chapter 2 we noted that speculative behavior ought to link the forward rate to the expected future spot rate. If the forward rate equaled the expected future spot rate, then expected excess returns would be zero. We will try to get a sense of how closely the two are tied together.

Suppose the forward rate equals the expected future spot rate. That is, assume the uncovered interest parity hypothesis.

\[ f_t^{\text{UIP}} = s_{t+1}^e \]

(11.8)

Under UIP we can therefore write

\[ f_t = s_{t+1} - \epsilon_{t+1} \]

(11.9)

where you will recall \( \epsilon_{t+1} \) is the spot-rate-forecast error.

\[ \epsilon_{t+1} \overset{\text{def}}{=} s_{t+1} - s_{t+1}^e \]

(11.10)

If the forecast error is white noise, we then have a natural regression equation.

\[ s_{t+1} = \beta_0 + \beta_1 f_t + \epsilon_{t+1} \]

(11.11)
11.1. EXCESS RETURNS

We expect to find an estimate $\hat{\beta}_1 = 1$. Do we? Figure 11.3 suggests that we do.

![Figure 11.3: Predicting Spot with Forward Rates: US/UK, 1973–1992](source: Data from Hai et al. (1997))

Note that if $\beta_1 = 1$, we can alternatively try the regression equation

$$\Delta s_{t+1} = \beta_0 + \beta_1(f_t - s_t) + \epsilon_{t+1}$$  \hspace{1cm} (11.12)

and again expect to find an estimate $\hat{\beta}_1 = 1$. Do we? Figure 11.3 suggests that we do not.

Table 11.1 confirms that the answers to these two questions are quite different. When the spot rate is regressed on the forward rate that should “predict” it, the results look quite as anticipated. However when the apparently equivalent model (11.12) is used, quite different results emerge.\footnote{Huisman et al. (1998) note that the large standard errors from such regressions imply that the hypothesis that $\beta = 1$ cannot always be rejected.} Apparently Tryon (1979) first noted these conflicting results, which have become known as the forward premium anomoly, and they have been repeatedly confirmed (Fama, 1984; Hodrick, 1987). In table 11.1, we discover large, negative estimates for $\beta_1$ from the second model.
Estimates of $\beta_1$ from Various Models

<table>
<thead>
<tr>
<th>Model: $s_{t+1} = \beta_0 + \beta_1 f_t + \epsilon_{t+1}$</th>
<th>USD/DEM</th>
<th>USD/GBP</th>
<th>USD/JPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{t+1} = \beta_0 + \beta_1 f_t + \epsilon_{t+1}$</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>$\Delta s_{t+1} = \beta_0 + \beta_1 f_t + \epsilon_{t+1}$</td>
<td>-4.20</td>
<td>-4.74</td>
<td>-3.33</td>
</tr>
<tr>
<td>$s_{t+1} - s_{t-1} = \beta_0 + \beta_1 (f_t - s_{t-1}) + \epsilon_{t+1}$</td>
<td>0.94</td>
<td>1.02</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Sample: 1978.01–1990.07  
Source: estimates from McCallum (1994).

Table 11.1: Testing UIP

11.1.2 Expected Changes In The Real Exchange Rate

Combine covered interest arbitrage ($fd = i - i^*$) with the definition of the risk premium ($rp = f - s^e = fd - \Delta s^e$) to write

$$rp = i - i^* - \Delta s^e$$

Defining the real interest rates $r = i - \Delta p^e$ and $r^* = i^* - \Delta p^{*e}$ implies

$$i - i^* = (r + \Delta p^e) - (r^* + \Delta p^{*e}) = (r - r^*) + (\Delta p^e - \Delta p^{*e})$$

We can therefore represent the risk premium as

$$rp = (r - r^*) - (\Delta s^e - \Delta p^e + \Delta p^{*e})$$  \hspace{1cm} (11.13)

The first term on the right of (11.13) is the real interest differential. The second term on the right is the expected change in the real exchange rate. Textbook presentation of the “monetary approach” often assume that both of these terms are zero, which implies that the risk premium is zero. Many tests for the existence of a risk premium have continued to invoke purchasing power parity, and evidence in favor of a risk premium has been interpreted as evidence of variations in $r - r^*$. However, equation (11.13) makes it clear that the presence of a risk premium may equally be due to expected changes in the real exchange rate. This
11.1. EXCESS RETURNS

Figure 11.4: Contemporaneous Spot and Forward Rates: US/UK 1973–1992

possibility was emphasized by Korajczyk (1985).

So another explanation of the forward rate bias emerges if we drop the purchasing power parity assumption (another cornerstone of the basic monetary approach). Let \( q = s + p^* - p \) be the deviation from absolute PPP. Then given covered interest parity and our definitions of the real interest rates, the risk premium can be written as

\[
 rp = r - r^* - \Delta q^e
\]

where \( \Delta q^e = \Delta s^e + \Delta p^{*e} - \Delta p^e \). So we find a risk premium whenever there is a real interest rate differential or an expected change in the real exchange rate.\(^3\) If real exchange rate

\[\begin{align*}
q - q^e &= (s - s^e) + (p^* - p^{*e}) - (p - p^e) \\
    &= (rp - fd) + \pi^{*e} - \pi^e \\
    &= (rp + i^* - i) + \pi^{*e} - \pi^e \\
    &= rp + (i^* - \pi^{*e}) - (i - \pi^e) \\
    &= rp + r^* - r
\end{align*}\]

\(^3\) Another common derivation works from the other end:

\[\begin{align*}
q - q^e &= (s - s^e) + (p^* - p^{*e}) - (p - p^e) \\
    &= (rp - fd) + \pi^{*e} - \pi^e \\
    &= (rp + i^* - i) + \pi^{*e} - \pi^e \\
    &= rp + (i^* - \pi^{*e}) - (i - \pi^e) \\
    &= rp + r^* - r
\end{align*}\]
changes are unpredictable—so that $\Delta q^e = 0$ as in the efficient markets version of PPP (7)—then the forward rate bias is evidence of a real interest rate differential. Real interest parity, on the other hand, implies that the forward rate bias is evidence that real exchange rate changes are anticipated. There is some evidence that real interest rates are not equal internationally (Mishkin, 1984). However Levine (JIMF, 1989) finds evidence that there is also a predictable component of real exchange rate changes. Thus it appears that both sources of forward exchange rate bias are operative. (Turning this around, (Huang, 1990) focuses on $\Delta q^e$: he finds that $r_p$ contributes more than real interest rate differentials to deviations in $\Delta q^e$.)

11.2 Diversification of Currency Risk

When we speak of the riskiness of an asset, we are speaking of the volatility of the control over resources that is induced by holding that asset. From the perspective of a consumer, concern focuses on how holding an asset affects the consumer’s purchasing power.

It might seem natural to view domestic assets as inherently less risky than foreign assets. From this perspective, domestic residents would demand a risk premium to hold a foreign asset. But clearly U.S. assets cannot pay a risk premium to Canadian residents at the same time that equivalent Canadian assets are paying a risk premium to U.S. residents. If a positive risk premium is paid in one direction, the risk premium must be negative in the other direction.

There are many possible sources of asset riskiness. For now we focus on currency risk. That is, we focus on how currency denomination alone affects riskiness. For example, we may think of debt issued in two different currency denominations by the U.S. government, so that the only clear difference in risk characteristics derives from the difference in currency.

So we find a risk premium

$$r_p = r - r^* - (q^e - q)$$
11.2. DIVERSIFICATION OF CURRENCY RISK

denomination.

11.2.1 Sources of Currency Risk

The basic sources of risk from currency denomination are exchange rate risk and inflation risk. Exchange rate risk is the risk of unanticipated changes in the rate at which a currency trades against other currencies. Inflation risk is the risk of unanticipated changes in the rate at which a currency trades against goods priced in that currency. For example, a Canadian holding assets denominated in U.S. dollars must face uncertainty not only about the rate at which U.S. dollars can be turned into Canadian dollars but also about the price of goods in Canadian dollars. Now for most countries exchange rates are much more variable than the price level, in which case exchange rate risk deserves the most attention. However there are exceptions, especially in countries relying heavily on monetary finance of a large fiscal deficit.

If we consider the uncovered real return from holding a foreign asset, it is

\[ r_{df} = i^* + \Delta s - \pi \]

(11.14)

So if \( \Delta s \) and \( \pi \) are highly correlated, the variance of the real return can be small—in principle, even smaller than the variance of the return on the domestic asset. Thus in countries with very unpredictable inflation rates, we can see how holding foreign assets may be less risky than holding domestic assets. This can be the basis of capital flight—capital outflows in response to increased uncertainty about domestic conditions.\(^4\) Capital flight can simply be the search for a hedge against uncertain domestic inflation.

The notion of the riskiness of an asset is a bit tricky: it always depends on the portfolio to which that asset will be added. Similarly, the risk of currency denomination cannot be considered in isolation. That is, we cannot simply select a currency and then determine its

\(^4\)Cuddington (1986) discusses the role of inflation risk in the Latin American capital flight of the 1970s and early 1980s.
riskiness. We need to know how the purchasing power of that currency is related to the purchasing power of the rest of the assets we are holding. The riskiness of holding a DEM denominated bond, say, cannot be determined without knowing its correlation with the rest of my portfolio.

We will use *correlation* as our measure of relatedness. The *correlation coefficient* between two variables is one way to characterize the tendency of these variables to move together. An asset return is positively correlated with my portfolio return if the asset tends to gain purchasing power along with my portfolio. An asset that has a high positive correlation with my portfolio is risky in the sense that buying it will increase the variance of my purchasing power. Such an asset must have a high expected rate of return for me to be interested in holding it.

In contrast, adding an asset that has a low correlation with my portfolio can reduce the variance of my purchasing power. For example, holding two equally variable assets that are completely uncorrelated will give me a portfolio with half the variability of holding either asset exclusively. When one asset declines in value, the other has no tendency to follow suit. In this case diversification “pays”, in the sense that it reduces the riskiness of my portfolio.

From the point of view of reducing risk, an asset that is negatively correlated with my portfolio is even better. In this case there is a tendency of the asset to offset declines in the value of my portfolio. That is, when the rest of my portfolio falls in value, this asset tends to rise in value. If two assets are perfectly negatively correlated, we can construct a riskless portfolio by holding equal amounts of each asset: whenever one of the assets is falling in value, the other is rising in value by an equal amount. In order to reduce the riskiness of my portfolio, I may be willing to accept an inferior rate of return on an asset in order to get its negative correlation with my portfolio rate of return.

If we look at an asset in isolation, we can determine its expected return and the variance of that return. A high variance would seem on the face of it to be risky. However we have seen that the currency risk and inflation risk of an isolated asset are not very interesting to
consider. We may be interested in holding an asset denominated in a highly variable foreign
currency if doing so reduces the variance of our portfolio rate of return. To determine
whether the asset can do this, we must consider its correlation with our current portfolio.
A low correlation offers an opportunity for diversification, and a negative correlation allows
even greater reductions in portfolio risk. We are willing to pay extra for this reduction in
risk, and the risk premium is the amount extra we pay. If adding foreign assets to our
portfolio reduces its riskiness, then the risk premium on domestic assets will be positive.

11.2.2 Optimal Diversification

Consider an investor who prefers higher average returns but lower risk. We will capture
these preferences in a utility function, which depends positively on the average return of
the investors portfolio and negatively on its variability, \( U(\mathcal{E}_r^p, \text{var}_r^p) \). We can think of
portfolio choice as a two stage procedure. First we determine the portfolio with the lowest
risk: the minimum-variance portfolio. Second, we decide how far to deviate from the
minimum-variance portfolio based on the rewards to risk bearing.

If the domestic asset is completely safe, then of course the minimum variance portfolio
does not include any risky foreign assets. For the moment, consider portfolio choice under
the additional assumption that we can treat nominal interest rates as certain. In this special
case, all uncertainty is exchange rate uncertainty. As shown in section 11.2.2, our optimal
portfolio can then be represented by (11.15).

\[
\alpha^d = -\frac{i - i^* - \Delta s^e}{RRA \text{var}_s^\Delta s} \tag{11.15}
\]

Here \( \alpha^d \) is the fraction of your portfolio that you want to allocate to foreign assets, and \( \text{var}_s^\Delta s \) is
the variance of the rate of depreciation of the domestic currency. \( RRA \) is a measure of
attitude toward risk: the more you dislike taking risks, the larger is \( RRA \), which is called
the coefficient of relative risk aversion. Finally, \( \text{var}_s^\Delta s \) is the variance of the rate of
depreciation of the spot rate.

Recall that we can write the covered interest arbitrage condition as $f_d = i - i^*$, and that we defined $r_p = f_d - \Delta s^e$. That is, the forward discount on domestic currency can be decomposed into two parts: the expected rate of depreciation of the domestic money, and the deviation of the forward rate from the expected future spot rate. The latter we call the risk premium on domestic currency. So we can use covered interest parity to write the risk premium as

$$r_p = i - i^* - \Delta s^e$$

We can therefore write our last solution for $\alpha$ as

$$\alpha^d = -\frac{r_p}{RRA \varDelta s}$$  (11.16)

Equation (11.16) says that we need a negative risk premium on the domestic currency (i.e., a positive risk premium on foreign currency) to be willing to hold any of the foreign asset. Exchange rate variance is also important of course: the higher is $\varDelta s$, the lower is $\alpha$, which just says that fewer foreign assets are held as they become riskier. Finally, attitude toward risk must also play a role. Here $RRA$ is a measure of risk aversion, i.e., of how much an investor dislikes risk. An investor with a higher $RRA$ is more risk averse and less inclined to hold the risky foreign asset.

It proves informative to turn this reasoning around. Let $\alpha$ be the outstanding proportion of foreign assets supplied in financial markets. In equilibrium, outstanding asset supplies must be willingly held: $\alpha = \alpha^d$. Treating $\alpha$ as exogenous, along with $RRA$ and $\varDelta s$, we can determine the risk premium.

$$r_p = -\alpha RRA \varDelta s$$  (11.17)

This is the risk premium that must be paid in order for the asset markets to be in equilibrium.
11.2. DIVERSIFICATION OF CURRENCY RISK

Of course if investors are risk neutral (so that \( RRA = 0 \)) then no risk premium is required. But if investors are risk averse, then a risk premium must be paid and it will vary with asset supplies. For example, as the U.S. floods international markets with dollar denominated debt, we can expect a rising risk premium to be paid on dollar assets.

This suggests that to understand variations in the risk premium we might turn to variations in asset supplies. Yet there appears to be considerable short-run variation in the risk premium, while asset accumulation is, in the short run, very small in comparison to the outstanding stocks of assets. In addition, Frankel (1986) has suggested that a 1% increase in world wealth would imply only a 0.2% per annum increase in the risk premium. This suggests that variations in asset stocks will not prove useful in explaining short-run variations in the risk premium.

Mean Variance Optimization

Let us return to our investor who prefers higher average returns but lower risk, as represented by the utility function \( U(\mathcal{E}r^p, \text{var}(r^p)) \). Domestic assets pay \( r = i - \pi \) and foreign assets pay \( r^{df} = i^* + \Delta s - \pi \) as real returns to domestic residents. The total real return on the portfolio \( r^p \) will then be a weighted average of the returns on the two assets, where the weight is just \( \alpha \) (the fraction of the portfolio allocated to foreign assets).

\[
r^p = \alpha r^{df} + (1 - \alpha) r
\]

For example if two-thirds of your portfolio is allocated to the domestic asset, then two-thirds of the portfolio rate of return is attributable to the rate of return on that asset (and the remaining one-third to the foreign asset). Therefore the expected value of the portfolio rate of return is

\[
\mathcal{E}r^p = \alpha \mathcal{E}r^{df} + (1 - \alpha) \mathcal{E}r
\]
There is another way to look at the same equations. The portfolio return can be though of as the domestic rate of return adjusted for the fraction of the portfolio in the foreign asset.

\[ r^p = r + \alpha (r^{df} - r) \]  
\[ \mathbb{E} r^p = \mathbb{E} r + \alpha (\mathbb{E} r^{df} - \mathbb{E} r) \]

(11.20)  
(11.21)

Using equations (11.18) and (11.19) along with the definition of variance, we can find the variance of the portfolio rate of return.\(^5\)

Consider how to maximize utility, which depends on the mean and variance of the portfolio rate of return. The objective is to choose \( \alpha \) to maximize utility.

\[
\max_{\alpha} U \left( \alpha \mathbb{E} r^{df} + (1 - \alpha) \mathbb{E} r, \right. \\
\left. \alpha^2 \text{var} r^{df} + 2\alpha (1 - \alpha) \text{cov} (r, r^{df}) + (1 - \alpha)^2 \text{var} r \right)
\]

(11.22)

Consider how changes in \( \alpha \) change total utility.

\[
\frac{dU}{d\alpha} = (\mathbb{E} r^{df} - \mathbb{E} r) U_1 \\
+ 2 \left( \alpha \text{var} r^{df} - (1 - \alpha) \text{var} r + (1 - 2\alpha) \text{cov} (r, r^{df}) \right) U_2
\]

(11.23)

As long as this derivative is positive, so that increasing \( \alpha \) produces and increase in utility, we want to increase alpha. If this derivative is negative, we can increase utility by reducing alpha. These considerations lead to the “first-order condition”: the requirement that \( dU / d\alpha = 0 \) at

---

\(^5\)Recall that variance measures how far you tend to be from the average: \( \text{var} r^p = (\mathbb{E} r^p - \mathbb{E} r^p)^2 \). Covariance measures the relatedness of two variables. That is, it tells us if one of the variables tends to be larger than average whenever the other is: \( \text{cov} (r, r^{df}) = \mathbb{E} (r - \mathbb{E} r)(r^{df} - \mathbb{E} r^{df}) \). Use these definitions along with (11.18) and (11.19) to produce

\[ \text{var} r^p = \alpha^2 \text{var} r^{df} + 2\alpha (1 - \alpha) \text{cov} (r, r^{df}) + (1 - \alpha)^2 \text{var} r \]
11.2. DIVERSIFICATION OF CURRENCY RISK

a maximum. We use the first-order necessary condition to produce a solution for $\alpha$.\(^6\)

$$
\alpha = \frac{(\mathbb{E}r_{df} - \mathbb{E}r) - \frac{2U_2}{U_1} (\text{var}(r, r_{df}) - \text{cov}(r, r_{df}))}{\frac{2U_2}{U_1} (\text{var}(r_{df}) + \text{var}(r) - 2\text{cov}(r, r_{df}))} \quad (11.24)
$$

Here $RRA = -\frac{2U_2}{U_1}$ (the coefficient of relative risk aversion) and $\sigma^2 = \text{var}(r_{df}) + \text{var}(r) - 2\text{cov}(r, r_{df})$. Recalling that $\mathbb{E}r_{df} - \mathbb{E}r = i^* + \Delta s_e - i = -rp$ we therefore have

$$
\alpha = -\frac{rp}{RRA \sigma^2} + \alpha_{\text{min var}} \quad (11.25)
$$

Here $\alpha_{\text{min var}} = \frac{(\text{var}(r - \text{cov}(r, r_{df}))}{\sigma^2}$ is the $\alpha$ that yields the minimum variance portfolio (Kouri 1978), so the rest can be considered the speculative portfolio share. Investors can be thought of as initially investing entirely in the minimum variance portfolio and then exchanging some of the lower return asset for some of the higher return asset. They accept some increase in risk for a higher average return. If the assets have the same expected return, they will simply hold the minimum variance portfolio.

For a moment, let us focus on the case where the covariance of the returns is zero. Diversification still reduces risk. (Compare $\alpha = 0$ to $\alpha = 1/2$ with equal variances of the two assets.) Thus there is an incentive to diversify even when one asset has a lower average return. Of course relative variance matters: ceteris paribus you wish to hold less of a more variable asset. In the extreme case when one asset has zero variance—say the domestic asset is viewed as completely safe—the other asset will be held only if it has a higher return.\(^7\)

Let us consider this case in a little detail. If the domestic asset is completely safe, then of course the minimum variance portfolio does not include any of the foreign asset. Suppose in addition we can treat nominal returns as certain, so that all uncertainty is exchange rate

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\(^6\)See problem 5.

\(^7\)Zero variability of the real return implies the domestic security is perfectly indexed to inflation or, equivalently, that inflation is perfectly predicted. However, corporations might be induced to act as if the domestic return were certain by accounting rules that emphasize nominal profits measured in the domestic currency.
uncertainty. Our optimal portfolio can then be represented by (11.26).

$$\alpha = \frac{i^\ast + \Delta s^e - i}{RRA \ var\Delta s}$$  \hspace{1cm} (11.26)

Here $RRA$ is a measure of attitude toward risk: the more you resist taking risks, the larger is $RRA$.

Recall that we can write the covered interest arbitrage condition as $fd = i - i^\ast$, and that $rp = fd - \Delta s^e$. That is, the forward discount on domestic currency can be decomposed into the expected rate of depreciation of the domestic money and the deviation of the forward rate from the expected future spot rate, which we call the risk premium on domestic currency. So we can use covered interest parity to write the risk premium as

$$rp = i - i^\ast - \Delta s^e$$

We can therefore write our last solution for $\alpha$ as

$$\alpha = \frac{-rp}{RRA \ var\Delta s}$$  \hspace{1cm} (11.27)

### 11.3 An Empirical Puzzle

Section 11.1.1 provided evidence that if foreign exchange markets are efficient then part of the forward discount on domestic currency is a risk premium. Risk averse investors will insist on a higher return before taking a position in a risky currency. Such behavior offers a potential explanation of deviations of the forward rate from the expected future spot rate.

For example, suppose we find $0 < \beta < 1$ when we estimate (11.12). Then when the Canadian dollar is selling at a forward discount, we will interpret only a fraction $\beta$ of that discount as expected depreciation of the Canadian dollar. We attribute the remaining fraction $(1 - \beta)$ of the discount to a risk premium reflecting the perceived risk of holding Canadian dollars.
11.3. AN EMPIRICAL PUZZLE

This may be clearer if we recall the covered interest parity condition, which says that when foreign exchange sells at a forward discount then the interest rate paid on foreign currency must be above the domestic interest rate. Some of this higher rate of return just offsets expected depreciation, on average, but the rest of it offsets the perceived risk of having a position in foreign exchange.

\[ i_t - i^*_t = \Delta s^e_t + r_p t \]

Probably the most famous study of the properties of spot and forward rates is that of Fama (1984). Using monthly data he showed that \( f - s_{+1} \) has a larger standard deviation than \( \Delta s_{+1} \). In this sense we can say that the current spot rate is a better predictor of the future spot rate than is the current forward rate. Further, the autocorrelations in \( \Delta s \) are close to zero, and those in \( f - s_{+1} \) are also small, but the autocorrelations in \( f - s \) are large and show a slow decay. Since \( f - \Delta s \) is the risk premium plus expected depreciation, at least one of these appears to be autocorrelated.

The forward rate appears more closely pegged to the current spot rate than the future spot rate: the standard deviation of \( f - s \) is much smaller than that of \( f - s_{+1} \). This may suggest that most of the innovation in both is due to news.

11.3.1 Algebra

To simplify notation, we will drop time subscripts as long as this will not generate confusion. Define the forward discount \( fd = f - s \), the risk premium \( rp = f - s^e = fd - \Delta s^e \) where \( \Delta s^e = s^e_{+1} - s \) is the expected rate of depreciation of the spot rate, and the forecast error \( \epsilon = s_{+1} - s^e_{+1} = \Delta s - \Delta s^e \). Then if we project (regress) \( \Delta s \) on \( fd \), we will find\(^8\)

\[ \Delta s = \hat{\beta}_0 + \hat{\beta}_1 fd + \text{residual} \]

---

\(^8\)Some empirical studies replace \( fd \) with \( i - i^* \), which should be equivalent by covered interest parity.
where

\[ \hat{\beta}_1 = \frac{\text{cov}(fd, \Delta s)}{\text{var}fd} \]

\[ \hat{\beta}_1 = \frac{\text{cov}(fd, \Delta s^e) + \text{cov}(fd, \epsilon)}{\text{var}fd} \]

The second equality follows from \( \Delta s = \Delta s^e + \epsilon \). Now note that we can write \( \Delta s^e = fd - rp \), so that \( \text{cov}(fd, \Delta s^e) = \text{cov}(fd, fd - rp) = \text{var}(fd) - \text{cov}(fd, rp) \). It follows that

\[ \hat{\beta}_1 = \frac{\text{var}(fd) - \text{cov}(fd, rp) + \text{cov}(fd, \epsilon)}{\text{var}(fd)} \]

\[ = 1 - b_{rp} - b_{re} \]

where we define\(^9\)

\[ b_{re} = -\frac{\text{cov}(fd, \epsilon)}{\text{var}(fd)} \]

\[ b_{rp} = \frac{\text{cov}(fd, rp)}{\text{var}(fd)} \] (11.28)

Note that \( b_{rp} = 0 \) if the risk premium is constant (or otherwise uncorrelated with \( fd \)). Similarly, \( b_{re} = 0 \) if there are no systematic prediction errors in the model. Most empirical work has assumed that there are no systematic prediction errors in the foreign exchange markets, that \( \epsilon \) is a white noise error term uncorrelated with anything else.\(^{10}\) In this case, \( b_{re} = 0 \) and \( \hat{\beta}_1 = 1 - b_{rp} \).

\[ \hat{\beta}_1 = 1 - \frac{\text{cov}(fd, rp)}{\text{var}(fd)} \] (11.29)

In this case, if movement in \( fd \) are not reflecting movements in \( rp \) then \( \hat{\beta}_1 = 1 \). In addition,

\(^9\)Froot and Frankel (1989) make use of \( \text{cov}(fd, rp) = \text{var}(rp) + \text{cov}(rp, \Delta s^e) \) in their definition of \( b_{rp} \).

\(^{10}\)White noise is zero mean, finite variance, and uncorrelated.
11.3. AN EMPIRICAL PUZZLE

since

\[ \text{cov}(fd, rp) = \text{cov}(fd, fd - \Delta s^e) \]
\[ = \text{var}(fd) - \text{cov}(fd, \Delta s^e) \]  \hspace{1cm} (11.30)

we also know that if \( \Delta s^e \) is constant then \( \hat{\beta}_1 = 0 \). For example, if the spot rate is believed to follow a random walk, then \( \Delta s^e = 0 \). In this case, all of the forward discount is due to a risk premium, and we should find an estimated regression coefficient of zero.

So if \( rp \) is constant, \( \hat{\beta}_1 = 1 \); but if \( \Delta s^e \) is constant, \( \hat{\beta}_1 = 0 \).\(^{11}\) The empirical results don’t support either: in fact, we generally find \( \hat{\beta}_1 < 0 \).\(^{12}\) Now this result, that \( 0 > \text{cov}(fd, \Delta s) \), implies \( 0 > \text{cov}(rp, \Delta s^e) \) since

\[ 0 > \text{cov}(fd, \Delta s) = \text{var}(\Delta s^e) + \text{cov}(rp, \Delta s^e) \geq \text{cov}(rp, \Delta s^e) \]

Further, we must have \( \text{var} rp > \text{var} \Delta s^e \) since

\[ \text{var} fd = \text{var} \Delta s^e + 2 \text{cov}(rp, \Delta s^e) + \text{var} rp \]
\[ = [\text{var} \Delta s^e + \text{cov}(rp, \Delta s^e)] + [\text{cov}(rp, \Delta s^e) + \text{var} rp] \]
\[ > 0 \text{ and } < 0 \iff > 0 \]

\(^{11}\) This can also be seen from a common alternative derivation that begins with the assumption that \( \epsilon \) is white noise to argue

\[ \hat{\beta}_1 = \frac{\text{cov}(fd, \Delta s)}{\text{var} fd} \]
\[ = \frac{\text{cov}(\Delta s^e + rp, \Delta s^e + \epsilon)}{\text{var} (\Delta s^e + rp)} \]
\[ = \frac{[\text{var} \Delta s^e + \text{cov}(rp, \Delta s^e)]}{[\text{var} \Delta s^e + 2 \text{cov}(rp, \Delta s^e) + \text{var} rp]} \]

\(^{12}\) So if you move your money with the interest differential, that will tend to pay! (Of course, it will be a risky strategy.) Note that the focus here is on short run correlations: \( i - i^* \) tends to correctly forecast long run depreciation since higher inflation generates both higher interest rates and depreciation.

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11.4 Explanations Of The Puzzle

Fama (1984) first noted the implications that $\text{cov}(rp, \Delta s^e) < 0$ and that most of the variance in the forward discount is due to variance in the risk premium, and he conjectured that the maintained rationality assumption is too strong. Another possibility noted by Isard (1988) is that central banks peg $i$ and $i^*$ so that $f - s$ relatively constant. Exogenous increments in $rp$ would then be reflected in decrements to $\Delta s^e$. More generally, as pointed out by Boyer and Adams (1988), interest elastic money supply or demand suffices for this result. This is discussed in the next section.

11.4.1 Peso Problem

Evidence of systematic forecast errors is puzzling: it suggests that financial market participants repeatedly make the same mistakes. The “peso problem” shows how even in the presence of rational expectations we may turn up such evidence. The idea is that low frequency expected events may lead to long periods of ex post excess returns (Krasker, 1980; Kaminsky, 1993). We can motivate this as agents having imperfect information about their economic environment.

During most of the 20th century, the MXP was a model currency.\textsuperscript{13} The oil crisis hit Mexico hard, however. In the late 1970s, (Krasker, 1980) observed a persistent interest rate differential in favor of the MXP, despite a fixed exchange rate. From a US perspective, we observed

$$i - i^* = f - s < 0 \quad (11.31)$$

We also observed

$$s_{t+1} = s \quad (11.32)$$

\textsuperscript{13}The nuevo peso, introduced January 1, 1993, has ISO code MXN. One MXN was worth 1.000 MXP at the time of the conversion. The word ‘nuevo’ was removed from the currency on January 1, 1996.
11.4. EXPLANATIONS OF THE PUZZLE

(because of the successfully fixed exchange rate), so we persistently have

\[ f < s_{t+1} \]  

(11.33)

In this sense, we have systematic forward rate forecast errors.

if agents know that the exchange rate is fixed so that

\[ s^e_{t+1} = s_{t+1} \]  

(11.34)

it seems we persistently have

\[ f < s^e_{t+1} \]  

(11.35)

However, following Mark, let us tell a different story. Suppose that each period there is a probability \( p \) of devaluation from \( \bar{s}_0 \) to \( \bar{s}_1 \). Then at each \( t \) prior to any devaluation we have

\[ s_{t+1} = \begin{cases} \bar{s}_1 & \text{with probability } p \\ \bar{s}_0 & \text{with probability } 1 - p \end{cases} \]  

(11.36)

This gives us a one period ahead expected value of

\[ \mathcal{E}[s_{t+1}] = p\bar{s}_1 + (1 - p)\bar{s}_0 \]  

(11.37)

The implied forecast error as long as the peg is maintained is

\[ \bar{s}_0 - \mathcal{E}_t[s_{t+1}] = p(s_0 - s_1) < 0 \]  

(11.38)

In this sense, we get a rational expectations explanation of the systematic forward rate forecast errors. Furthermore, our forecast errors are serially correlated, but they contain no information that would help us better predict the future.
Learning

Lewis (1989) pursues the peso problem logic one step further by adding learning. In this case, agents are not immediately certain whether a regime shift has taken place. (Consider for example the changes in Fed operating procedures in 1979 or 1982.) Lewis introduces these considerations into our basic monetary approach model

\[ s_t = \frac{1}{1 + \lambda} \tilde{m}_t + \frac{\lambda}{1 + \lambda} \mathbb{E}_t s_{t+1} \]  \hspace{1cm} (11.39)

Assume that fundamentals follow a random walk with drift:

\[ \tilde{m}_t = \mu_0 + \tilde{m}_{t-1} + v_t \]  \hspace{1cm} (11.40)

where \( v_t \sim N(0, \sigma_v^2) \) is white noise. Recall that (using the method of undetermined coefficients, for example) you can show that in our basic monetary model this has the solution

\[ s_t = \lambda \mu_0 + \tilde{m}_t \]  \hspace{1cm} (11.41)

Now introduce the possibility of a regime shift to \( \mu'_0 > \mu_0 \). This changes the expected future fundamentals:

\[ \mathbb{E}_t \tilde{m}_{t+1} = p \mu_0 + (1 - p) \mu'_0 + \tilde{m}_t \]  \hspace{1cm} (11.42)

Solving again (e.g., using the method of undetermined coefficients), we get

\[ s_t = \tilde{m}_t + \lambda [p_t \mu_0 + (1 - p_t) \mu'_0] \]  \hspace{1cm} (11.43)

After some manipulation, this implies

\[ \mathbb{E}_t s_{t+1} = \tilde{m}_t + (1 + \lambda) [p_t \mu_0 + (1 - p_t) \mu'_0] \]  \hspace{1cm} (11.44)
This gives us a forecast error of

\[ s_{t+1} - \mathcal{E}_t s_{t+1} = \lambda \left[ (\mu'_0 - \mu_0)(p_{t+1} - p_t) \right] + \mu'_0 + v_{t+1} - \left[ \mu_0 + (\mu'_0 - \mu_0)(1 - p_t) \right] \]  \hspace{1cm} (11.45)

### 11.4.2 Exogenous Risk Premia

In order to relax the standard monetary approach assumption of risk neutrality, we will now consider the following model due to Boyer and Adams (1988).

\[
\begin{align*}
  s & = q + p - p^* \\
  i - i^* & = f - s \\
  f & = s^e + rp \\
  h - h^* - (p - p^*) & = \phi(y - y^*) - \lambda(i - i^*)
\end{align*}
\]  \hspace{1cm} (11.46) (11.47) (11.48) (11.49)

The model is a simple version of the monetary approach model developed in chapter 3: (11.46) is absolute purchasing power parity, (11.47) is covered interest parity, (11.48) defined the risk premium \( rp \), and (11.49) is money market equilibrium. The variables \( s, p - p^*, i - i^*, f \) are endogenous. This is a fairly standard monetary approach model. However, we allow \( f \neq s^e \) so that there can be an \textit{exogenous} risk premium: \( rp \overset{\text{def}}{=} f - s^e \).\(^{14}\)

The solution procedure is largely unchanged from chapter 3. The only real change is that we will distinguish two data generating processes: one for the risk premium, and one for the remaining exogenous determinants of the exchange rate. Recall that \( rp \overset{\text{def}}{=} f - s^e \), so that

\(^{14}\)Note that we call \( rp \) the risk premium; it is the risk premium paid by domestic assets. In the literature \( rp^* \equiv - rp \) is often called the risk premium as well; it is the risk premium paid by foreign assets. To avoid confusion, just keep in mind that the risk premium is a real return differential and ask yourself: who is being paid to bear what risk? Here, \( rp \) is the expected extra cost (the premium) of buying future foreign exchange without risk. Note that—like the real interest rate—the risk premium is an ex ante concept.
\[ f - s \equiv rp + s^e - s. \] Thus relative prices can be written as

\[ p - p^* = h - h^* - \phi(y - y^*) + \lambda(i - i^*) \]
\[ = h - h^* - \phi(y - y^*) + \lambda(f - s) \]
\[ = h - h^* - \phi(y - y^*) + \lambda(rp + s^e - s) \quad (11.50) \]

Recalling that \( s = q + p - p^* \), we therefore have

\[ s = q + h - h^* - \phi(y - y^*) + \lambda(rp + s^e - s) \quad (11.51) \]

So, if we combine terms in \( s \) (and let \( \tilde{m} = h - h^* - \phi(y - y^*) \) to reduce notation),

\[ s = \frac{1}{1 + \lambda}(\tilde{m} + \lambda rp) + \frac{\lambda}{1 + \lambda}s^e \quad (11.52) \]

Equation (11.52) can be solved using the recursive substitution procedure developed in chapter 4. Adding time subscripts for clarity, we find

\[ s_t = \frac{1}{1 + \lambda} \sum_{i=0}^{\infty} \left( \frac{\lambda}{1 + \lambda} \right)^i \left( \varepsilon_t \tilde{m}_{t+i} + \lambda \varepsilon_t r_{p_{t+i}} \right) \quad (11.53) \]

To explore this, suppose \( \tilde{m} = \tilde{m}_{t-1} + u_t \) where \( u_t \) and \( rp_t \sim \)White Noise. Note that we are treating \( rp \) as *exogenous*! Then

\[ s_t = \tilde{m}_t + \frac{\lambda}{1 + \lambda} rp_t \]
\[ \implies s_{t+1} = \tilde{m}_{t+1} + \frac{\lambda}{1 + \lambda} r_{p_{t+1}} \]
\[ \implies \Delta s_{t+1} = \Delta \tilde{m}_{t+1} + \frac{\lambda}{1 + \lambda} \Delta r_{p_{t+1}} \]
\[ \implies \Delta s_{t+1}^e = -\frac{\lambda}{1 + \lambda} rp_t \]
11.4. EXPLANATIONS OF THE PUZZLE

From these relations we can derive

\[ \text{cov}(\Delta s^e, rp) = -\frac{\lambda}{1 + \lambda}\text{var}rp < 0 \]  \hspace{1cm} (11.54)

\[ \text{var}\Delta s^e = \left(\frac{\lambda}{1 + \lambda}\right)^2\text{var}rp < \text{var}rp \]  \hspace{1cm} (11.55)

Keeping in mind the rational expectations hypothesis under which this analysis is conducted, so that the exchange rate forecast error is uncorrelated with any current information, we can also derive

\[ \text{cov}(\Delta s, rp) = \text{cov}(\Delta s^e + \epsilon, rp) \]
\[ = -\frac{\lambda}{1 + \lambda}\text{var}rp < 0 \]  \hspace{1cm} (11.56)

\[ \text{cov}(\Delta s, fd) = \text{cov}(\Delta s^e + \epsilon, \Delta s^e + rp) \]
\[ = \text{var}\Delta s^e + \text{cov}(\Delta s^e, rp) \]
\[ = -\frac{\lambda}{(1 + \lambda)^2}\text{var}rp < 0 \]  \hspace{1cm} (11.57)

Note that equation (11.57) explains the negative coefficient Fama (1984) and others found in the unbiasedness regressions discussed in section 11.3. In addition, Fama (1984) regressed \( fd - \Delta s \) on \( fd \). This supplies no additional information since

\[ \text{cov}(fd - \Delta s, fd) = \text{var}fd - \text{cov}(\Delta s, fd) \]

However, since \( fd(= f - s = rp + \Delta s^e) = rp/(1 + \lambda) \), treating \( rp \) as exogenous suggests that the regression

\[ fd = a + brp + v \]

should yield \( 0 < b < 1 \), and Boyer and Adams find \( \hat{\beta}_1 = .033 \).

They also note that we observe \( rp^x = fd - \Delta s = rp - \epsilon \) instead of the true risk premium,
creating an errors in variables problem.\textsuperscript{15} Fama (1984) ran $rp^e = a' + b'fd$. Boyer and Adams suggest 1) this reverses the exogeneity and 2) this ignores errors in variables. Correcting for both problems,\textsuperscript{16} Boyer and Adams find $\hat{\beta}_1 = .129$, which implies $\lambda \approx 6.8$ and a resulting interest rate elasticity between $.14 + .68$ (see their paper for the details). This estimate is consistent with other work on money demand.

### 11.4.3 Irrational Expectations

Survey data on exchange rate expectations uses the reported forecasts of participants in the foreign exchange market. Recently, such data have been increasingly exploited to address questions of the source of the rejection of unbiasedness. Surveys allow us to actually collect data on $\Delta s_{t+1}$ and $rp$. For example, Cavaglia et al. (1994) estimate the regression

$$\Delta s_{t+1}^e = \alpha + \beta fd_t + \epsilon_{t+1}$$

using survey data. Perfect substitutability between domestic and foreign assets would imply $\alpha = 0$ and $\beta = 1$. If there is a risk premium but it is zero on average, then $\alpha = 0$. They generally find evidence against perfect substitutability, but they cannot reject the risk premium being zero on average. They also find the risk premium tends to be autocorrelated, in line with some theoretical predictions.\textsuperscript{17}

Survey data allows us to consider the possibility that the problem lies in the rational expectations assumptions. Early discussions of this possibility include Bilson (1981), Longworth (1981), ?, and Fama (1984). However, economists were reluctant to accept this in-

\textsuperscript{15}That is, the ex post measure of the risk premium most often used in empirical work is

$$rp^e = f - s_{t+1} = f - (s_{t+1}^e + \epsilon) = rp - \epsilon$$

However, survey data may offer direct measurements of $rp$.

\textsuperscript{16}However, if we take their model seriously Fama’s procedure in fact gives consistent and more efficient estimates of $\lambda$ than their errors in variables procedure. This is because under the rational expectations hypothesis the forecast error is not correlated with the forward discount.

\textsuperscript{17}Nijman, Palm, and Wolff (1993) show that a certain class of models predicts that the risk premium follows a first order autoregressive process. Cavaglia et al. (1994) offer some support for this prediction.
interpretation. It seems to imply that speculators are overlooking ready profit opportunities, and economists have been inclined to view financial markets as particularly likely to efficiently exploit all available information. However, alternatives to the rational expectations hypothesis have drawn increasing attention since the dollar’s 65% real appreciation in the mid 1980s and its subsequent offsetting depreciation sent economists scrambling for possible explanatory fundamentals.\(^1\) Ito (1990) has found considerable evidence in the survey data that expectations are not rational—individuals appear to have “wishful” expectations, and their short run and long run expectations are not consistent. We will examine Froot and Frankel (1989), who also address this question using survey data on the expectations of foreign exchange traders; they find the forecast error to be correlated with the forward discount. While this may reflect learning or “peso problems”, it remains a provocative problem for the REH.\(^2\) In particular, it suggests that \textit{ex post} data do not provide a good econometric proxy for the expectations of market participants.

Recall our regression results

\[
\Delta s = \hat{\beta}_0 + \hat{\beta}_1 fd + \text{residual}
\]

\(^1\)Froot and Thaler (1990, p.185): “From late 1980 until early 1985, dollar interest rates were above foreign rates so the dollar sold at a forward discount, implying that the value of the dollar should fall. However, the dollar appreciated (more or less steadily) at a rate of about 13 percent per year. Under the risk-premium scenario, these facts would suggest that investors’ (rational) expectation of dollar appreciation was strongly positive (perhaps even the full 13 percent), but that the risk premium was also positive. Therefore, according to this view, dollar-denominated assets were perceived to be much riskier than assets denominated in other currencies, exactly the opposite of the “safe-haven” hypothesis which was frequently offered at that time as an explanation for the dollar’s strength.

The subsequent rapid fall in the value of the dollar would conversely imply a reversal in the risk premium’s sign, as investors in 1985 switched to thinking of the dollar as relatively safe. Something very dramatic must have happened to the underlying determinants of currency risk to yield such enormous swings in the dollar’s value: during the appreciation investors must have been willing to give up around 16 percent per year (13 percent from dollar appreciations plus 3 percent from an interest differential in favor of the dollar) in order to hold the “safer” foreign currency, whereas during the later depreciation phase they must have been willing to forego about 6 percent in additional annual returns (8 percent average annual depreciations minus the 2 percent average interest differential) in order to hold dollars. These premia are very large. It is hard to see how one could rely on the risk-premium interpretation alone to explain the dollar of the 1980s.

\(^2\)Lewis (1989) suggests slow learning about monetary policy can account for some of the US forward rate anomaly, but she notes that the errors seem to persist. Another problem for learning and peso problem explanations, as noted by Froot and Thaler (1990) is cross country consistency of the anomalous results.
where
\[ \hat{\beta}_1 = 1 - b_{re} - b_{rp} \]

Here we define \( b_{re} = -\frac{\text{cov}(fd, \epsilon)}{\text{var}(fd)} \) and \( b_{rp} = \frac{\text{cov}(fd, rp)}{\text{var}fd}. \)

Recall also that \( b_{rp} = 0 \) if the risk premium is constant (or otherwise uncorrelated with \( fd \)). Similarly, \( b_{re} = 0 \) if there are no systematic prediction errors in the model. With survey data, both \( b_{re} \) and \( b_{rp} \) can be estimated.

You can estimate \( b_{re} \) by regressing the forecast error on the forward discount. The rational expectations hypothesis implies that \( b_{re} = 0 \), since the forecast error should not be predictable based on current information. However survey data tend to reject this implication (Frankel and Froot, 1987; Cavaglia et al., 1994). In fact Froot and Frankel not only find support for systematic prediction errors but also find that these prediction errors are the primary source of bias in the forward discount: they could not reject \( b_{rp} = 0 \). These are the opposite conclusions of the earlier work that assumed rational expectations!

More recently, Cavaglia et al. (1994) found a significant contribution of both irrationality and the risk premium to forward discount variability.

Recall
\[ b_{rp} = \frac{\text{cov}(fd, rp)}{\text{var}fd} = \frac{\text{cov}(\Delta s^e + rp, rp)}{\text{var}(\Delta s^e + rp)} = \frac{\text{cov}(\Delta s^e, rp) + \text{var}rp}{\text{var}\Delta s^e + \text{var}rp + 2\text{cov}(\Delta s^e, rp)} \]

So \( b_{rp} > 0.5 \) iff \( \text{var}rp > \text{var}\Delta s^e \). Cavaglia et al. (1994) use this direct test of the relative variance of the risk premium and expected depreciation and find at best modest support. Note

---

\(^{20}\)Froot and Frankel (1989) make use of \( \text{cov}(fd, rp) = \text{var}(rp) + \text{cov}(rp, \Delta s^e) \) in their definition of \( b_{rp} \).

\(^{21}\)Recall that under rational expectations \( b_{re} = 0 \). This implies \( b_{rp} > 1 \) since \( \hat{\beta}_1 < 0 \) and also \( \text{cov}(fd, \Delta s^e) < 0 \) since \( 1 - b_{rp} = \frac{\text{cov}(fd, \Delta s^e)}{\text{var}fd} \).

\(^{22}\)We should allow that concluding irrationality from the predictability of the forecast error is problematic. For example, a rational allowance for a small probability of a large exchange rate movement could manifest \textit{ex post} as biased forecasts. (This is the “peso problem” of Krasker (1980).)
that unlike the test offered by Fama (1984), this test does not invoke rational expectations.

**Problems for Review**

1. What do figures 11.3 and 11.3 tell us, and what do we learn from the difference between them?

2. Use your own words to define in economic terms the risk premium in the international assets markets \( r_p \).

3. If the forward rate exceeds the expected future spot rate

   (a) domestic assets pay a risk premium.
   
   (b) foreign assets pay a risk premium.
   
   (c) covered interest parity is violated.
   
   (d) domestic and foreign assets are perfect substitutes.
   
   (e) none of the above.

4. Which are true statements about currency risk?

   (a) A foreign investor will always consider the dollar risky if its exchange rate vis-a-vis the foreign currency is uncertain.
   
   (b) The currency risk of an asset depends on the correlation of the real return on the asset with the return on the investor’s portfolio.
   
   (c) In a country with unstable monetary policy and a highly variable price level, the domestic assets may be riskier than foreign currency denominated assets.
   
   (d) b. and c.
   
   (e) all of the above
5. Check the second-order sufficient condition for the mean-variance optimization problem above. (Remember that the partial derivatives of $U$ are still functions of $\alpha$.)
Bibliography


Appendix

11.5 Quadratic Programming

Mean-variance optimization is an application of quadratic programming with equality constraints. In this section, we focus on a specialized problem: minimize $x'Ax$ subject to $Cx = b$ where $A$ is symmetric and positive definite. We form the Lagrangian

$$\mathcal{L}(x, \lambda) = x'Ax - 2\lambda'(Cx - b)$$

(11.58)

Differentiating yields

$$\frac{\partial \mathcal{L}}{\partial x} = (A + A')x - 2C'\lambda$$

(11.59)

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -2(C'x - b)$$

(11.60)

When $A$ is symmetric (as in mean-variance optimization), our first-order conditions become

$$Ax - C'\lambda = 0$$

(11.61)

$$Cx = b$$

(11.62)

or

$$\begin{bmatrix} A & C' \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ -\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

(11.63)

Recalling that in this application $A$ is symmetric positive definite, assume the leftmost matrix is invertible. (I.e., assume that $C$ has full rank: $C_{M \times K}$ has rank $M$.) Then we can produce
an inverse and solve

\[
\begin{bmatrix}
  x \\ \\
  \lambda
\end{bmatrix}
= 
\begin{bmatrix}
  A^{-1}[I - C'(CA^{-1}C')^{-1}CA^{-1}] & A^{-1}C'(CA^{-1}C')^{-1} \\
  (CA^{-1}C')^{-1}CA^{-1} & -(CA^{-1}C')^{-1}
\end{bmatrix}
\begin{bmatrix}
  0 \\ \\
  b
\end{bmatrix}
\] (11.64)

This gives us our solution for \( x \) as

\[
x = A^{-1}C'(CA^{-1}C')^{-1}b
\] (11.65)

Consider the resulting value of our objective function. (Recall that \( A \) is symmetric, and thus so is \( A^{-1} \).)

\[
x'Ax = b'(CA^{-1}C')^{-1}CA^{-1}AA^{-1}C'(CA^{-1}C')^{-1}b
= b'(CA^{-1}C')^{-1}b
\] (11.66)

Consider any other value, \( x + dx \), that also satisfies the constraint. This means

\[
C(x + dx) = b
\] (11.67)

\[
Cx + C\, dx = b
\] (11.68)

\[
C\, dx = 0
\] (11.69)

Note that

\[
(x + dx)'A(x + dx) = x'Ax + x'A\, dx + dx'A\, x + dx'A\, dx
\] (11.70)
By the symmetry of $A$

$$x' A dx + dx'A x = 2x' A dx$$

$$= b'(CA^{-1}C')^{-1}CA^{-1}A dx$$

$$= b'(CA^{-1}C')^{-1} C dx$$

$$= 0$$

So

$$(x + dx)' A(x + dx) = x'A x + dx'A dx > x'A x$$

since $A$ is positive definite. Thus we have found a minimizer.

### 11.5.1 Mean-Variance Optimization

An investor can hold any linear combination of $n$ assets, with random returns $R$. Here $R \sim (\mu, \Sigma)$ where $\mu = \mathbb{E} R$ and $\Sigma = \text{cov} R = \mathbb{E}(R - \mu)(R - \mu)'$. Note that $\text{cov} R$ is nonnegative definite; if it is also positive definite then the investor cannot eliminate all risk.

The portfolio weights are $\omega$, and the weights must sum to 1. The portfolio return is $\omega R$. The mean and variance of the portfolio return are therefore and the variance is

$$\mathbb{E}(\omega'R) = \omega' \mu$$

$$\text{var}(\omega'R) = \mathbb{E}[\omega'(R - \mu)(R - \mu)'\omega]$$

$$= \omega'\Sigma\omega$$

Assume $\Sigma$ is positive definite. Then from our work in section 11.5, we know the minimum variance portfolio is

$$\omega_0 = \Sigma^{-1} 1 (1'\Sigma^{-1}1)^{-1}$$

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To explore the efficient frontier we add a second constraint:

\[ \mathcal{E}\omega R = \mu_0 \]  

(11.76)

Let us stack the constraints as

\[
\begin{bmatrix}
1' \\
\mu'
\end{bmatrix} \omega = 
\begin{bmatrix}
1 \\
\mu_0
\end{bmatrix}
\]  

(11.77)

or \( W\omega = [1\mu_0]' \). Assuming not all mean returns are equal, we find the minimum variance portfolio to be

\[ \omega(\mu_0) = \Sigma^{-1}W(W\Sigma^{-1}W)^{-1} \begin{bmatrix} 1 \\
\mu_0
\end{bmatrix} \]  

(11.78)

Evidently this satisfies our two linear constraints. The portfolio variance is

\[ \sigma(\mu_0) = \omega'\Sigma^{-1}\omega \]

\[ = \begin{bmatrix} 1 & \mu \\ \mu & \Sigma^{-1}\omega \end{bmatrix} (W\Sigma^{-1}W)^{-1} \begin{bmatrix} 1 \\
\mu
\end{bmatrix} \]  

(11.79)

To show this is the minimum variance portfolio for this level of return, we again consider \( d\omega \) such the \( Wd\omega = 0 \). (The proof is almost identical.) Note that we end up with a variance that is quadratic in \( \mu \). Naturally the unique minimum is at the mean return of the minimum variance portfolio.

The efficient frontier minimizes portfolio variance subject to an average return constraint.

### 11.5.2 Characterizing the Data

Gather together \( T \) observations on the return on \( K \) assets into a \( T \times K \) matrix \( \tilde{X} \). Store the mean returns for the assets in \( \mu = \text{rowsum}(\tilde{X})/T \). Create the matrix \( X \) of deviations from the mean for each variable. (So \( X_{tk} = \tilde{X}_{tk} - \mu_k \).) Store the covariance matrix for the assets.
\[ [\sigma_{rk}] = \Sigma = \frac{1}{T} X^\top X \]  \hspace{1cm} (11.80)

(This is also called the variance-covariance matrix.) It is common to standardize the covariance matrix by creating the correlation matrix

\[ \rho_{rk} = \sigma_{rk} / \sqrt{\sigma_{rr} \sigma_{kk}} \]  \hspace{1cm} (11.81)

Naturally the correlation matrix has ones along its diagonal. Additionally every element is in the interval \([-1, 1]\).

The following is based on an example in Halliwell (1995). We begin with the means and covariances of three asset classes:

\[
\begin{bmatrix}
0.129 \\
0.053 \\
0.043
\end{bmatrix}
\begin{bmatrix}
0.042025 & 0.00466375 & -0.0002296 \\
0.00466375 & 0.004225 & 0.0002912 \\
-0.0002296 & 0.0002912 & 0.000784
\end{bmatrix}
\]  \hspace{1cm} (11.82)

Based on this data, we compute the minimum variance portfolio weights: [0.011, 0.098, 0.891], which produces and expected return of 0.045 (i.e., 4.5% annually). The variance of this return is 0.0007, implying a standard deviation of about 4.5%. The resulting efficient frontier is shown in Figure 11.5.

Figure 11.5: Efficient Frontier


• Frankel, Jeffrey, 1980, SEJ

• Isard, Peter, 1988


On the failure of rational expectations:


Optimal Diversification


- Dornbusch

- Niehans 7, 8, 10

- Aliber, R. Z., “The Interest Rate Parity Theorem: A Reinterpretation”, JPE Nov./Dec., 1977. (Examines the role of political risk.)


- Cumby, R., “Is It Risk? Explaining Deviations From Uncovered Interest Parity”, JME 22(2), Sep. 1988. (Empirical evidence that the consumption based asset pricing model is not an adequate model of the returns to forward speculation.)


• Schinasi, G., and P.A.V.B. Swamy, “The Out of Sample Performance of Exchange Rate Models when Coefficients are Allowed to Change”, I.F. Discussion paper #301, January 1987. (Improves on Meese and Rogoff.)

The Relationship between Spot and Forward Rates: Is There a Risk Premium?

Tests Based on Rational Expectations

We will discuss two approaches to testing the assumption of risk neutrality in efficient markets. We call a market efficient to the extent that (i) current market prices reflect market participants’ beliefs about future prices and (ii) these beliefs incorporate relevant information. The first condition requires the absence of significant market imperfections or high transactions costs. The second condition, that beliefs incorporate relevant information, is known as the rational expectations hypothesis.

In the absence of market imperfections and transactions costs, risk neutrality will ensure that the forward rate for the purchase of foreign exchange next period $f_t$ will equal the expected value of next period’s spot rate $s_{t+1}^e$. This is because risk neutrality implies that investors care only about their average rate of return, and market equilibrium must then reflect this in an equal average rate of return for all investments.

$$f_t = s_{t+1}^e \quad (83)$$

This is the assumption of a zero risk premium. We can also write this as

$$f d_t = \Delta s_{t+1}^e \quad (84)$$

---

23In the literature, the “efficient markets hypothesis” is sometimes defined to include a third element: a zero risk premium. Such a definition is not concerned with market efficiency per se. We can distinguish weak, semi-strong, and strong efficiency as $J_t$ comprises past prices, all publicly known information, or all relevant information (Fama). Since the foreign exchange market is huge, with trade volumes around forty times world trade volumes in goods and services and twenty time US GNP, we expect that it is highly liquid and at least semi-strong efficient.
Let the information set used by market participants at time $t$ be $\mathcal{I}_t$. We will represent the rational expectations hypothesis by the following assumption:

$$s^e_{t+1} = \mathcal{E}\{s_{t+1} \mid \mathcal{I}_t\}$$ \hspace{1cm} (85)

where $\mathcal{I}_t$ is the publicly available information at time $t$ and $\mathcal{E}$ is the mathematical expectation operator. This is the rational expectations assumption. Let us define the forecast error by

$$\epsilon_{t+1} = s_{t+1} - s^e_{t+1}$$ \hspace{1cm} (86)

and note that the rational expectations assumption implies that the expected forecast error is zero. We can also write

$$\epsilon_{t+1} = \Delta s_{t+1} - \Delta s^e_{t+1}$$ \hspace{1cm} (87)

which implies that on average depreciation is what it is expected to be.

**Implication 1: Unbiased Prediction**

We have seen that in the absence of a risk premium, the forward rate is an unbiased predictor of the future spot rate. Equivalently, the forward discount on domestic money is an unbiased predictor of the rate of depreciation of the spot rate. By “unbiased” we simply mean that the forecast error is zero on average. The forecast error is $\epsilon_{t+1}$, which is zero on average, in the following equations.

$$s_{t+1} = f_t + \epsilon_{t+1}$$ \hspace{1cm} (88)

$$\Delta s_{t+1} = fd_t + \epsilon_{t+1}$$ \hspace{1cm} (89)

This unbiasedness lends itself readily to empirical tests. Econometric tests are often
based on the following regression equations.

\[ s_{t+1} = \alpha + \beta f_t + \epsilon_{t+1} \]  \hfill (90)

\[ \Delta s_{t+1} = \alpha + \beta f d_t + \epsilon_{t+1} \]  \hfill (91)

Equations (90) and (91) allow for a more general empirical relationship between the spot and forward rate than equations (88) and (89). Comparing the two equations, we see that in the absence of a risk premium, we should find that \( \alpha = 0 \) and \( \beta = 1 \). Most attention has been paid to \( \beta \): estimates are generally less than one, often as low as 0.5, and it is not uncommon to find estimates near or even below zero.\(^{24}\) Estimates near zero imply that the forward rate contains no information about the future spot rate: rather than base a prediction on the forward rate, it is better to guess the exchange rate will not change. That is, the current spot rate is then the best guess for the future spot rate; the exchange rate follows a martingale.

So given rational expectations, we reject the absence of a risk premium. Of course it may be the rationality hypothesis that is the basis of this rejection. However until recently most economists have been willing to maintain the assumption of rationality and view this as evidence of a risk premium. We will have more to say about this in chapter 11.

Consider the implications of finding \( \beta = 0.5 \) in (89). Then if the U.S. dollar sells at a forward premium of 2%/year against the Canadian dollar, our best guess is that the dollar will appreciate against the Canadian dollar by 1%/year. On the other hand if \( \beta = 0 \), then the forward premium would contain no information about expected future movement of the exchange rate.

\(^{24}\)For example, Froot and Thaler (1990) report an average coefficient across several dozen studies of -0.88. Note however that survey evidence generally suggests that currencies at a forward discount are also expected to depreciate (Frankel and Froot 1987; Cavaglia et al. 1994).
Implication 2: Uncorrelated Forecast Errors

If $\mathcal{E}\{x_{t+1} \mid J_t\} = x_t$ for $t \geq 0$, then we say $x_t$ is a martingale with respect to $J_t$. Define $\epsilon_{t+1} = x_{t+1} - x_t$. Then $\epsilon_t$ has a zero unconditional mean and is serially uncorrelated: $(\mathcal{E}\epsilon_t = 0)$ and $(\mathcal{E}\epsilon_{t+n}\epsilon_t = 0)$.

Suppose your period $t$ forecast of the period $\tau$ spot rate is $\mathcal{E}\{s_\tau \mid J_t\}$. If information sets only increase, i.e., $J_t \subset J_{t+1} \subset J_{t+2} \ldots$, then your forecasts of the period $\tau$ spot rate will follow a martingale. ($\mathcal{E}\{s_\tau \mid J_t\}$ will be a martingale with respect to $J_t$ since your best guess of your next period’s best guess of a future spot rate is just your best guess of that future spot rate.) Let

$$\mathcal{E}\{s_\tau \mid J_\tau\} = s_\tau$$

so that the current spot rate is in the current information set. Then it is easy to see that non-overlapping spot rate forecast errors should be serially uncorrelated. For example, if the time $t$ one period ahead forward rate is equal to the market participants’ expectation of next period’s spot rate (so that $f_t = \mathcal{E}\{s_{t+1} \mid J_t\}$), we should find that $\epsilon_{t+2} = s_{t+2} - f_{t+1}$ is uncorrelated with $\epsilon_{t+1} = s_{t+1} - f_t$.

In the early 1980s, economists began to document serial correlation in $s_{t+1} - f_t$ (Frankel, 1980; Hansen and Hodrick, 1980). The bias in the forward rate prediction of the future spot rate attracted considerable attention for two major reasons. First, it was evidence against the joint hypothesis of no risk premium ($f_t = s_{t+1}$ due to perfect substitutability of domestic and foreign assets) and rational expectations ($s_{t+1} = \mathcal{E}\{s_{t+1} \mid J_t\}$). This joint hypothesis had been used in much of the research on the popular monetary approach to exchange rate determination. The implication was that either market participants were making persistent

\footnote{So we can write $x_{t+n} = x_t + \sum_{j=1}^{n} \epsilon_{t+j}$, and the $n$-period forecast error is $x_{t+n} - \mathcal{E}\{x_{t+n} \mid J_t\} = \epsilon_{t+n}$.

\footnote{A widely used special case is the random walk. We say that $x_t$ follows a random walk with respect to $J_t$ if $x_{t+1} - x_t = \epsilon_{t+1}$ and $\epsilon_t \sim \text{i.i.d.}(0, \sigma^2)$. If $x_{t+1} - x_t = c + \epsilon_{t+1}$ where $c \neq 0$, we say that $x_t$ follows a random walk with drift. The variance of the forecast error is then $n\sigma^2$ (which obviously increases without bound with the forecast horizon). Note that while the forecast errors $\epsilon_t$ must be uncorrelated for a martingale, they are independent for a random walk. If $\mathcal{E}\{x_{t+1} - x_t \mid J_t\} > 0(< 0)$ we say $x_t$ is a sub (super) martingale with respect to $J_t$.}
forecast errors of there was a risk premium in the market for foreign exchange. Suspicion fell on the former, since economists are naturally reluctant to give up the rationality hypothesis. The serial correlation is taken to be evidence of imperfect substitutability of domestic and foreign assets. Second, imperfect substitutability is of policy interest since it suggests that sterilized intervention can be effective and that even a small open economy under flexible exchange rates can effectively use fiscal policy. Empirically, the risk premium seems to be about 6-8% annually.\(^{27}\)

**Technical Analysis of Exchange Rates**

Most academic work on exchange rate forecasting has focussed on structural models rather than technical analysis, and this book places a corresponding emphasis on structural modeling. In practice, however, forecasters use many approaches to predicting exchange rates.

Many forecasters focus on trends in the recent behavior of exchange rates, basically ignoring the kinds of models we emphasize in this book. This approach is often called “technical analysis”, although for “chartists” the extrapolation techniques may literally involve no more than hand drawn charts. Moving average models often recommend buying a currency when its short run moving average rises above its long run moving average. Momentum models recommend buying a currency on sustained increases. This tendency to extrapolate recent trends can add instability to the foreign exchange markets.

Exchange rate predictions vary widely. For example, Frankel and Froot (1990) show that at a 6-month horizon, forecasts vary over a range averaging more than 15%. Whether traders rely on technical or fundamental analysis, they lose money on many trades.\(^{28}\) Yet to profit, traders need only a slight edge over chance in predicting spot rate movements.

However Schulmeister and Goldberg (1989) and Goodman (1979) offer some evidence that moving average and momentum models can in fact be profitable. This may explain

\(^{27}\)This conflicts with the much smaller predictions of many models: see Engel (1990).

\(^{28}\)Even when most traders lose money we may observe only profitable traders in the market if losers leave the market.
the shift in the early 1980s among foreign exchange forecasting firms from fundamental to technical analysis. However, fundamentals are generally given a role for forecasts over longer horizons (say, 6 months or more).


**On chartists and noise trading:**


Efficient Markets


• Kouri, P., 1978
We will describe the excess demand of hedgers in the spot market by

\[ T_t = \beta(S_t - S^*) \]  \hspace{1cm} (93)

Comment: if the foreign exchange market only contains hedgers, then we will have

\[ F = S(1 + i_{US})/(1 + i_{UK}) \]  \hspace{1cm} (94)

and the forward exchange market will be redundant.