The Structure of Neoclassical Consumer Theory

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This paper offers a relatively non-technical introductory survey of the application of abstract choice theory to consumer theory. In the process, the paper identifies—somewhat more carefully than is usual in the literature—the relatively small role of rationality postulates and the relatively large role of *ad hoc* assumptions that are invoked to produce the minimal structure of neoclassical consumer theory. (For choice under certainty, this structure is that Marshallian demand obeys the weak axiom of revealed preference in budget data.) Finally, this paper serves as a reminder that economists should abandon the behaviorist rhetoric and aspirations that they have repeatedly, but incorrectly, associated with abstract choice theory.

1 Introduction

Microeconomics fundamentally concerns choices: their structure, and their implications. Firms are characterized as choosing what to produce and how to produce it. The structure imposed on the firm by neoclassical economics is that these choices be profit maximizing. Consumers are characterized as choosing what to consume and how to allocate their time. The structure imposed on these choices by neoclassical economics is roughly that they be utility maximizing.[1] Abstract choice theory studies such structures.

Since neoclassical economics imposes structure on choice, two natural questions arise. The first is whether the imposed structures are methodologically justifiable. For example, underpinning a theory with structure that simply embodies our understanding of the nature of preference, or possibly our core notions of rationality, might be viewed as methodologically reasonable. The second question is whether these structures are empirically supported. The empirical standing of the structure might be expected to reflect on its methodological acceptability, although this is controversial for neoclassical economists. Alternatively, if economists give these structures the status of axioms, the second question may never arise. For example, the applied economist analyzing consumer demand behavior might be satisfied with empirical techniques that simply do not allow the data to suggest violations of the theory's *a priori* structure. (Note that when the axioms of consumer theory are treated as empirically contestable, violations are common.)

Abstract choice theory bears on both questions. Instead of starting with the *a priori* structures assumed by the neoclassical paradigm, abstract choice theory begins with a relatively unstructured characterization of choice. We can then develop some necessary and sufficient conditions for moving from this unstructured characterization to the structured characterizations of neoclassical economics. This proves extremely informative. We learn what structure is required to turn an abstract actor into a neoclassical actor on the microeconomic stage.

Since this paper introduces abstract choice theory in terms of its relationship to neoclassical consumer theory, we should note that abstract choice theory is something of a misnomer: it is not a theory of anything. Rather it is a set-theoretic vocabulary for discussing theories of choice. There is a single neoclassical theory of the consumer: the theory of preference-based choice. This theory is generally presented in terms of three basic modeling strategies: utility "theory," binary-preference "theory," and abstract choice "theory." Most economists are familiar with the first two strategies. This paper explores some advantages of the third.[2]

1.1 Behavior and Preference

Neoclassical economics treats as primitive the concepts of action, state, and outcome. Call these the ontological primitives. It also generally treats as primitive two intentional states: belief and desire (Jeffrey 1983). Call these the intentional primitives. Some economists have proposed that economics might do without the intentional primitives, and abstract choice theory has been their primary vehicle for exploration of this possibility.

Traditionally, however, the intentional primitives are treated as unproblematic. Preferences over the alternative actions are characterized as induced by desires over the possible outcomes along with beliefs about their likelihood. Rational action is characterized as preferential. That is, rational action involves a special relationship between the intentional primitives and the ontological primitives.

> To say of an action that it is rational is to put forward an empirical hypothesis and a critical appraisal. The hypothesis is to the effect that the action was done for certain reasons, that it can be *explained* as having been motivated by them; these reasons will include certain ends the agent sought to attain, and his beliefs about available means of attaining them. And the *critical appraisal* implied by the attribution of rationality is to the effect that, judged in light of the agent's beliefs, his action constituted a reasonable or appropriate choice of means for the attainment of his ends. (Hempel 1962, p.5)

[1] A bit more precisely, in treating choice under certainty, a minimal standard set of requirements is that consumer demand be characterizable in terms of complete, transitive, continuous, locally insatiable preferences.

[2] Since we will focus on consumer theory, we will neglect one important advantage of abstract choice theory: its use to expose common structures across different theories of choice, such as consumer theory, the theory of the firm, and social choice theory.

Most contemporary economists are accustomed to characterizing rational consumer behavior as "utility" maximizing. This characterization has its roots in the notion that a bundle of goods has an intrinsic utility, or an intrinsic ability to produce welfare, which has been traced to Gossen (1854), Jevons (1871), and Edgeworth (1881). However the meaning of 'utility' became problematic when Pareto (1909) noted a measurability problem with the concept: demand behavior is invariant to monotonically increasing transformations of the utility function. Cardinal utility had been dealt a crippling blow; ordinal utility rose to take its place (Hicks and Allen 1934).

As Wong (1978, p.29) notes, cardinal utility was abandoned in favor of ordinal utility not on the grounds that it was a psychological concept but rather because the cardinality had no observable correlates. This stress on observability soon led economists to argue that there was no need to make any reference to the notion of utility at all (Samuelson 1938; Little 1949; Houthakker 1950). They saw reference to things psychological as unnecessarily burdening economic science, and they saw the nascent abstract choice theory—which ironically soon became known as revealed preference theory—as dispensing with such unobservables.

This notion still persists. For example, Mas-Colell, Whinston, and Green (1995, p.5) suggest that "[the choice-based approach] makes assumptions about objects that are directly observable (choice behavior), rather than about things that are not (preferences). Perhaps most importantly, it makes clear that the theory of individual decision making need not be based on a process of introspection but can be given an entirely behavioral foundation." This is a claim that abstract choice theory fully reconciles economics and behaviorism. Such claims are ill-founded, as we will see. Nevertheless there is some justification for hoping that abstract choice theory can aid economists who wish to develop the "explanation sketches" of neoclassical economics into full-blown explanations worthy of an empirical science (Hempel 1942).

2 Choice

Abstract choice theory characterizes the structure in the selection procedure which an actor uses to select one alternative from a menu in a given choice situation.[3]

Definition 1 (Menu) *A* menu *is an non-empty, exhaustive listing of the mutually-exclusive available alternatives to which a selection procedure is applied.*

In consumer theory the actor is called the *consumer* and menu is called the *budget set*, which is the set of feasible consumption bundles. It is often claimed that abstract choice theory offers an alternative to the preference-based model of choice that is standard in consumer theory, where the "primitive" becomes the choice function rather than preferences.[4] This is extremely misleading. Consumer theory is a theory of preference-based choice. Abstract choice theory is simply a convenient way to model the structure that ("rational") preference imposes on the relationship between choices across a variety of choice situations.

The kernel of truth in the claim is that abstract choice theory encourages us to begin talking about preference-based choice without presuming that preferences are invariant to the choice situation. For example, standard textbook treatments simply assume that the consumer has fixed preferences that are well represented in any choice situation by a context-independent weak order on the entire choice space. Abstract choice theory reminds us to focus on the particular choice situation faced by an actor, which in turn reminds us that to assume that rankings of alternatives in a particular menu will persist in another menu is to impose structure on consumer choice.

[3] Although we may talk of the actor "using" a selection procedure, at the formal level the actor often simply "is" the selection procedure. Since the abstract actor is fundamentally a selection procedure, she may be a consumer, a household, a group, a firm, or an organization. McFadden and Richter (1990, p.164) even allow a mechanical device. However abstract choice theory is commonly introduced as part of a foundational investigation in consumer theory or social choice theory.

[4] For example, McClennen (1990, p.21) suggests that abstract choice theory "takes the concept of an abstract choice set as the primitive and simply defines preference in terms of it." Mas-Colell, Whinston, and Green (1995, p.9) suggest that "choice behavior itself is taken to be the primitive object of the theory", where choice behavior is "represented" by an abstract choice rule. Kreps (1990, pp.26–27) is more careful: "that the consumer has preferences that explain choice ... is our *model* of her behavior, and it would make more sense to take her behavior as primitive and ask, When are her choices consistent with our preference-based model of choice? ... The approach is not entirely satisfactory because of the primitive we use; we assume we have our hands on the consumer's entire *choice function*."

Consider a consumer facing a menu, A.[5] The consumer judges a subset of alternatives (perhaps a single element) to be acceptable; all other alternatives are "rejected". We call this *choosing*, and the set of acceptable alternatives is called the choice set.

Definition 2 (Choice Set) *The* choice set *from a given menu is the subset of the available alternatives that the consumer does not reject.*

So some alternatives in the menu are judged acceptable, while some are rejected—perhaps because the consumer judges them to be "unacceptable". Two questions for consumer theory arise immediately: are such judgments corrigible, and does the partition into acceptable and unacceptable alternatives exhaust the menu? The evident answers to these questions—that all judgments are corrigible and that some alternatives will not have been judged—are not the answers given by neoclassical consumer theory. Rather the consumer incorrigibly judges all consumption bundles in a budget set, accepts only the "best" of these, and rejects the rest. We will return to this shortly.

Choosing is the first stage in the application of the consumer's selection procedure. Selection procedures generally involve a second stage as well, which we will call *picking*. The consumer must select a single alternative from A, but more than one alternative may be "acceptable". Picking is the process of selecting one alternative from the choice set. Like the neoclassical theories of the consumer and the firm, abstract choice theory has much to say about choosing and little to say about picking. For example, if a consumer has two "best" bundles available in her budget set, economic theory has nothing to say about how this "indifference" is resolved into a behavioral outcome (Ullmann-Margalit and Morgenbesser 1977). A consumer picks an alternative when she has no reason on which to base a choice among some alternatives.[6]

Furthermore, as Armstrong (1950) stresses, the distinction between picking and choosing immediately enmeshes the economist in references to mental activity, since only a single pick from possibly many acceptable alternatives can be observed as a behavioral outcome. However, the hope remains that we might be able to deduce *properties* of the selection procedure, which mediates between choice situations and outcomes, without worrying about the *psychology* of the selection process. If this hope were realized, applied economists might simply search for empirical regularities in the relationship between choice situations and outcomes, regularities that are causally attributed to the consumer without invoking any psychological speculations. While it is true that economic theory refers to choice sets, which are generally not observed, this would become purely a taxonomic convenience for the applied economist who is searching for structural regularities in the relationship between choice situations and outcomes.

For any particular choice situation, it is a natural to speak of accepted alternatives as ranked above rejected alternatives. In the abstract theory of choice, we are free to do this as long as we are not thereby introducing a primitive notion of ranking. In particular, it must not be taken to imply that the apparent ranking would also appear in a different choice situation. Such persistence across choice situations is a *structure* that the concept of choice does not immediately imply. Similarly, for any particular choice situation, one may find it natural to speak of the consumer as indifferent among the acceptable alternatives. Again this raises no problems as long as we do not thereby surreptitiously introduce some primitive notion of indifference, especially one that would apply across choice situations. To assume common rankings across choice situations is to impose structure on the selection procedure. A key motivation for the study of abstract choice theory is that it forces us to be as explicit as possible about the points at which we introduce such structure.

Consideration of the nature of choice situations raises many difficulties for efforts to abandon reference to mental processes. For example, there may be influences on choice besides the set of available alternatives. We will call these influences the *context*.[7]

[5] For the moment, think of the menu simply as the set of alternatives *percieved* by the consumer to be available. We will return to this below.

[6] Ignoring an implied regress in the concept of indifference, we might say a consumer picks from a set when she is indifferent to the mechanism that selects one member of the set.

[7] We might also call the context the 'choice setting' or 'environment'. The set of available alternatives may depend on the context. For example, in neoclassical consumer theory, prices determine the set of available alternatives (the budget set), but we can imagine relative prices having additional effects on choice (e.g., through conspicuous consumption or relative wage effects). Since the idea of context is extremely general, it can in principle accommodate dependence of choices on the time of

Definition 3 (Context) *The* context *of a choice situation comprises the values of a set of variables that influence the choice set from the given menu.*

The context and the menu together constitute the choice situation.

Definition 4 (Choice Situation) *A* choice situation *consists of a menu and a context.*

Where does the knowledge of the choice situation reside? This is a serious issue for the empirical content of economic theory. Even the element picked is observed only in some weak sense: we do not know the consumer's perceptions of and beliefs concerning the "observed" choice. Yet these perceptions and beliefs are presumably a basis of the choice behavior. This raises the question once again: might abstract choice theory allow us to ignore the psychological basis of choice behavior and focus just on alternatives and choices as characterized by the applied economist?[8]

While abstract choice theory is an extremely general framework, most economists are interested in relatively concrete applications. For example, consumer theory characterizes the structure in the consumer's economic behavior in different choice situations, i.e., in her choices. Neoclassical utility theory implies that the consumer's choices in different choice situations share some common structure. Abstract choice theory models these regularities by imposing structure on a more general choice theoretic framework.

For example, neoclassical utility theory treats choice as determined by specific aspects of the choice situation. We begin to impose this structure by assuming that the consumer's choices depend only on the menu and the context.

Assumption 1 (Stability) *The consumer's choices can be characterized by a function, $C(\cdot)$, which depends only on the choice situation.*[9]

Given stability, the consumer has a unique choice set for each choice situation. In the abstract this assumption may seem plausible, but once we move to empirical characterizations of choice situations it will prove quite strong. For example, it rules out the consumer dealing with a large number of alternatives by randomly assessing only a few of them. It also rules out revised judgments about the acceptability of alternatives.

Furthermore, there is a conceptual anomaly here: stable choice seems to embody the idea of repeated observations, which inherently brings in the notion of time. If we accommodate this by simply including the date t as part of the context of the choice situation, then we abandon our search for structures that persist across choice situations. This implies that economists must invoke some kind of time independence, limiting the extent to which the past can condition the present. The neoclassical consumer who maximizes lifetime utility chooses a single lifetime consumption bundle, makes a single choice, and gives us a single observation. Notions of consistency across choices become vacuous (Pollak 1990; Anand 1990). Empirical work on the theory of the consumer therefore always embroiders the neoclassical interpretive framework with some kind of dynamic stability and temporal separability.

Some economists would consider even the stability assumption too strong if choice situations are to be characterized in terms of observables. For example, Austrian and Post Keynesian economists are inclined to stress a creative element in choice, including spontaneity in the generation of the perceived available alternatives (Shackle 1958). This is obviously a problem for empirical work, which requires observability. It is a problem, however, that most economists dismiss by assumption.

Assumption 2 (Verifiable Menus) *In any given choice situation, the economist can fully characterize the menu faced by the consumer.*

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day, the framing of a choice situation, or past selections. Context thus encompasses the "hereditary influences" introduced by Georgescu-Roegen (1950) to rescue the constancy of economic laws. Related to this, context includes any "reference level" of consumption or utility (Rabin 1998; Helson 1964).

[8] This question is not asked at the theoretical level, where the answer is clearly "no". (Consumer theory is preference based.) Rather it is asked at the level of applied work, where a simple answer is much trickier to give.

[9] The domain and range of this function are discussed below. Since the choice situation includes an exhaustive list of the mutually-exclusive available alternatives, the range is implicitly subsets of the menu.

2.1 The Domain of Choice

Suppose we can specify a domain, $\mathcal{D}$, which is a collection of choice situations for which choice depends only on the choice situation. (Recall that by definition a choice situation includes a non-empty set of alternatives.) Then given a menu A and a context e, the consumers choice can be represented $C(A, e)$.

Definition 5 (Choice Function) *We call $C(A, e)$ a choice function over domain $\mathcal{D}$ if $C(A, e) \subseteq A$ and $C(A, e) \neq \emptyset$ for all $(A, e) \in \mathcal{D}$.*

That is, a choice function selects a non-empty subset of any (non-empty) menu (in its domain).[10] For example, the choice function might be the Marshallian demand correspondence with domain equal to the set of budget sets. (In this case, there is no direct dependence on the context.)

Recall that the *choice set* $C(A, e)$ can have more than one element, although in an actual choice situation we will generally observe only a single choice. E.g., suppose $A = \{x, x', x''\}$ and $C(\{x, x', x''\}, e) = \{x, x'\}$. (We will have more to say about this later.) $C(A, e)$ is the subset of elements of A any one of which the consumer might pick in an actual choice situation.

Often it is convenient to refer an exhaustive list of potential selections.

Definition 6 (Choice Space) *The* choice space*, a set X, is the union of the sets of alternatives in the domain.*[11]

3 Preference

If we say no more than that the consumer is characterized by a choice function with some domain, then we have a single prediction in our consumer theory: some choice set is selected from any menu faced by the consumer. If we wish to say more than this, we need to impose more structure on the choice function. A key question is then, What is the source of this structure?

In consumer theory, economists have traditionally looked for structure in the notions of preference and rationality. Specifically, the standard model of the consumer is that the consumer has *preferences* that explain her choices. For example, in consumer theory, observed choices are generally characterized as manifesting the preferences of a rational consumer.

Note how problematic this is for any behaviorist project. In principle the behaviorist project would be simply to discover structural regularities in the relationship between choice situations, as characterized by the economist, and outcomes, as observed by the economist.

> Sidestepping all considerations about whether the decision-maker possesses a mind (it might be a committee, a mechanical device, a random device, etc.), we may simply ask whether its choice behavior could have been generated by "maximizing" a single preference ... Indeed, we may wish to know ... whether it can be rationalized by a particular kind of preference, for example by a transitive, or a total preference. (McFadden and Richter 1990, p.162)

However, this begs the question of why any regularity is even suspected. And this is quite obvious in the literature that applies abstract choice theory to consumer theory: the structures that are proposed for the choice function universally rely on intuitions about preferential choice.[12] We therefore adopt for consumer theory an ontological assumption.

[10] This definition of 'choice function' is not universal. For example, Kreps (1990, p.27) and McClennen (1990, p.23) define a choice function as having as its domain the set of all non-empty subsets of the choice space X. Kreps also allows $\emptyset$ in the range, although he then adds his Assumption 3 to exclude that possibility.

[11] Alternatively we can treat the choice space as primitive, and construct sets of alternatives as subsets of the choice space. For example, neoclassical consumer theory often begins with the choice space defined as a K-dimensional Euclidian space ($\Re^K_+$) and then constructs sets of alternatives (budget sets) in this space.

[12] Sen (1973, p.244) recognizes this explicitly: "The rationale of the revealed preference approach lies in this assumption of revelation and not in doing away with the notion of underlying preferences, despite occasional noises to the contrary." Wong (1978) perspicaciously documents this reliance in the early revealed preference literature. Suzumura (1983, p.22) takes the same stand: "If the choice behavior of an agent is guided systematically by some underlying preferences, that fact will infallibly reveal itself in his actual choices, so that by observing his choices under alternative specifications of environmental conditions, we may possibly reconstruct his underlying preferences." (Note that, in contrast to usage in the present paper, Suzumura's reference to alternative environmental conditions refers to no more than menu variations.)

Assumption 3 (Diachronically Persistent Preferences) *Consumers have preferences, which persist over time.*

Preference and rationality are notions that prove rather intimately intertwined. However, Herzberger (1973, part 2) suggests that three of Nozick's (1963, III.2, p.85) "conditions of sanity" offer a natural boundary between the concept of preference and that of rational preference. The conditions of sanity are:

- N1. Indifference is symmetric.
- N2. Strict preference is asymmetric.
- N3. Strict preference and indifference are disjoint relations.

For example, if someone tells us that they strictly prefer x to y, and that they are indifferent between x and y, this seems to be a misuse of language rather than a violation of rationality. (As conditions of sanity, these are intended as synchronic (point of time) characterizations.)

3.1 Are Preferences Binary?

Nozick's terminology derives from the literature on binary relations. Characterizations of preference and indifference typically invoke the notion of pairwise comparisons. Pairwise comparisons in turn are the natural territory of binary relations.

Definition 7 (Binary Relation) *A binary relation R on a set X is a set of* ordered *pairs drawn from X.*[13] *A binary relation is called "symmetric" if $x\,R\,y \implies y\,R\,x$. A binary relation is called "asymmetric" if $x\,R\,y \implies \neg y\,R\,x$.*

Example 1 *The universal binary relation on a set X is the set of all ordered pairs that can be constructed from X. We can write the universal binary relation as $\{(x, x') \mid x, x' \in X\}$ or equivalently as $X \otimes X$. The universal binary relation is symmetric.*

The concept of preference lends itself readily to characterization in terms of binary relations. For example, let $x, y \in A$ be two elements of menu A in a specific choice situation (A, e). Consider a judgment applying to these two alternatives. Let's write $x\,P\,y$ when x is judged strictly preferred to y and $x\,I\,y$ when x is judged indifferent to y. Our three conditions of sanity might then be interpreted as follows.

- N1'. $x\,I\,y \implies y\,I\,x$
- N2'. $x\,P\,y \implies \neg y\,P\,x$
- N3'. $x\,P\,y \implies \neg x\,I\,y$

The first of these is a bit odd, since ordinary language does not order the concept of indifference. That is, we do not usually draw a distinction between saying x is judged indifferent to y and saying y is judged indifferent to x. But in this sense it can be considered a reflection of this use of language when alternatives are ordered.

Now it is clear that N1', N2', and N3' cannot be used to *define* preference, for as noted by Herzberger (1973, p.193), these three conditions are satisfied by the symmetric and asymmetric parts (i.e., subrelations) of any binary relation (see below).[14] So a concept of preference is presupposed. In some sense, we are treating these three conditions as deducible from the concept of preference. Do notions of rationality clearly fall on the other side of this boundary? If not, even these three "conditions of sanity" invoke some notions of rationality (as suggested by Nozick's nomenclature).

[13] So $R \subseteq X \otimes X$ is a binary relation on X if $(x, x') \in R \Rightarrow x, x' \in X$. We generally write $x\,R\,x'$ for $(x, x') \in R$ and $\neg x\,R\,x'$ for $(x, x') \notin R$.

[14] Nozick includes reflexivity of indifference as a sanity condition. Herzberger sees this as somewhat stronger than the other three, since it does not follow immediately from taking strict preference and indifference to be the asymmetric and symmetric subrelations of an arbitrary binary relation. Schwartz (1976) gets around this by specifying reflexive indifference relative to the domain of choice.

3.2 Preferential Choice

Hume argued that "Where a passion is neither founded on false supposition, nor chuses means insufficient for the end, the understanding can neither justify nor condemn it." Nozick (1993, p.140) suggests that we may draw a line between the "conditions of sanity" and other "axioms of rationality" as follows: "Contemporary decision theory takes this one step beyond Hume: although it does not say that any individual preference is irrational, it does say that a group of them together can be." So we will say that when we apply the conditions of sanity to a single preference, we do not cross over into considerations of rationality. As we will see, this implies that economists have been too hasty in invoking "rationality axioms" when applying abstract choice theory to neoclassical consumer theory.

The idea of preferential choice is absolutely central to the neoclassical paradigm. We have just considered a suggestion that an action be interpreted as preferred if it is chosen. It is worth pausing for a moment to ask how reasonable this is, especially since many economists find it so obvious as to be axiomatic. "We content ourselves therefore with the following simple economic postulate: *Each individual acts as he desires*" (Fisher 1892). "Acting man chooses between various opportunities offered for choice. He prefers one one alternative to others" (von Mises 1966, p.94).

Sen (1997) offers several illustration of possible gaps between preference and choice. These include reputation and indirect effects, social commitment and moral imperatives, direct welfare effects of the process of choice, and conventional rule following. For example, "you may prefer mangoes to apples, but refuse to pick the last mango from a fruit basket, and yet be very pleased if someone else were to 'force' that last mango on you." The instinct of most economists is to expand the description of outcomes to encompass each of these cases, but to do so increasingly involves the economist in focusing not just on observable outcomes but increasingly on mental states.

3.3 Revealed Preference

Of course we cannot directly observe judgments or preferences. What we potentially observe, we denote choices. If preference is a disposition to choose one thing over another (but see Nozick 1963), then we can treat choices as revelatory of preferences. Preferential choice is then the result of these preferences, a choice situation, and the satisfaction of certain preconditions: "being alive, having the capacity to make a choice, being able to effectuate an action toward a chosen alternative, facing no interference with these capacities that makes it impossible to exercise them" Nozick (1993, p.142).

Behaviorism is not rescued by arguing that consumer theory hypothesizes only that consumers behave "as if" their choices were preference-based. Such a move actually increases the economist's dependence on introspection. After all, in principle one can ask a consumer what they prefer. If the consumer becomes no more than a black box, the proposed structure of choice must come entirely from the economist's intuition about the nature of preference. If one tries to go further and *define* preference in terms of observed choice, the "theory" would have literally no predictions. The economist might nevertheless document any apparent structures in the choices made across choice situations, but such structures would be merely a convenient summary of past observations. What reason could the economist offer for such structure to persist? In any choice situation, what is observed is, after all, simply observed.

In practice, economists introduce structure by combining concepts of preference and rationality in the presumption of preference-based choice.

Assumption 4 (Purposive Choice) *The consumer rejects any alternative to which a more preferred alternative is known to be available.*

We will take this as our first "rationality postulate." In this we follow Robbins (1935, p.93):

> [T]here is a sense in which the word rationality can be used which renders it legitimate to argue that at least some rationality is assumed before human behaviour has an economic aspect—the sense, namely, in which it is equivalent to "purposive". . . . [I]t is arguable that if behaviour is not conceived of as purposive, then the conception of the means-ends relationships which economics studies has no meaning. So if there were no purposive action, it could be argued that there were no economic phenomena.

The assumption of purposive choice links beliefs and desires to actions. It is still the case the beliefs and desires remain epistemically interdependent, in the sense that we cannot use a consumer's choices to provide independent information about her beliefs and about her desires (Davidson, Suppes, and Siegel 1957; Hempel 1962). Economists have tended to impose considerable *a priori* structure on beliefs in order to allow information about desires to be extracted from choice behavior.

The assumption of purposive choice ensures that chosen alternatives are not inferior to any known alternative. But we need more than this is we are to hope that preference-based choices will exhibit some systematic structure. Economists usually proceed by assuming away all computational and epistemological difficulties in assessing the complete menu.

Assumption 5 (Completeness) *In any given choice situation, for each alternative in the menu, the consumer knows which alternatives in the menu are more preferred.*

This rules out consumer failure to choose the best option out of ignorance of the available options. (It does not rule out the choice of options that prove inferior *ex post*, when additional information may be revealed.) Together, completeness and purposive behavior ensure that only "best" (in the sense of undominated) alternatives are chosen.

Suzumura (1983, p.19) is rather unusual among economists in accepting this as the basic content of the concept of rationality: "*choice behavior is rational if it is made in accordance with the optimization of some preference relation, irrespective of whether or not this underlying preference relation is transitive.*" Note that whereas Robbins offers purposiveness as a necessary condition for rationality, Suzumura suggests this is sufficient.[15] We will return to this below.

Purposiveness is a theoretical link between behavior and preferences. For example, when applied to Marshallian demand it implies that the consumer purchases in the market a most preferred consumption bundle.[16] We will capture the idea of preference-based choice by defining two context dependent "revealed" preference relations as follows.[17]

Definition 8 (Context Dependent Revealed Preference) *Alternative x is* revealed strictly preferred *to alternative y in choice situation $\{A, e\}$ iff x is chosen and y is rejected. Alternative x is* revealed indifferent *to alternative y in choice situation $\{A, e\}$ iff x is chosen and y is chosen.*

We will adopt the notation $P_C^{A,e}$ and $I_C^{A,e}$ for these two concepts, where

$$x\, P_C^{A,e}\, y \Leftrightarrow x \in C(A,e)\ \&\ y \in A/C(A,e) \tag{1}$$

$$x\, I_C^{A,e}\, y \Leftrightarrow x \in C(A,e)\ \&\ y \in C(A,e) \tag{2}$$

Suppose we attempt to interpret preference as (context dependent) revealed preference. Then our three conditions of sanity follow directly from the existence of a choice function. In this sense, the conditions of sanity are preconditions to the concept of preferential choice. The important thing for our purposes is that no additional structure is imposed upon a choice function by assuming both that choice is preferential and that these three "conditions of sanity" hold.

There is a subtler background question that can be raised, however. Have we imposed any structure on our understanding of preference by the adoption of abstract choice theory? The answer is clearly yes: the assumption that a choice function exists rules out certain representations of preferences under the standard understanding of preference-based choice.

[15] Suzumura (1983, p.21) retreats somewhat by introducing the notion of "full rational" choice functions, which have transitive binary representations. He does not discuss the semantic suggestion that this qualifies his previous characterization of "rational" choice behavior.

[16] Note that misspecification of the domain can still affect inference. For example, indivisible goods, quantity discounts, and market power are not acknowledged in the competitive model.

[17] There are many revealed preference concepts in the literature. See Sen (1971) for a discussion. We will work with what, hopefully, is the most obvious meaning of 'choice reveals preference'.

To explore this a bit, let $\hat{R}_C^{A,e}$ be the union of context dependent revealed strict preference and context dependent revealed indifference, which we will call context dependent revealed weak preference in $\{A, e\}$.[18]

$$x\hat{R}_C^{A,e}y \Leftrightarrow x\, P_C^{A,e}\, y \text{ or } x\, I_C^{A,e}\, y \tag{3}$$

Note that by construction choice can be represented by context dependent revealed weak preference. That is,

$$C(A, e) = \{x \in A | x\hat{R}_C^{A,e} y\ \forall y \in A\} \tag{4}$$

This suggests that we might take seriously the notion that choice is determined by maximizing a pair-wise ranking of alternatives, represented by $R^{A,e}$. However, under this interpretation of preference-based choice, the existence of a choice function implies preferences are never represented by binary relations that cannot be maximized on A (due to, e.g., incompleteness or due to a cycle in the asymmetric subrelation). Nevertheless, given the existence of a choice function, we *always* have access to such a (context dependent!) binary representation.

4 Weakly Context Independent Choice

Preferences are seen by economists as exogenous to the choice situation, so that preferences implied in one choice situation will not be contradicted in other choice situations.[19] *Given* this persistence of preference across choice situations, rationality postulates are then offered as a restriction on preferences.

We can distinguish several kinds of selection procedures. For example, given a set of alternatives A, absolute selection procedures evaluate each alternative without reference to other elements of A (in consumer theory, a utility function), in contrast to relative selection procedures, which provide rankings relative to other elements of A (in consumer theory, a preference ordering). Unidimensional selection procedures are procedures that always imply a ranking of any two evaluated alternatives in a choice situation, while multidimensional selection procedures are procedures that can rank evaluated alternatives by individual criteria but do not guarantee a resolution of disparate rankings (on different criteria) into a ranking of alternatives. *Weakly context independent* selection procedures respond only to the set of available alternatives, while context dependent selection procedures depend on other aspects of the choice situation (the environment). We qualify the context independence as "weak" in that we have not ruled out selection procedures that depend directly on the menu. That is, the menu can still provide a context for choice. (See section 5.)

Consider any theory that predicts that an action $a \in A$ will be chosen in a given choice situation, which is characterized by the context $e \in E$ and the feasible menu $A(e) \in \mathcal{A}$. Here the notation $A(e)$ indicates that the set of feasible alternatives generally depends on the context. (For example, a consumer's budget set depends on her prices and income.) The neoclassical paradigm requires that the particular choices associated with elements of $\mathcal{A} \otimes E$ have three properties: context independence, menu independence, and transitivity. These three properties are necessary for choice to be characterized as the result of maximizing a utility function $U(a)$.

Menu independence and transitivity are discussed below. Weak context independence means that conditioning on the choice context is irrelevant for the characterization of choice.

Definition 9 (Weakly Context Independent Choice) *A choice function $C(A, e)$ is called* context independent *if we can find a function $C(A)$ such that $C(A) = C(A, e)$ for all $(A, e) \in \mathcal{D}$.*

This may be because context simply does not matter, or it may be that choice function is restricted to a domain where the context does not vary. (For example, we may view competitive markets as providing a context in which consumption choices are made.) In a weakly context independent characterization of choice, variations in e may matter only by affecting A, not directly. If we must control for the context, we might

[18] Equivalently, $\hat{R}_C^{A,e} = \cup_{A\in\mathcal{D}, e\in E} C(A, e) \otimes A$.

[19] This restriction is seldom articulated, although Hicks (1939, p.55) noted the need for preferences to be independent of prices.

still hope for a utility representation of the form $U(a;e)$. (For example, demand may depend on relative prices directly due to "conspicuous consumption" effects.) This will not be helpful if the set of feasible alternatives must be included in the environment, and it will be problematic for the applied economist if key environmental variables have no observable proxies.

Note that if preference-based choice is context independent, preference is "revealed" by menu choices independently of the context of the choice situation. For example, if $\{A, e\}$ and $\{A, e'\}$ are in the domain of the choice function, $P^{A,e} = P^{A,e'}$. We capture this by by defining

Definition 10 (Context Independent Revealed Preference) *Alternative x is* revealed strictly preferred *to alternative y in A iff there is some context in which x is chosen from A when y is rejected. Alternative x is* revealed indifferent *to alternative y in A iff there is some context in which both are chosen in A.*

We will adopt the notation P_C^A and I_C^A for these two concepts. The union of revealed strict preference in A and revealed indifference in A will be called revealed weak preference in A, represented as $\hat{R}_C^A$.

$$x\, P_C^A\, y \Leftrightarrow \exists e \text{ such that } x \in C(A,e) \;\&\; y \in A/C(A,e) \tag{5}$$

$$x\, I_C^A\, y \Leftrightarrow \exists e \text{ such that } x \in C(A,e) \;\&\; y \in C(A,e) \tag{6}$$

$$x \hat{R}_C^A y \Leftrightarrow x\, P_C^A\, y \text{ or } x\, I_C^A\, y \tag{7}$$

Note that with context free choice we can write

$$C(A,e) = \{x \in A | x \hat{R}_C^A y \;\forall y \in A\} \tag{8}$$

That is, the preferences that are revealed are also context independent. Furthermore, since context independent choice implies that P_C^A and I_C^A are just the asymmetric and symmetric subrelations of $\hat{R}_C^A$, they satisfy Nozick's three conditions of sanity (for any *given* menu, A).

4.1 An Illustration: Consumer Choice Under Certainty

Recall that the menu, A, is the set of alternatives from which the consumer makes a selection. We usually think of the available alternatives as possible actions, where actions are linked causally to outcomes. In the theory of choice under certainty, however, the basic identity between the actions and the outcomes allows a simplification: the actor is characterized as applying a selection procedure directly to outcomes. In this case A is just an exhaustive list of the mutually exclusive available outcomes.

For example in consumer theory X is the commodity space, $\mathcal{D}$ is the set of competitive budget sets, and $C(\cdot)$ generates the demand correspondence.[20] The consumer problem considered in this section is standard: a price-taking consumer with income w can purchase goods and services for known prices p. We focus on the Marshallian demand function as a representation of the consumer's choice rule.

The budget correspondence B specifies the affordable consumption bundles at any strictly positive prices (for the K price commodities) and income: $B : \Re_{++}^{K+1} \to \Re_{+}^{K}$. We represent the competitive budget sets as follows.

$$B(p,w) = \{x \in \Re_{+}^{K} \mid px \leq w\} \tag{9}$$

The budget correspondence is homogeneous of degree zero. That is, $B(\alpha p, \alpha w) = B(p,w)\; \forall \alpha > 0$. This means that a proportional change in all prices does not change the set of alternatives available to the consumer.

The consumer's choice situation consists of a budget set and an environment. The consumer selects a consumption bundle from her budget set. Given the stability assumption we can then characterize her demand behavior as $C[B(p,w), e]$.

[20] We usually characterize C as the choice *function*, where $C : [X] \to [X]$. The demand *correspondence*, in contrast, is usually implies a mapping $[X] \mapsto X$. This is just a matter of semantic convenience.

Definition 11 (Marshallian Demand) *The Marshallian demand correspondence $x(p, w)$ returns the consumer's acceptable choices from competitive budget set at each set of prices and income.*

Clearly the existence of a Marshallian demand correspondence implies an effective restriction of the choice context: only prices and income—or context functionally dependent on prices and income, such as the budget set—can both influence choice and vary in the domain of the choice function. We can therefore write

$$x(p, w) = C[B(p, w), c(p, w)] \tag{10}$$

4.1.1 Context Independence and Homogeneous Demand

Recall that choice is weakly context independent if, over the domain of the choice function (which may be context delimited), it can be characterized as depending only on the available alternatives and not on the environment. Slightly abusing notation, Marshallian demand is context independent if choices can be characterized as $C[B(p, w)]$ rather than $C[B(p, w), c(p, w)]$. That is, the consumer's behavior is determined solely by the budget sets. Prices and income matter only insofar as they determine the budget set.

Theorem 1 (WCI = Homogeneity) *Marshallian demand is context independent iff it is homogeneous of degree zero in (p, w).*

> Proof: Since budget sets are homogeneous of degree zero, it is clear that context independence is sufficient for Marshallian demand to be homogeneous of degree zero. Necessity follows from the fact that two budget sets are equal iff all prices and income change in the same proportion. Since $B(p, w) = B(p', w')$ iff $p/w = p'/w'$, a violation of context independence implies a violation of homogeneity. To see this, suppose $\exists x \in x(p, w)$ but $x \notin x(p', w')$ although $B(p, w) = B(p', w')$. Then homogeneity would imply $\exists x \in x(p/w, 1)$ such that $x \notin x(p'/w', 1)$. But this cannot be, since by equality of the budget sets $p/w = p'/w'$.

The homogeneity of Marshallian demand is fundamental to neoclassical consumer theory. At the same time, the context dependence of individual choice is well documented.[21] How uneasy should this make us? Perhaps clearest defense of neoclassical consumer theory is to actually invoke the importance of context. Neoclassical consumer theory is developed in a particular institutional context of impersonal choice of commodity bundles in competitive markets (without the interpersonal, reciprocal relationships of "customer markets"). This suggests that while budget studies may offer an appropriate setting for studying neoclassical consumer theory, many laboratory studies of individual choice will be irrelevant. Of course this justification undercuts the project of "economic imperialists" who wish to extend neoclassical microeconomic reasoning to all areas of human behavior. In addition it is at best a defense of an approximation, since consumer choice is never fully depersonalized. (E.g., consumer purchases are often made face to face from retailers, who in addition may be known to the consumer; fashion is known to be important in apparel sales; boycotts involve shifts in consumer demand.) Nevertheless, given the project in the current paper of characterizing the structure of neoclassical consumer theory, we will assume context independent choice from now on.

Assumption 6 (Context Independence) *Choice depends only on the available alternatives.*

This is our first assumption on the structure of the choice function. It is effectively a domain restriction: we assume the choice function can be written as $C(A)$.

5 Menu Independent Choice Functions

If choice is context independent, then preference can be revealed in a context independent way. This is already a strong (and falsified) assumption about choice, although as argued above, domain assumptions are

[21] For example, Kahneman and Tversky (1979, 1984) find that framing matters for choice. Empirically, humans often respond to deviations from a reference level rather than to the absolute characteristics of the situation (Rabin 1998; Helson 1964). Related to this, Tversky and Kahneman (1991) show the dependence of choice on initial endowment. Institutions can also provide context, as Bowles (1998) discusses in the context of preference endogeneity.

crucial to its interpretation. Nevertheless, neoclassical consumer theory insists upon considerable additional structure. We will refer to that structure as menu independent ranking.

Evidently choice itself cannot be menu independent. The concept of menu independence can applied to a selection procedure, but not to choice. The idea of a menu independent selection procedure offers the weakest concept of menu independent choice.

Definition 12 (Weak Menu Independence) *A choice function is* weakly menu independent *if it can be characterized by an algorithm that is not conditional on characteristics of the specific choice menu.*

Characteristics of the choice menu include such things as the number of alternatives in the menu or a list of specific alternatives in the menu. The concept is roughly that the selection procedure proceeds in the same way in regardless of the specific characteristics of the choice menu. Weak menu independence thus seems capture the very concept of having *a* selection procedure: menu dependence of the choice function is formally equivalent to having many selection procedures, one for each menu.

Here are three examples of weakly menu independent choice functions: minimize the maximum regret from the chosen action; based on a menu independent ranking of alternatives, choose the alternatives with the highest rank; based on a menu independent ranking of alternatives, choose the alternatives with the second highest rank. Neoclassical consumer theory imposes the second of these procedures, thereby ruling out the other two.

5.1 Revealed Preference

Neoclassical consumer theory is a theory of preference-based choice. It assumes that preferences are not only context independent, they are also menu independent. Although one encounters arguments that the theory proposes only that the consumer behaves *as if* her choices were preference based, this is quite misleading. The claim captures the truth that empirically we can only test for an *as if* compatibility between our theory of preference based choice and actual consumer behavior. But this should not obscure the point that the *only* reason we have for predicting that choices will display certain structures is our theory that choice is in fact preference-based. (What other reason might we offer for believing that the consumer will behave "as if" her choices were preference-based?) The concept of preference is therefore "primitive" in neoclassical theory, and we cannot unpack the implications of menu-independent preference independently of an explicit concept of preference.

In addition to assuming choice is preference-based, neoclassical theory assumes that choices *reveal* preferences. The applied economist assumes that in any given choice situation the consumer's selection is one of her "best" alternatives.

Consider three possible (roughly stated) approaches to the idea of a "menu independent preference".

- Choice can be characterized by a menu independent binary relation.
- Preferences "revealed" in any given choice situation are never reversed in any other choice situation.
- Preferences "revealed" in any family of choice situations are never reversed in any other choice situation.

Sen (1997) proposes the first of these, which we will call binariness, as a characterization of menu independence in the choice function. But as we have seen, choice functions are reasonably considered menu independent under much weaker conditions. Comparison with the second and third suggest that binariness is in turn too weak to be taken seriously as a characterization of the menu independence of *preference*. We will call the second of these the weak axiom of revealed preference (WARP). We will call the third of these Houthakker's axiom of revealed preference (HARP). Is the decision among these (and other possible) options as representing the phrase "menu independent preference" merely a matter of semantics? Before addressing that question, consider a problem faced by all three.

5.1.1 Menu Independent Preference

Is menu independent preference a reasonable axiom of choice? Economists have generally treated this menu independence as a rationality requirement. This would make it an axiom of rational choice, rather than of choice in general. But even this status is quite dubious.

Most obviously, information about the nature of the alternatives may be revealed by menu changes. Consider an examples due in its essentials to Tullock (1964). The choice-space consisting of three shades of turquoise paint: g_1, g_2, g_3. The object is to choose the bluest shade for a room painting project. Consider the following choices. (For readability, set braces are dropped and no note is made of the revealed indifference of a chosen element to itself.)

Example 2 (Imperfect Discrimination)

A	$C(A)$	*Revelation*	$CS(A, R_C)$
g_1, g_2, g_3	g_3	$g_3 \, P_C \, g_1$,$g_3 \, P_C \, g_2$	g_2, g_3
g_1, g_2	g_1, g_2	$g_1 \, I_C \, g_2$	g_1
g_2, g_3	g_2, g_3	$g_2 \, I_C \, g_3$	g_2
g_1, g_3	g_3	$g_3 \, P_C \, g_1$	g_3

Why does the consumer choose only g_3 from $\{g_1, g_2, g_3\}$ but also chooses g_2 from $\{g_2, g_3\}$? The answer is simple: the shades are close enough together that in a pair-wise comparison the consumer cannot tell g_1 from g_2 or g_2 from g_3. But with all three shades in the menu, the consumer can *deduce*—from her own knowledge of her limits to discrimination—that g_2 falls between the other two. Thus menu independent preference implies that the menu does not reveal information about the alternatives.

Assumption 7 (Noninformative Menus) *The menu yields no information about the alternatives other than their (mutually-exclusive) availability.*

Even without such effects, however, apparently plausible descriptions of rational choice behavior can conflict with menu independence (Sugden 1985). The classic illustration involves the minimax rule, and following example is drawn from McClennen (1990, pp.26–27). Consider a consumer faced with three gambles, g_1, g_2, g_3, over three states of the world, E_1, E_2, E_3. There is a state-dependent outcome (payoff) for each gamble. For each gamble, the consumer calculates her regret for each state of the world as the difference between the outcome of her gamble and the best outcome (in that state). As the two tables show, if the consumer minimizes her maximum regret, her choices will be menu dependent: $C(\{g_1, g_2, g_3\}) = \{g_3\}$, but $C(\{g_2, g_3\}) = \{g_2\}$. Although the minimax rule completely orders the alternatives in a given menu, the ordering of alternatives is not menu independent.

Example 3 (Minimax vs. MI)

	Outcomes			*Regret*			*Maximum Regret*
gamble\state	E_1	E_2	E_3	E_1	E_2	E_3	*(across states)*
g_1	*10*	*5*	*1*	*0*	*5*	*9*	*9*
g_2	*0*	*10*	*4*	*10*	*0*	*6*	*10*
g_3	*5*	*2*	*10*	*5*	*8*	*0*	*8*

	Outcomes			*Regret*			*Maximum Regret*
gamble\state	E_1	E_2	E_3	E_1	E_2	E_3	*(across states)*
g_2	*0*	*10*	*4*	*10*	*0*	*6*	*6*
g_3	*5*	*2*	*10*	*5*	*8*	*0*	*8*

The problem lies is the (missing) link between outcomes and alternatives. If the menus of gambles is replaced with menus of regrets, there is no reordering of alternatives as the menu changes. Any link between preferences and alternatives derives from the link between alternatives and outcomes. For choices among alternatives to reveal preference, therefore, we must assume a menu independent link between alternatives and outcomes.

Assumption 8 (Menu Independent Outcomes) *The outcome(s) caused by the selection of an alternative is independent of the menu from which the alternative is selected.*

For the applied economist, understanding this requirement can affect the characterization of the choice space. But it can also raise difficulties of observability, since preferences over actions are naturally *derived* from preferences over *anticipated* outcomes. Difficulties arise whenever an ordering of anticipated outcomes does not imply a menu-independent ordering of actions.

5.2 Menu Independent Revealed Preference

This section develops the notion of menu independent revealed preference as meaning that preferences revealed in any given choice situation are assumed never to be reversed in any other choice situation.[22] We begin by defining the menu independent revealed preference relations P_C and I_C.

$$x \, P_C \, y \;\Leftrightarrow\; \exists x, y, A, e \quad \text{such that} \quad x \in C(A,e) \;\&\; y \in A/C(A,e) \tag{11}$$

$$x \, I_C \, y \;\Leftrightarrow\; \exists x, y, A, e \quad \text{such that} \quad x \in C(A,e) \;\&\; y \in C(A,e) \tag{12}$$

We read $x \, P_C \, y$ as "x is revealed strictly preferred to y". We read $x \, I_C \, y$ as "x is revealed indifferent to y". In both cases we make no reference to the context or the menu. We also define the revealed weak preference relation as the union of revealed strict preference and revealed indifference.[23]

$$x \hat{R}_C y \;\Leftrightarrow\; \exists x, y, A, e \quad \text{such that} \quad x \in C(A,e) \;\&\; y \in A \tag{13}$$

We read $x \hat{R}_C y$ as "x is revealed weakly preferred to y".

We define menu independent choice in terms of the weak axiom of revealed preference (WARP).

Definition 13 (WARP) *A choice function $C(A,e)$ satisfies the* weak axiom of revealed preference *iff $x \hat{R}_C y \implies \neg y \, P_C \, x$.*

Definition 14 (Menu Independent Choice) *A choice function $C(A,e)$ is* menu independent *iff it satisfies WARP.*

Satisfaction of WARP means that whenever x is revealed weakly preferred to y then it is not the case that y is revealed strictly preferred to x (and vice versa). Note that menu independence implies that the choice function can be represented by weak revealed preference. That $C(A) \subseteq CS(A, R_C)$ is immediate, from the definition of R_C. To show that $C(A, R_C) \subseteq CS(A)$, suppose we could find $x \in C(A, R_C)$, so that $x \hat{R}_C y \; \forall y \in A$, but $x \notin C(A)$, so that $\exists y \in A$ such that $y \, P_C \, x$. Then for some $y \in A$ we would have $x \hat{R}_C y$ and $y \, P_C \, x$, which violates menu independence.

WARP implies a nice relationship to the conditions of sanity, but this time it takes a little work to show it.

Theorem 2 *A choice function $C(A,e)$ is menu independent iff revealed strict preference and revealed weak preference satisfy Nozick's three conditions of sanity.*

[22] Note that for compactness of the presentation, we are ignoring the important case where choice depends on the choice context but not on the menu (in the sense defined in the present section). However the considerations for that case are precisely the same as those raised in the present section.

[23] It is important to note that revealed strict preference and revealed indifference are *not* generally the asymmetric and symmetric subrelations of revealed weak preference. The reasons for this usage will become clearer later in our discussion, but for now note that the asymmetric and symmetric subrelations of R_C would satisfy the conditions of sanity by definition.

Proof:
From the definition of $\hat{R}_C$, $x\hat{R}_C y \implies$ either $x\ P_C\ y$ or $x\ I_C\ y$ (or possibly both). Note $x\ P_C\ y \implies \neg y\ P_C\ x.$ is a sanity condition. Further $x\ I_C\ y \implies y\ I_C\ x$ is a sanity condition, implying by the last sanity condition that $\neg y\ P_C\ x$. Therefore we know from the conditions of sanity that $x\hat{R}_C y \implies \neg y\ P_C\ x$.
Now suppose $x\hat{R}_C y \implies \neg y\ P_C\ x$. It follows immediately that $x\ P_C\ y \implies \neg y\ P_C\ x$, yielding the asymmetry of strict preference. Symmetry of revealed indifference follows from the definition of I_C. Using that symmetry and the definition of $\hat{R}_C$ we know $x\ I_C\ y \implies y\ I_C\ x \implies y\hat{R}_C x$, and since menu independence states that $y\hat{R}_C x \implies \neg x P_C y$, menu independence implies disjointness ($x\ I_C\ y \implies \neg x\ P_C\ y$).

How then do we interpret a violation of WARP, knowing that it implies a violation of the sanity conditions? We could say the consumer is insane. Or we could say the consumer's preferences have changes, either because of menu dependence or context dependence. The latter alternative is clearly preferable, but it raises its own difficulties: we cannot rely on choice to reveal preference *and* to reveal changes in preferences (Robinson 1962, p.50;Hicks 1974;Wong 1978).

Under this interpretation, the conditions of sanity impose considerable structure. Now the three conditions of sanity imply that if x is ever chosen over y, y is never chosen with or instead of x in any choice situation. (So, for example, if a preference is "revealed" by choice over a pair of alternatives, it is never reversed in larger sets of alternatives.) Unfortunately this structure is so substantive that it is far from obvious. For example, it assumes that preferences are not changing over the domain of the choice function.

While this structure is very strong, economists have completely accepted it. Typically choice displaying such structure is justified as "rational". Note how the equivlance of WARP and the sanity conditions calls this into question, however. We require no "rationality axiom" to justify WARP. Rather, what we require is that menu independent preferences be revealed in choices. Neither menu independence nor revelation are natural rationality restrictions.

5.3 Weak Axiom of Revealed Preference for Marshallian Demand

Once again we return to neoclassical consumer theory. We work with the Marshallian demand correspondence, as introduced in section 4.1: $x(p, w)$ represents the consumer's choices from the competitive budget set determined by prices p and income w. The domain of choice is the collection of budget sets. In this setting, we can give a specialized formulation of the weak axiom of revealed preference. Let us begin with a situation of weak revealed preference: $x' \in x(p', w')$ and $p' \cdot x \leq w'$. That is, x' is chosen when x is affordable: $x'\ R_C\ x$. In this circumstance, WARP implies that x can never be chosen when x' is affordable but not chosen: $x \in x(p, w) \implies p \cdot x' > w$ or $x' \notin x(p, w)$. That is, if x is affordable when x' is chosen, then x is only chosen without x' when x' is not affordable.

How can the applied economist know whether or not an affordable bundle was chosen but not picked? There is really no way: many bundles might be indifferent, but only the bundle picked is observed. Once again the economist must turn to the nature of preferences to suggest enought structure on consumer behavior. Here are two possibilities:

- The consumer has a unique choice in any budget set. (Univalence)
- Demand exhausts income (Walras' Law)

Univalence is a widely adopted restriction, which has the advantage of implying that the choice set is revealed in any choice situation. Yet it is obviously *ad hoc*.[24] Walras' Law can be motivated by an appeal to the structure of underlying preferences: any of a number of insatiability assumptions on preferences imply the satisfaction of Walras Law.

[24] That is, the preference restrictions that produce univalence, such as strict convexity, are driven by the theoretical convenience of univalence rather than the plausibility of the restriction.

5.3.1 Walras' Law

If the consumer spends all her wealth, placing her choice on the *budget hyperplane*:

$$\{x \in \Re_+^K \mid px = w\} \tag{14}$$

The slopes of the budget hyperplane reflect the rates of exchange between commodities. Consider any bundle $\bar{x}$ on the budget hyperplane. If we draw the price vector from this point, it must orthogonal to any vector drawn from this point to any other point on the hyperplane. This is because along the hyperplane

$$p \cdot \Delta x = 0 \tag{15}$$

We can now state Walras Law: $x \in x(p,w) \implies p \cdot x = w \quad \forall w > 0, p \gg 0$. That is, the consumer spends all her income.

When the Marshallian demand correspondence satisfies Walras Law, we will refer to it as a Walrasian demand correspondence.[25]

5.3.2 WARP for Walrasian Demand

Suppose $x \in x(p,w)$ & $x' \in x(p',w')$. Note that Walras' Law and WARP together imply the following: if $p \cdot x' < w$ then $p' \cdot x > w'$. That is because when x' is strictly cheaper we know $x' \notin x(p,w)$, by Walras Law.

5.3.3 Law of Compensated Demand

Definition 15 (LCD) *We say a Marshallian demand correspondence satisfies the* Law of Compensated Demand (LCD) *if* $x \in x(p,w)$ *and* $x' \in x(p', p' \cdot x)$ *implies* $(p' - p) \cdot (x' - x) \leq 0$.

The law of compensated demand, that income compensated demand curves slope downward, is a centerpiece of consumer theory. This section states a well-known relationship between the law of compensated demand and the weak axiom of revealed preference. Suppose we observe $x \in x(p,w)$, so that $p \cdot x \leq w$. Now suppose we change prices to p' but also change income to $w' = p' \cdot x$, so that the previous consumption bundle is just affordable. What does consumer theory predict happens to demand?

We call such an income change a *Slutsky income compensation*. We want to show that, given a Slutsky income compensation, price and quantity changes are negatively correlated: $(p' - p) \cdot (x' - x) \leq 0$. If $x' = x$ this is satisfied trivially, so we will consider the cases where $x' \neq x$. Note that

$$(p' - p) \cdot (x' - x) = p' \cdot (x' - x) - p \cdot (x' - x) \tag{18}$$

Under the Slutsky income compensation, we know

$$p' \cdot x' \leq p' \cdot x \tag{19}$$

so $p' \cdot (x' - x) \leq 0$. Therefore we just need to show that $p \cdot (x' - x) \geq 0$.

If $x' \notin B(p,w)$, the result is immediate: $p \cdot x' > w \geq p \cdot x$. This gives us $(p' - p) \cdot (x' - x) < 0$, satisfying the law of compensated demand (with strict inequality). If only a single price p_k changes, then this simplifies to

$$0 < (p_k' - p_k)(x_k' - x_k) = \Delta p_k \Delta x_k \tag{20}$$

[25] As a counter-example, note that

$$x_1 = \frac{p_2}{p_1 + p_2} \frac{w}{p_1} \tag{16}$$

$$x_2 = \frac{\beta p_1}{p_1 + p_2} \frac{w}{p_2} \tag{17}$$

satisfies first degree homogeneity but does not satisfy Walras Law (unless $\beta = 1$).

We say that own price effects are negative, or compensated demand curves slope downward. Note that we did not use WARP for this result.

If $x' \in B(p, w)$, WARP assures us that $x' \in x(p, w)$. But this is not enough: we need to know that at prices p, x' costs at least as much as x. If so, then $p \cdot x' \geq p \cdot x$ and we have LCD. The problem is that x and x' have been revealed indifferent and there is nothing in the nature of the choice problem to ensure that the consumer will pick the cheapest of them. WARP must therefore be supplemented to get the law of compensated demand. We therefore invoke Walras' Law, which assures us that for $x, x' \in x(p, w)$

$$p \cdot x' = p \cdot x \tag{21}$$

It also implies we can write the (compensated) budget constraint as

$$p' \cdot x' = p' \cdot x \tag{22}$$

we then have

$$\Delta x \cdot \Delta p = 0 \tag{23}$$

In sum, the law of compensated demand states that

$$\Delta x \cdot \Delta p \leq 0 \tag{24}$$

For a Walrasian demand correspondence, we find that WARP implies LCD, with strict inequality iff $x' \notin B(p, w)$.

6 Rational Preferences

It should be clear that economists' interest in rationality derives from the hope that rationality imposes useful structure on the choice function. Notions of rationality are normative, and this structure is intended to reflect these norms. Following McClennen (1990), we can distinguish three approaches to justifying such normative imposition of structure.

1. foundationalist: the structure codifies fundamental, "unquestionable" principles
2. coherentist: the structure consistently codifies existing norms regarding evaluation and choice among alternatives
3. pragmatic: the structure codifies necessary conditions for the successful pursuit of goals

As McClennan notes, the debate over the standard principles in utility theory and expected utility theory seems to cut equally well against the foundationalist and coherentist approaches.

In the application of abstract choice theory to consumer theory, economists have relied largely on the foundationalist approach. That is, they have engaged in a search for principles that would find universal acceptance as requirements of rational choice. Primarily this has meant a search for principles that embody core intuitions about the ways in which preferences should "hang together".

For example, the following collection of choices is generally seen as problematic.

Example 4 ("Odd" Preferences)

A	$C(A)$	*Revelation*	$CS(A, R_C)$
x, y	x	$x \, P_C \, y$	x
y, z	y	$y \, P_C \, z$	y
x, z	z	$z \, P_C \, x$	z

A	$C(A)$	*Revelation*	$CS(A, R_C)$
x, y	x	$x \, P_C \, y$	x
y, z	y	$y \, P_C \, z$	y
x, z	x, z	$z \, I_C \, x$	x, z

When preferences are menu independent, and choice reveals preference, it seems inherent in the concept of preference that such outcomes not occur.[26] That is, it seems inherent to the nature of preference that strict preference be transitive. (This is known as the "quasi-transitivity" of preference.)

[26] Note that the second set of "odd" preferences is allowed under Sen's definition of "menu independence", since if $\{x, y\} = C(\{x, y, z\})$, the choice function can be represented by weak revealed preference.

6.1 Transitivity

Definition 16 (Transitivity) *A binary relation R on a space X is called* transitive *if for all $x, y, z \in X$, $x\,R\,y$ & $y\,R\,z \implies x\,R\,z$. If a binary relation R has a transitive asymmetric subrelation, then R is called* quasitransitive.

Note that transitivity is a property of triples. However, it is readily extended to longer chains. Also note that transitivity implies quasi-transitivity: if R is transitive, then it is quasi-transitive. Many economists have presumed that rational choices should display certain transitivities (Houthakker 1950; Samuelson 1950). Quasi-transitivity is particularly intuitive if we think of the binary relation as representing weak preference, so that the asymmetric subrelation represents strict preference. Because human perception is limited, however, it is easy to imagine $x\,R\,y$ & $y\,R\,z$ but $\neg x\,R\,z$. This is just a matter of increments too small to discriminate individually but large enough to discriminate in conjunction. Although there is a literature on intransitive indifference that addresses this, we will not pursue this interesting topic in any detail.[27] Note that in experimental situations, individuals are observed to sometimes make intransitive choices (May 1954; Rose 1957). However, when this is explained to them, they tend to revise the choices (now seen as mistakes—but is experimenter influence on this perception adequately controlled?). Marschak (1960) has compared these "mistakes" to arithmetical errors: individuals make arithmetical errors but do not deny the laws of arithmetic. Savage (1953) appears to have a related position: "Whenever I examine such a triple of [intransitive] preferences on my own part, I find that it is not at all difficult to reverse one of them. In fact, I find on contemplating the three alleged preferences side by side that at least one among them is not a preference at all, at any rate not any more." (As quoted by Herzberger (1973, p.195).)

Herzberger (1973, p.195) claims that the transitivity of strict preference is "almost beyond reproach" as a normative condition. "Under the conception of the rational agent as an ideal systematizer, if he strictly prefers one alternative to a second and also the second to a third, he clearly ought to bring these separate evalutations to bear on one another; so he ought to strictly prefer the first to the third. ... Whatever evidence supports the claim that a certain agent has an indirect strict preference between two alternatives, could equally well be brought in as evidence for a direct strict preference between them."

There are two ways in which one might conceive the transitivity condition, which are not generally separated in neoclassical consumer theory. One might think of preference as binary, and see transitivity as a constrain on binary preferences in a given choice situation. Alternatively, one might see transitivity as a kind of consistency relationship that should pertain across choices at different points in time. The bridge between the two is preference-based choice, with constant preferences. Preference constancy is assumed in neoclassical consumer theory, although it is rejected by some econmists. For example, (von Mises 1966, p.103) sees examples of intransitive preference as showing only that "value judgments are not immutable and that therefore a scale of value, which is abstracted from various, necessarily nonsynchronous actions of an individual, may be self-contradictory." Tullock (1964, p.404) also focuses on inconstancy in such cases, invoking Occam's razor along with the claim that "the hypothesis that the subjects had transitive preference schedules but changed their minds in the simpler one." (This move is somewhat ironic in the context of a literature that has attempted to use Occam's razor to excise preferences altogether.)

In terms of revealed preference, the transitivity of strict preference implies that revealed preference will satisfy the following condition, which we denote the strong axiom of revealed preference (SARP).

Assumption 9 (SARP) $x^n\,P_C\,x^{n-1}$ & ... & $x^2\,P_C\,x^1 \implies \neg x^1\,R_C\,x^n$

In fact, most economists wish to go the extra mile and say weak preference is also transitive. For example, Arrow (1967, p.5) states, "The idea of transitivity clearly corresponds to some strong feeling of the meaning of consistency in our choice. Economists have traditionaly identified the concept of rationality with the notion of choices derivable from an ordering." In terms of revealed preference, this implies the following condition, which we denote Houthakker's axiom of revealed preference (HARP).[28]

[27] See van Acker (1990) for a recent discussion. Also see Georgescu-Roegen (1936) and Armstrong (1950).

[28] Since Houthakker (1950) and Samuelson (1950) worked with single-valued demand functions, some interpretive issues are raised in applying these designations to set-valued choice functions (Richter 1966, p.637). However, with Suzumura (1983), we find this usage more useful than misleading.

Assumption 10 (HARP) $x^n \, R_C \, x^{n-1} \; \& \; \ldots \; \& \; x^2 \, R_C \, x^1 \implies \neg x^1 \, P_C \, x^n$

Clearly a violation of WARP implies a violation of SARP, and a violation of SARP implies a violation of HARP. In this sense it seems that we have three proposed structures of choice in ascending restrictiveness.

Should we adopt 9 and/or 10 as a rationality axiom? Before addressing this question, a few comments are in order. First note that 9 and 10 collapse to the same statement when the choice function is single valued. This is the context in which the early work on revealed preference took place.

Suppose all triplets of elements in the choice space are in the domain of a given choice function.

Definition 17 (T-Domain) *We say a choice function is defined on a T-domain if*

$$x, y, z \in X \implies \{x, y, z\} \in \mathcal{D}$$

On a T-domain, WARP implies transitivity of weak revealed preference. This collapses all our axioms of revealed preference (Arrow 1959; Sen 1973).

Theorem 3 (WARP $\implies$ HARP) *A choice function that satisfies WARP on a T-domain also satisfies HARP (and therefore SARP).*

Proof: Since HARP follows immediately from WARP and the transitivity of R_C, we just prove that WARP implies transitivity. The "proof of transitivity" is similar to that in Tullock (1964). Suppose $x \, R_C \, y \; \& \; y \, R_C \, z$, and consider $C(\{x, y, z\})$. By WARP, we must have $x \in C(\{x, z\})$, so that $x R_C z$ and weak revealed preference is transitive. (If $x \notin C(\{x, y, z\})$, WARP implies $y \notin C(\{x, y, z\})$, and WARP then implies $z \notin C(\{x, y, z\})$. But $C(\{x, y, z\}) \neq \emptyset$ since the choice function is defined on a T-domain.)

6.2 Marshallian Demand and the Domain of Choice

The Marshallian demand correspondence is generally defined on the set of budget sets. Since this is not a T-domain, WARP has been generally presumed to be a weaker assumption on Marshallian demand that SARP or HARP. Sen (1973, section 3) strongly challenged the logic of this presumption. "[I]t has been customary to assume, usually implicitly, that the Weak Axiom holds only for those choices that can be observed in the market and not necessarily for other choices. ... Presumably, the argument lies in the fact that ... the Weak Axiom can be verified only for those choices and not for others that cannot be observed in the market." But as Sen argues, verification is a red herring, since we can at best observe a finite number of budget situations. If we are going to assume WARP holds across budget situations we cannot observe, then we might as well assume it holds across triples as well. Our motivation for asserting WARP is our intuition about preferences, not its empirical verification.

If we are addressing the axiomatic foundations of consumer theory, Sen's argument is sound. No one has offered a reason to believe that consumers are capable of making choice from infinite budget sets but incapable of choosing from triples, nor that their preferences over infinite budget sets are more likely to be context independent than their preferences over triples. This point is two and a half decades old, but it is still overlooked. For example, Mas-Colell, Whinston, and Green (1995, p.35) suggest, "For Walrasian demand functions, the theory derived from the weak axiom is weaker than the theory derived from rational preferences." They can reach this conclusion only by the odd assumption that the weak axiom applies only to choices over budget sets and not to choices over finite sets.

Nevertheless, the distinction between WARP, SARP, and HARP does not become pointless on this account. For actual budget data, if we offer an empirical proxy for strict preference, each condition can be checked. A consumer who generates budget data from a choice function that satisfies WARP on an T-domain will of course also satisfy SARP and HARP. But checking that the data satisfy HARP is nevertheless much more demanding than simply checking if the data directly satisfy WARP.

Putting this another way, WARP is the basis of the *predictions* of consumer theory. HARP offers no additional predictions once consumer theory is reasonably characterized (so that WARP is satisfied over a T-domain). One of our predictions is the budget data will satisfy HARP. This implies, but is not implied by, the satisfaction of WARP by the budget data. To see this, consider an example offered by Hicks (1956).

Example 5 (WARP $\not\Rightarrow$ HARP) *There are three commodities and the consumer's income is 8. At* $p^1 = (2,1,2)$ *she consumes* $x^1 = (1,2,2)$*. At* $p^2 = (2,2,1)$ *she consumes* $x^2 = (2,1,2)$*. At* $p^3 = (1,2,2)$ *she consumes* $x^1 = (2,2,1)$*.*

Hicks assumes single-valuedness of the Marshallian demand correspondence, so that revealed preference is always strict. Since $p^3 \cdot x^2 = 8$*,* $x^3 \, P_C \, x^2$*. Since* $p^2 \cdot x^1 = 8$*,* $x^2 \, P_C \, x^1$*. Since* $p^1 \cdot x^3 = 8$*,* $x^1 \, P_C \, x^3$*. This cycle of strict preference is not ruled out by the pairwise comparison's suggested by WARP, but it is of course ruled out by HARP (and SARP).*

Now it is of course true that we can ask, What would be chosen from $\{x^1, x^2, x^3\}$? We discover from this that there is an implicit violation of WARP. But that is just the point: checking budget data for empirical violations of HARP *is* just checking for implicit violations of WARP.

6.2.1 Generalized Axiom of Revealed Preference

Suppose we observe a finite set of choices:

$$x^1 \in C[B(p^1, y^1)], x^2 \in C[B(p^2, y^2)], x^3 \in C[B(p^3, y^3)], ..., x^n \in C[B(p^n, y^n)]$$

Suppose we want to test for the satisfaction of HARP. We immediately face the problem that we do not observe the entire choice set. At best the picks are observed, and even this comes by assumption.

Assumption 11 (Causality) *The outcome observed by the economist in any given choice situation is caused by the consumer's pick from her choice set.*

This means that we have access only to the empirically revealed part of the revealed preference relationship. For example, it is certainly the case that picked bundles are revealed weakly preferred to all other affordable bundles. So let us define R_C^o by

$$x^i \, R_C^o \, x^j \iff x^j \in B(p^i, y^i)$$

where x^i is the pick from $B(p^i, y^i)$. Note that $R_C^o \subseteq R_C$

In contrast to our access to at least part of the weak revealed preference relation, we have no direct access to strict preference. Once again we will work with Walrasian demand, so that the consumer's pick is revealed strictly preferred to bundles in the interior of the budget set. Define P_C^o by

$$x^i \, P_C^o \, x^j \iff p^i x^j < w^i$$

where once again x^i is the pick from $B(p^i, w^i)$. Note that $P_C^o \subseteq P_C$.

The generalized axiom of revealed preference is just a restatement of HARP in terms of the empirical revealed preference relationship.

Definition 18 (GARP) *A collection of budget data satisfies the* generalized axiom of revealed preference *iff*

$$x^n \, R_C^o \, x^{n-1} \; \& \; \cdots \; \& \; x^2 \, R_C^o \, x^1 \implies \neg x^1 \, P_C^o \, x^n$$

So any cycle of revealed preference in the data cannot include a strict revealed preference. Clearly a violation of GARP implies a violation of HARP, which (assuming choice is defined on a T-domain) implies a violation of WARP.

In section 5.2 we saw that satisfaction of WARP ensures that a choice function can be represented by revealed preference. In section 6.1 we saw that satisfaction of WARP on a T-domain enures the transitivity revealed preference. We will now show that if a choice function can be represented by a complete and transitive binary relation, then it satisfies WARP.

Definition 19 *A binary relation* R *on a space* X *is called* complete *if*

$$x, y \in X \implies x \, R \, y \text{ or } y \, R \, x$$

Note that this is an inclusive 'or', and that x and y and not assumed to be distinct.

Theorem 4 *Suppose $C(A,e)$ is represented by R, which is complete and transitive on X. Then $C(A,e)$ satisfies WARP.*

Proof:
Recall representability means that for any menu A in the domain of the choice function, $C(A,e) = \{x \in A | x\,R\,y\ \forall y \in A\}$. Suppose for the purpose of contradiction that we can find $x, y \in X$ such that $y\,R_C\,x$ and $x\,P_C\,y$. Since x cannot be chosen over itself, x and y must be distinct. By representability, $x\,R\,y\ \&\ y\,R\,x$. So there must be a choice situation (A,e) where a third element z prevents y from being chosen along with x: $x, y, z \in A; x \in C(A,e); y \notin C(A,e); A \in \mathcal{D}$. That is, y is not chosen in A because $\not\exists\, y\,R\,z$. By completeness, we then have $z\,P\,y$. By transitivity, $z\,P\,y\ \&\ y\,R\,x \implies z\,P\,x$.[29] But then, by representability, x cannot be chosen in the presence of z.

We have uncovered the basic complementarity between WARP are "rational" preferences.

7 Conclusion

There is a single neoclassical theory of consumer demand: the theory of preference-based choice. There are three popular representations of the theory: representations in terms of utility functions, representations in terms of preference relations, representations in terms of choice functions. While abstract choice theory offers the most general characterization of choice in general, when applied to consumer theory it is neither more "general" nor more "empirically oriented" than representations in terms of preference relations. However, it does offer a pedagogical advantage, in that it assists in pinning down the series of assumptions on the general structure of choice that are invoked in developing the neoclassical theory of consumer demand. It this sense, abstract choice theory is an aid to the discovery of the structure of neoclassical consumer theory.

Neoclassical consumer theory has considerable structure. It is usually characterized as the theory of rational choice. We have seen that rationality postulates get us nowhere near the degree of structure required by the theory, however. In addition, we required a large number of auxiliary hypotheses which range from the plausible to the incredible. The most plausible case for neoclassical consumer theory comes from restricting it to choices made in competitive markets, where the context of competitive markets may plausibly be treated as dominant and relatively constant.

The fact that the neoclassical paradigm invokes an incredible amount of structure makes methodological justification problematic. The assumptions are completely contrary to what we know of actual humans, and this is verified by both casual empirical reflection and actual empirical work. We might therefore find the explorations of the present paper prompt additional methodological speculations related to neoclassical consumer theory.

It is not that the neoclassical model of consumer behavior is "wrong": a model need not be "right" to be useful. But viewing neoclassical consumer theory throught the lens of abstract choice theory undercuts many of the traditional rhetorical ploys in the former's defense, such as the claim that it is the study of rational behavior. This suggests that the methodological justification of neoclassical economics must be strictly pragmatic, which in turn opens the door to methodological pluralism. The assumptions most useful for understanding how economies function at the aggregate level may be completely different than those for understanding individual behavior, and either or neither may be those of neoclassical economics.

We have seen that neoclassical consumer theory requires much more structure than a few "rationality" postulates, and that much of this structure is implausible. Going far beyond the topics of the present paper, we can make more provocative claims. Experimental economics has decimated neoclassical consumer theory at the individual level. Well known results from general equilibrium theory tell us further that the neoclassical project of providing microfoundations for macroeconomics is impracticable (Kirman 1989,

[29] Note that if $x\,R\,x'$ and $x'\,P\,x''$, transitivity implies $x\,R\,x''$; then we cannot have $x''\,R\,x$ or transitivity would imply $x''\,R\,x'$; so $x\,P\,x''$. Call this result RPP. Similarly, if $x\,P\,x'$ and $x'\,R\,x''$, transitivity implies $x\,R\,x''$; then we cannot have $x''\,R\,x$ or RPP (which is implied by transitivity) would imply $x''\,P\,x'$; so $x\,P\,x''$.

1992). While neoclassical theory continues to offer a coherent description of imaginary economies, it is becoming increasingly difficult to articulate exactly what neoclassical economists are doing when they rely on neoclassical theory at any level of applied work.

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