FISCAL POLICY AND THE NATURAL RATE

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Abstract

Unemployment trends may depend both on the policy stance of the fiscal authority and on the current state of the economy. Key to this “unnatural rate” hypothesis is the observation that the bargaining strength of labor is affected by the economy’s unemployment history as well as the current unemployment rate. Following a suggestion of Hargreaves-Heap (1980), Cottrell (1984-85), and Coe (1988), we formalize this dependence in a dynamic Post Keynesian macromodel. Historical conditions and fiscal policy prove to be fundamental determinants of unemployment trends, and a case for fiscal activism emerges.

1 INTRODUCTION

The natural rate hypothesis of Phelps [1967], Friedman [1968], and Lucas [1972] still dominates the formulation of contemporary macroeconomic policy. Simply stated, the natural rate hypothesis predicts that unemployment trends to a level that is independent of current economic conditions and current macroeconomic policy. Despite occasional reservations expressed by mainstream theorists, this hypothesis remains part of the ‘hard core’ of mainstream macroeconomic theory. In contrast, Post Keynesian macroeconomists are inclined to view natural rate doctrine as inhibiting the conduct of sensible macroeconomic policy: belief in a natural rate encourages resignation in the face of persistent high unemployment.

This paper scrutinizes the natural rate hypothesis from a Post Keynesian perspective. Skepticism about the natural rate arises from the observation that the bargaining strength of labor depends on the economy’s unemployment history as well as the current unemployment rate. Such dependence can generate hysteresis in unemployment. Although the possibility of hysteresis in unemployment has received recent theoretical and empirical attention in the mainstream literature (Cross, 1988), the most popular theoretical models remain the insider-outsider models in the tradition of Blanchard and Summers (1986). Insider-outsider models target union behavior as the source of labor market underperformance; unions are modeled as raising wages high enough to exclude hiring beyond current employment levels. Post Keynesian writers take a different view of the likely source of hysteresis, focussing instead on the erosion of job skills and the discouraged worker effects attributable to extended unemployment. Just how such effects can generate hysteresis in unemployment is explored in this paper.

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The natural rate hypothesis currently faces a number of well known empirical challenges. The central goals of this paper are dual: to complement these recent empirics with a theoretical challenge to mainstream beliefs about the natural rate of unemployment, and to bolster the case for fiscal activism. While most economists would probably grant that almost any aggregate behavior can be squeezed out of adequately complicated models, this paper shows that an extremely simple and plausible addition to a natural rate model can undermine the natural rate hypothesis.

The model below is highly stylized, subsuming or concealing many macroeconomic issues that, although inherently important, are tangential to the current goals. Examples of issues that are not addressed include international interdependencies, learning and expectations formation, the effects of money on the real economy, the proper role of microfoundations, and the effects of the accumulation of government debt and real capital. The paper focusses on an aggregate macromodel of the goods and labor markets of a closed economy. The model is Post Keynesian in flavor. In contrast to models in the neoclassical mainstream, income distribution is treated as an important determinant of aggregate demand, the traditional expectations augmented Phillips curve is taken to be an appropriate characterization of the the influence of labor market conditions on the distribution of income, and the possibility of sustained involuntary unemployment is treated seriously.

Section 2 of this paper presents a simple, stylized macromodel that, despite its familiar Post Keynesian features, does imply a natural rate of unemployment. In Section 3, a distinction is drawn between the reference rate of unemployment (RRU) in the wage bargaining process and the concept of a natural rate of unemployment. Section 4 develops the main results of the paper, showing how endogeneity of the RRU vitiated the natural rate hypothesis. Unemployment now trends to a level that depends fundamentally on historical circumstances and on fiscal policy. Section 5 explores the conduct of fiscal policy in such circumstances, and Section 6 concludes.

2 ADJUSTMENT DYNAMICS WITH A “NATURAL” RATE OF UNEMPLOYMENT

This section introduces a Kaleckian characterization of goods market equilibrium and a Phillips curve characterization of bargaining outcomes in the struggle over income shares. Despite such Post Keynesian features, the model of this section implies the existence of a natural rate of unemployment that is unaffected by fiscal policy. This contrasts with the model developed in section 4, which suggests that hysteresis in the labor market can permit fiscal policy a sustained influence on the level of unemployment.

There is a general agreement among Post Keynesians that the distribution of income is an important determinant of the aggregate demand for goods and services. The Kaleckian model of Asimakopoulos (1975) is a popular and particularly tractable characterization of such distributional effects, and the treatment of goods market equilibrium in this paper is essentially a simple variant of the Asimakopoulos model. However in addition to the standard Kaleckian result that increases in the profit share reduce aggregate demand by shifting income toward individuals with a higher propensity to save, this paper recognizes that increased profitability can stimulate aggregate demand by increasing desired investment expenditures. Equation (1) summarizes these influences in a particularly compact fashion.

\[ U = \sigma \Pi - g \] (1)

Here \( U \) is the unemployment rate and \( \Pi \) is the profit share. The net effect of changes in the profit share is captured by the parameter \( \sigma \), where \( \sigma > 0 \) implies that unemployment is increasing in the profit share, while \( \sigma < 0 \) implies that unemployment is decreasing in the profit share. In the spirit of Bhaduri and Marglin (1990), denote these two cases ‘stagnationism’ and ‘exhilarationism’. In addition, unemployment is decreasing in autonomous demand, movements of which are captured by
the parameter $g$.\footnote{A more formal justification of equation (1) and a more precise definition of $g$ are offered in Appendix A.}

The unemployment rate affects the bargaining power of workers. Since the ability of workers to maintain or increase the wage share of national income depends on their relative bargaining strength, the unemployment rate affects wage outcomes and thereby the profit share. We capture these standard Phillips curve considerations in equation (2).\footnote{The link to the Phillips curve can be seen by noting that $-\Pi \approx \ln(1 - \Pi) = \ln(W/PA) = w - p - a$. Here $W$ is the nominal wage, $P$ is the aggregate price level, and $A$ is average labor productivity. Thus $-\Pi \approx \dot{w} - \dot{p} - \dot{a}$, an approximation that is excellent near equilibrium. Use of the Phillips curve in Post Keynesian macromodels is discussed in Skott (1989, Chapter 8) and in Isaac (1991). Note that to forestall concern about the role of expectational errors in generating the results of this paper, expectations formation has been modeled according to the rational expectations hypothesis.}

$$\ddot{\Pi} = (\dot{a} - \dot{w}_c) + \phi_1(U - RRU) \tag{2}$$

Equation (2) characterizes the dependence of the change in the profit share on the growth of productivity, $\dot{a}$, autonomous growth in real wage claims, $\dot{w}_c$, and the unemployment gap, $U - RRU$. (Lower case letters indicate natural logarithms and an overdot indicates a time derivative.) The reference rate of unemployment for the wage bargaining process, $RRU$, is just that rate which yields no anticipated real wage growth beyond the autonomous component; thus if $U = RRU$ and autonomous claims growth matches productivity growth then the profit share is stationary. This reference rate is often referred to as the non-accelerating inflation rate of unemployment (NAIRU) or natural rate. In Section 4 of this paper we do not treat $RRU$ as exogenous: the reference rate of unemployment is found to depend crucially on the history of the economy and on fiscal policy. For the moment, however, we continue to explore the model under the traditional assumption that the $RRU$ is exogenous.

A final ingredient in the characterization of the outcomes of bargaining over income shares derives from the suspicion that workers strike more successful bargains during expansions than during contractions of the aggregate economy. An extensive theoretical and empirical literature models this effect by shifting the Phillips curve when the unemployment rate is changing. This effect has been included in empirical Phillips curves at least since Lipsey (1960), and Tobin (1972) includes it in his well known theoretical summary of the Phillips curve orthodoxy. Boddy and Crotty (1974) also stress the importance of this effect for formal modeling, and more recently Flaschel and Kruger (1984) have motivated it as reflecting “endogenous aspirations”: expansions and contractions affect worker wage aspirations and thereby affects the bargaining strength of workers. In keeping with the present paper’s emphasis on modeling simplicity, equation (3) captures such endogenous aspirations by allowing increases in autonomous wage claims to exceed productivity increases when unemployment is falling.

$$\dot{w}_c = \dot{a} - \phi_2 \dot{U} \tag{3}$$

Substituting (3) into (2) yields our final characterization of bargaining outcomes in the labor market.

$$\ddot{\Pi} = \phi_1(U - RRU) + \phi_2 \dot{U} \tag{4}$$

Equations (1) and (4) are the key ingredients in our first Post Keynesian macromodel. Following the usual practice of treating the reference rate of unemployment as a constant, we can derive equation (5) by using (1) to substitute for $U$ and $\dot{U}$ in (4).

$$\ddot{\Pi} = \phi_1(\sigma \Pi - g - RRU) + \phi_2 \sigma \dot{\Pi} \tag{5}$$

Solving (5) for $\ddot{\Pi}$ yields equation (6).

$$\ddot{\Pi} = -\phi \sigma \dot{\Pi} + \phi(g + RRU) \tag{6}$$
Here $\phi = \phi_1/(\phi_2 \sigma - 1)$. The stability condition $\phi \sigma > 0$ is satisfied when $\sigma < 0$ or $\sigma \phi_2 > 1$, but $\phi \sigma < 0$ if $0 < \sigma \phi_2 < 1$. One could formulate an attack on the natural rate hypothesis based on dynamic instabilities, but—since dynamics are inherently more fragile than the attracting equilibria—this paper focuses on the stable cases. Thus, in the absence of investment shocks or fiscal policy changes, the economy converges steadily to the reference unemployment rate $\text{RRU}$ and the associated profit share $\Pi_\infty = (g + \text{RRU})/\sigma$. As the development of the model to this point has shown, the $\text{RRU}$ can be chosen as a natural rate of unemployment given an exogenous $\text{RRU}$. However, as argued in the next section, there is little reason to believe in the exogeneity of the $\text{RRU}$.

3 RECONSIDERING THE NATURAL RATE HYPOTHESIS

The neoclassical treatment of unemployment has always been problematic, and the difficulties remain evident in the contemporary mainstream literature. This literature often treats unemployment as a kind of optimal vacation or even as a definitional inconvenience. For example, Nickell (1990) considers a model in which unemployment is the gap between an exogenously given labor force and desired labor supply. On the whole, frictional unemployment in many guises appears to be the preferred mainstream explanation of unemployment. (Consider the kind reception of the “sectoral shifts” hypothesis despite the well known conflicting empirical evidence of a negative correlation between job leaving and aggregate unemployment.) Unemployment becomes the voluntary, even optimal, joblessness associated with efficient job search. Such a description is empirically deficient. The unemployed spend relatively little time in search, and there is little evidence that job search while unemployed is more effective than job search while employed.

In the standard textbook trichotomy of frictional, structural, and cyclical unemployment, frictional and structural rates of unemployment are usually taken as constituting an exogenous “natural” rate of unemployment. Unemployment trends toward this natural rate, and the effectiveness of fiscal policy is limited to—at best—short run stabilization. Operationally, the natural rate becomes the residual unemployment after fluctuations directly attributable to business cycles are averaged out.

Estimates of the natural rate reflect this thinking in a startling fashion: natural rate estimates are essentially moving averages of actual unemployment rates. Thus the 1970s and 1980s saw the accepted natural rate of unemployment rise with the actual rate of unemployment. Such a practice allows policy authorities to evade accountability: substandard job creation is eventually attributed to a shift in the natural rate of unemployment. The situation deteriorates when policy makers actually believe this story: chary of using demand management to lower unemployment below estimates of the natural rate, they are destined to constrain their policy options in ways that perpetuate existing unemployment. Through the support it has lent to a fatalistic acceptance of high unemployment, the natural rate hypothesis may be the single most destructive macroeconomic policy legacy of mainstream economics.

Empirically, the evidence against the natural rate hypothesis mounted throughout the 1970s and 1980s as the U.S. economy experienced sustained shifts in the domestic unemployment rate. Attempts at demographic rationalizations of a shifting natural rate proved unconvincing, as demonstrated by the empirical work of Summers [1986], Krashevski [1988], Coen and Hickman [1988], and others. For example, under the assumption of constant age- and sex-specific natural rates, demographic changes account for only a small part of persistent unemployment changes.

Over the same period, the natural rate hypothesis also failed to predict the sustained rises in European unemployment rates, many of which doubled or tripled after the 1960s. The European unemployment rate

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3 The characterization of goods market equilibrium in Appendix A suggests that $\sigma \Pi \approx 1$ near equilibrium. In this context, $\sigma \phi_2 > 1$ if $\phi_2$ exceeds the equilibrium profit share. Given a current (broadly defined) profit share near 0.3, $\phi_2$ is large enough if the annual percentage increase in wages is reduced by a third of a percent in the face of an annual percentage increase in the unemployment rate. Unfortunately, empirical specifications of the Phillips curve vary tremendously, and the empirical importance of this term can be debated. Readers uncomfortable with the crucial role of $\phi_2$ in ensuring stability in the stagnationist case may prefer to focus on the exhilarationist case.
experience is often described in terms of shifting natural rates of unemployment, but explanations of these shifts remain speculative and unpersuasive. Traditional labor market descriptions have also been challenged by persistent high unemployment in otherwise strong labor markets (Blanchard and Summers 1986, 1987, 1988). For example, toward the end of the 1980s Britain experienced sustained real GNP growth, low permanent and temporary layoffs, increased overtime, and real wage growth simultaneously with high unemployment rates.

An alternative to the natural rate hypothesis is the hypothesis of unemployment hysteresis. Unemployment hysteresis exists when the current unemployment rate depends on the past economic performance of the economy (Isaac, 1992). For example, Phelps (1971) speaks of the “historical residues” on the unemployment rate that might be left by an economic boom, and Cottrell (1984-85) argues that the erosion of job skills in the labor force that results from extended unemployment has a sustained impact on the labor market. Similarly, DeLong (1990) argues that an experience of sustained unemployment may hinder subsequent re-employment. The present paper defines unemployment broadly enough to include discouraged workers, so such arguments also apply to workers who drop out of a more narrowly defined labor force in the face of persistent unemployment. Workers leaving the labor force lose their influence on the wage bargains struck in the labor market, and this can help explain the possibility of strong wage performance in the presence of high unemployment.

In sum, current labor market conditions depend intimately on past labor market conditions. The next section proposes a simple modification of the theoretical model of the section 2 that can illustrate the resulting fragility of the natural rate hypothesis.

4 ENDOGENEITY OF THE “NATURAL” RATE

Section 2 showed that Phillips curve dynamics may produce a natural rate of unemployment given an exogenous RRU, even in a Post Keynesian macromodel. In order to accommodate the theoretical arguments and empirical evidence discussed above, this section adopts a formalization of the “deceleration hypothesis” of Hargreaves-Heap (1980). Hargreaves-Heap (1980) offers a stylized characterization of the response of the reference rate of unemployment to the current rate of unemployment, which is also adopted by Cottrell (1984-85) and Coe (1988). Equation (7) is a continuous time version of the discrete time formalization adopted by these authors.

\[ \dot{RRU} = \beta(U - RRU) \]

This simple addition to the model of Section 2 has profound implications for the behavior of the economy. In particular, in accordance with the empirical evidence discussed in the section 3, equation (7) weakens the links between high wage inflation and low unemployment. In another paper (Isaac, forthcoming), I explore the implications of equation (7) for the conduct of monetary policy. The present paper focuses on the case for fiscal activism.

Using the solution for unemployment from the goods market given by equation (1), equation (7) can be rewritten as

\[ \dot{RRU} = -\beta g - \beta RRU + \beta \sigma \Pi \]

For comparison we rewrite (6) here as

\[ \dot{\Pi} = \phi g + \phi RRU - \phi \sigma \Pi \]

Our model of the endogeneity of the natural rate is summarized by equations (8) and (9), a linear first order system of two differential equations in RRU and \( \Pi \). Since changes in the profit share and changes in the reference rate of unemployment both depend on the current unemployment gap, this system does not have any tendency to a unique reference rate of unemployment. This suggests that no single level of unemployment can be selected as a natural rate.

4 Tano [1991] has questioned the behavioral relevance of the distinction between unemployed workers and discouraged workers for a large part of the labor force.
It is true that—given satisfaction of the stability condition $\beta + \sigma \phi > 0$—the unemployment gap disappears over time, and it is true that—given the initial profit rate and RRU—there is some level toward which both the unemployment rate and the reference unemployment rate move over time. One may choose to call this level $(\text{RRU}_\infty)$ the natural rate of unemployment, but one will lose most of the connotations ordinarily associated with such terminology. In particular, this “natural” rate depends crucially on the history of the economy as summarized by the current profit share and reference unemployment rate. In the absence of such information, we know only that the economy tends to the levels $\Pi_\infty$ and $\text{RRU}_\infty$ such that equation (10) holds.

$$\text{RRU}_\infty = \sigma \Pi_\infty - g$$  \hspace{1cm} (10)

Equation (10) simply reflects goods market equilibrium and elimination of the unemployment gap: a relationship between RRU and $\Pi$ is determined, but not their levels. It is necessary to add information about the current state of the economy, $\Pi_o$ and $\text{RRU}_o$, in order to determine the longer term tendencies of the economy. As shown in Appendix B, the influence of the initial conditions never fades. This is apparent in the solutions (11) and (12).

$$\Pi_\infty = \Pi_o \beta / (\beta + \sigma \phi) + \text{RRU}_o \phi / (\beta + \sigma \phi) + g \phi / (\beta + \sigma \phi)$$  \hspace{1cm} (11)

$$\text{RRU}_\infty = \Pi_o \sigma \beta / (\beta + \sigma \phi) + \text{RRU}_o \sigma \phi / (\beta + \sigma \phi) - g \beta / (\beta + \sigma \phi)$$  \hspace{1cm} (12)

Thus the “natural” rate of unemployment, that rate toward which the economy tends if undisturbed, depends intimately on the fiscal policy stance as well as the historically given state of the economy. This prediction of the model might appropriately be termed an “unnatural” rate hypothesis.

5 A FISCAL EXPANSION

This section illustrates the effects of a fiscal expansion. The results of the previous section suggest that, in addition to its short term employment benefits, a more expansionary fiscal policy stance succeeds in lowering the “natural” rate of unemployment and thus engenders a permanent reduction of the unemployment rate. These effects are illustrated in Figure 1.

Let the economy initially be stationary at point $A$ with $g$ being the initial level of autonomous expenditures. A fiscal expansion, raising autonomous expenditures to $g'$, stimulates the economy and
lowers the unemployment rate. This is indicated by point B. Neither II nor RRU respond immediately, however, so point A still indicates their current levels. As in our natural rate model, the immediate effect of fiscal policy is to drive the unemployment rate below the reference rate of unemployment, but this low level of unemployment is not sustainable. As unemployment is drawn toward the reference rate along the new goods market equilibrium curve, the behavior of this economy continues to parallel the natural rate economy. In the stagnationist case, rising unemployment gives firms enough leverage to raise their profit share despite the lower overall level of unemployment. In the exhilarationist case, the effects of lower current unemployment on labor’s bargaining power dominates: firms find themselves forced to negotiate a lower profit share. In contrast to the natural rate model, however, the initial decline in unemployment generated by the fiscal expansion influences the reference rate of unemployment. RRU falls as newly employed workers acquire valuable new job skills and the bargaining power that comes with tenure. This movement is captured by the movement from point A to point C instead of, as in the model of section 2, from A to E.

The economy has no tendency to return to the pre-expansion level of employment: a fiscal expansion can permanently lower the unemployment rate. In addition, the initial position of the economy also determines the unemployment rate toward which the economy tends at point C. For example, if the economy had begun with a higher rate of unemployment, say at point D, the economy would be drawn toward point E, a position of higher unemployment than that prevailing at point C. Thus historical circumstances as well a macroeconomic policy influence the entire macroeconomic evolution of the economy, and this influence is not diminishing over time.

In the example just considered, a reversal of the fiscal policy change will undo the economy’s employment gains. However, the model is also capable of illustrating a more extreme form of hysteresis: permanent change in response to a temporary policy. Reconsider the exhilarationist economy’s response to the fiscal expansion as it moves between points A and C, and suppose the falling profit share leads to a temporary wage restraint policy such that \( \dot{\Pi} = 0 \).

Although the profit share is static, the reference rate of unemployment will continue to fall over time as the fiscal expansion sustains a lower level of unemployment. Removal of both the fiscal stimulus and the wage restraint will of course cause unemployment to rise once more, but it will now tend to a level lower than that initially prevailing at point A. A temporary policy change can therefore succeed in lowering the apparent “natural rate” of unemployment.

6 CONCLUSION

Macroeconomic policy for the last two decades has been influenced by the broad acceptance of the natural rate hypothesis among mainstream economists. Over the same period, considerable empirical evidence has accumulated against the notion of a constant natural rate of unemployment. This paper adds a theoretical argument against the natural rate, supporting the alternative hypothesis that an undisturbed economy tends to a level of unemployment that depends both on fiscal policy and the current state of the economy. This proposed dependency, which might be termed an unnatural rate hypothesis, is illustrated in a simple Kaleckian macromodel. The key ingredient of the model is a formalization of the observation that the bargaining strength of labor is affected by the economy’s unemployment history as well as the current unemployment rate. Historical conditions and macroeconomic policy prove to be fundamental determinants of the level of unemployment over any time horizon. In particular, the model implies—without invoking expectational errors—that expansionary fiscal policy can permanently lower the rate of unemployment. The case for fiscal activism has been strengthened.

\[ ^5 \text{An anonymous referee is gratefully acknowledged for suggesting this policy experiment.} \]
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APPENDIX A

The purpose of this appendix is to show that equation (1) in the text can be derived from a simple Kaleckian model of the goods market. The focus in this appendix is on the stagnationist case. This derivation can be repeated with minor changes for the exhilarationist case, but that case really begs for the introduction of important non-linearities in the investment function.6

Use the profit share, Π, to divide total real national income, $Y$, into profits, $ΠY$, and wages, $(1 − Π)Y$. Letting all saving out of national income be done by capitalists at marginal rate $s$, and ignoring taxes for simplicity, we can characterize total consumption as $(1 − sΠ)Y$. Goods market equilibrium can then be given the familiar characterization in equation (A.1).

$$Y = G + I + (1 − sΠ)Y$$  \hspace{1cm} (A.1)

Here $G$ is all autonomous expenditure on final goods and services and $I$ is the induced component of gross private domestic investment. The proper characterization of the determinants of investment expenditure remains controversial, but in addition to the basic accelerator mechanism the Post Keynesian tradition tends to emphasize the positive influence of profits. (Exogenous movements in expectations

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6It is worth noting that such non-linearities are implicit in the treatment of exhilarationism offered by Bhaduri and Marglin (1990). To see that, note that their equation (10) cannot imply exhilarationism for a linear investment function, given the restriction in their inequality (11).
are also recognized as being important but are not emphasized in this paper.) A very simple formal representation of these influences is \( I = \iota \Pi Y \), where the parameter \( \iota \) is the sensitivity of desired investment to profits. Solving for equilibrium national income, we find that it depends on autonomous expenditures and the profit share.

\[ Y = G/(s-\iota)\Pi \]  
(A.2)

The formulation in (A.2) allows ready comparison with the standard Keynesian result, \( Y = G/(s-\iota) \), which arises when saving and investment are functions of income directly rather than through its influence on profits. This comparison makes it clear that the influence of income distribution on aggregate demand in the Kaleckian formulation is absolutely crucial to the dynamics considered in this paper.

For simplicity, we basically follow Asimakopulos in treating government expenditure as autonomous over the model’s horizon; but since we allow for some labor force and productivity growth over the model horizon, we take \( G/Y_f \) as fixed rather than \( G \).\(^7\)

Here \( Y_f \) is the full employment level of national income. Rewrite equation (A.2) as equation (A.3).

\[ Y/Y_f = G/\kappa(s-\iota)\Pi \]  
(A.3)

Here \( G = \kappa G/Y_f \), and \( \kappa \) is a constant chosen so that \( \kappa(s-\iota)\Pi \approx 1 \) near equilibrium. Equation (A.3) suggests a simple logarithmic expression of goods market equilibrium.

\[ y - y_f = g - \sigma \Pi \]  
(A.4)

Here \( y - y_f = \ln(Y/Y_f) \) is the GNP gap, \( \sigma = \kappa(s-\iota) \), and \( g = \ln(G) - 1 \).\(^8\) We use a simple version of Okun’s law to relate the unemployment rate, \( U \), to the GNP gap.\(^9\)

\[ U = y_f - y \]  
(A.5)

Equation (A.5) can be combined with (A.4) to get equation (1) in the text. Also, the definitions of \( \kappa \) and \( \sigma \) given above verify the claim in footnote 3 of the text.

**APPENDIX B**

The first order system developed in the text

\[ \dot{RRU} = -\beta g - \beta RRU + \beta \sigma \Pi \]  
(B.1)

\[ \dot{\Pi} = \phi g + \phi RRU - \phi \sigma \Pi \]  
(B.2)

has two characteristic roots, 0 and \(- (\beta + \phi \sigma) \). Since the zero root corresponds to a constant in the solution, the system does indeed converge to stationary levels of RRU and \( \Pi \) over time. Unlike more familiar steady state solutions, however, the dependence of the system on the initial values of RRU

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\(^7\)That is, autonomous expenditure must appropriately reflect the size of the economy. Note that adding an induced component to \( G \) does not qualitatively affect the results in this paper.

\(^8\)Since we ignore the effects of the accumulation of government debt and of real capital, the model is strictly a short run business cycles model rather than a long run growth model. However, the basic argument of Section 4 can easily be adapted to a long run growth setting.

\(^9\)We have used the fact that \( \ln(1 + x) \approx x \) to approximate \( \ln[\kappa(s-\iota)\Pi] \) as \( \kappa(s-\iota)\Pi - 1 \). Recall that the constant \( \kappa \) was chosen so that this approximation is a good one.

\(^{10}\)This simplified formulation of Okun’s Law is readily motivated given a fixed level of labor productivity. Let \( A \) be labor productivity measured as output per labor hour, and let \( N_f \) be the total number of labor hours available for employment in the economy. We can then define \( Y_f = AN_f \). Actual worker hours are \( N = Y/A \), expressed logarithmically as \( n = y - a \), so \( -U = [(N - N_f)/N_f] = [(Y - Y_f)/Y_f] \approx y - y_f \) since \( \ln(1 + x) \approx x \). This gives us equation (A.5).
and $\Pi$ does not disappear. To see this, it is convenient to examine an explicit solution for the system given initial values $RRU_o$ and $\Pi_o$. First note that any values that satisfy the relationship

$$RRU = \sigma \Pi - g$$  \hspace{1cm} (B.3)

are a solution to our differential equation system. Choose any such pair of constants, $RRU^p$ and $\Pi^p$, as a particular solution to the system. We can then use any standard technique to find a general solution to the system. For example the adjoint matrix technique, as found in Murata (1977, p.70), suggests writing the general solution in the form

$$RRU_t = -\sigma\phi k_1 + \beta k_2 e^{-(\beta + \sigma\phi)t} + RRU^p$$  \hspace{1cm} (B.4)

$$\Pi_t = -\phi k_1 - \phi k_2 e^{-(\beta + \sigma\phi)t} + \Pi^p$$  \hspace{1cm} (B.5)

where $k_1$ and $k_2$ are constants to be determined by the given initial values $RRU_o$ and $\Pi_o$. Note that the second term in each solution gradually loses influence on the solution. We can therefore write

$$RRU_\infty = -\sigma\phi k_1 + RRU^p$$  \hspace{1cm} (B.6)

$$\Pi_\infty = \beta k_1 + \Pi^p$$  \hspace{1cm} (B.7)

This is the first suggestion that the initial state of the system will always matter, for $k_1$ generally depends on the initial state. Noting that (B.4) and (B.5) must be satisfied at $t = 0$, we can exhibit this in more detail by solving for $k_1$ and $k_2$ from

$$\Pi_o = -\phi k_1 - \phi k_2 + \Pi^p$$  \hspace{1cm} (B.8)

$$RRU_o = -\sigma\phi k_1 + \beta k_2 + RRU^p$$  \hspace{1cm} (B.9)

Substitute the solution for $k_1$

$$k_1 = (\Pi^p - \Pi_o)\beta/\phi(\beta + \sigma\phi) + (RRU^p - RRU_o)/(\beta + \sigma\phi)$$  \hspace{1cm} (B.10)

into (B.6) and (B.7) to get

$$RRU_\infty = (\Pi_o - \Pi^p)\sigma\beta/(\beta + \sigma\phi) + RRU_o\sigma\phi/(\beta + \sigma\phi) + RRU^p\beta/(\beta + \sigma\phi)$$  \hspace{1cm} (B.11)

$$\Pi_\infty = \Pi_o\beta/(\beta + \sigma\phi) + \Pi^p\sigma\phi/(\beta + \sigma\phi) + (RRU_o - RRU^p)\phi/(\beta + \sigma\phi)$$  \hspace{1cm} (B.12)

validating our suspicion that the influence of the given initial values $RRU_o$ and $\Pi_o$ is not eliminated over time. A remaining puzzle is the apparent influence of our particular solution, given the arbitrariness of its selection. Recall, however, that $RRU^p$ and $\Pi^p$ are intimately linked according to (B.3). This allows expression of $RRU_\infty$ and $\Pi_\infty$ without reference to the particular solutions.

$$RRU_\infty = \Pi_o\sigma\beta/(\beta + \sigma\phi) + RRU_o\sigma\phi/(\beta + \sigma\phi) - g\beta/(\beta + \sigma\phi)$$  \hspace{1cm} (B.13)

$$\Pi_\infty = \Pi_o\beta/(\beta + \sigma\phi) + RRU_o\phi/(\beta + \sigma\phi) + g\phi/(\beta + \sigma\phi)$$  \hspace{1cm} (B.14)

This is not surprising: any arbitrariness allowed in the choice of a particular solution does not introduce arbitrariness in the definite solution. Again, the solution supports the unnatural rate hypothesis that the economy tends to a level of unemployment that depends both on macroeconomic policy and the current state of the economy.