The Real Interest Differential Model after Twenty Years

Alan G. Isaac*and Suresh de Mel

July 28,1999

Abstract

It has been twenty years since Frankel (1979) offered the classic empirical support for the Dornbusch (1976) overshooting model against the simple monetary approach model, and almost that long since Driskill and Sheffrin (1981) uncovered some important inconsistencies between Frankel's theoretical framework and his empirical implementation. Frankel's RID model nevertheless spawned a huge literature in international monetary economics. In this paper, we replicate and update the Frankel (1979) and Driskill and Sheffrin (1981) results, in order to offer a retrospective and a reëvaluation of this literature. We also explain why the model estimated by Driskill and Sheffrin (1981) cannot underpin a critique of Frankel (1979), a point which is not generally recognized. While specialists in international finance generally recognize that the initial promise of Frankel's research has not been kept, we believe that many will be surprised nevertheless by our stark findings.
JEL: F31, F40, C13

*Please address all correspondence to:
Alan G. Isaac, Associate Professor of Economics, American University, Washington, DC 20016.

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1 Introduction

The simple monetary approach to the determination of flexible exchange rates scored some notable early successes (Frenkel 1976; Bilson 1978), but its promise faded as experience with the generalized float accumulated. The extreme simplicity of the model, which was initially seen as a strength, fell under suspicion. Drawing upon the strong evidence against short-run purchasing power parity, a body of research emerged that discarded the monetary approach's assumption of continuous purchasing power parity.

Dornbusch (1976) presents the key theoretical innovation. This classic exposition of the "sticky price" approach to exchange rate dynamics shows that price inertia can be an important source of large real exchange rate movements. The key empirical paper is Frankel (1979), which applies the Dornbusch overshooting model—slightly modified to allow for secular inflation—to the USD/DEM exchange rate. Frankel finds striking support for the Dornbusch model against the simple monetary approach model: he reports statistically significant and reasonably sized estimated coefficients, which are signed as predicted by his "real-interest-differential" (RID) model.

Unfortunately, Driskill and Sheffrin (1981) argue that Frankel's coefficient estimates are inconsistent. (Frankel estimates a single equation that is only a partially reduced form of the Dornbusch (1976) model.) They also stress that Frankel ignores the possibility of testing the overidentifying restrictions imposed upon the model by the rational expectations assumption. Driskill and Sheffrin develop an explicit rational expectations version of the Frankel model, which allows the derivation of a true reduced form equation for the exchange rate. They firmly reject the RID model.

In this paper, we replicate and update the Frankel (1979) and Driskill and Sheffrin (1981) results, in order to offer a retrospective and a reëvaluation of this literature. Section 2 briefly presents the RID model and replicates some basic results from Frankel (1979). Section 3 briefly presents the model under rational expectations and discusses our efforts to replicate the Driskill and Sheffrin (1981) empirical results. We also explain why the empirical model estimated by Driskill and Sheffrin (1981) cannot underpin a critique of the RID model, a point which is not generally recognized. We then estimate their theoretical model, which we call the RIDRE model, and our results prove somewhat more favorable than those reported by Driskill and Sheffrin (1981). Finally, in section 4 we offer additional perspective on the RID and RIDRE models by reëstimating them over an updated sample.

It is twenty years since Frankel (1979) offered his exciting empirical support for a simple version of the Dornbusch (1976) overshooting model. His RID model remains, with the simple monetary approach model, a pedagogic staple in the field of international monetary economics. Although specialists in international finance generally recognize that the initial promise of Frankel's research has not been kept, many will be surprised nevertheless by our stark findings.

2 The 'Real Interest Differential' Model

We characterize Frankel's RID model in terms of four structural equations plus two simplifying auxiliary assumptions. The structural equations characterize uncovered interest parity, regressive expectations, long-run purchasing power parity, and a Classical model of long-run price determination. The auxiliary assumptions link long-run purchasing power parity to expected depreciation (see equation (6) below) and observed exogenous variables to their full-equilibrium levels (see section 2.2).

We begin with uncovered interest parity and regressive expectations.

$$i_t = s_{t+1}^e - s_t \tag{1}$$
$$s_{t+1}^e - s_t = \Delta \bar{s}_{t+1}^e - \theta(s_t - \bar{s}_t) + \varepsilon_t \tag{2}$$

Here s is the logarithm of the spot rate, s_{t+1}^e is the value of s_{t+1} expected at time t, $\bar{s}$ is the full-equilibrium value of s, and i is the nominal interest differential.[1] Additionally, θ is the speed at which the exchange rate is expected to move toward its full-equilibrium level, ε is a random deviation from the deterministic regressive expectations formulation, and $\Delta \bar{s}^e$ is the rate at which the full-equilibrium exchange rate is expected to change over time. (For example, the full-equilibrium spot rate would be expected to depreciate if the domestic country has relatively high inflation.)

The two remaining ingredients of the model are characterizations of *full-equilibrium* outcomes: long-run purchasing power parity, and a Classical model of long-run price determination. The assumption of long-run purchasing power parity provides a characterization of $\bar{s}$.[2]

$$\bar{s}_t = \bar{p}_t \tag{3}$$
$$\bar{p}_t = \bar{m}_t - \phi \bar{y}_t + \lambda \pi_t \tag{4}$$

Here $\bar{p}$ is the (log of) the full-equilibrium level of the relative price level (as determined by the simple Classical model of price determination), $\bar{m}$ is the (log of) full-equilibrium relative money supply, $\bar{y}$ is the (log of) full-equilibrium relative income, and π is the expected full-equilibrium inflation-rate differential.

2.1 Model Solution

Uncovered interest parity plus regressive expectations imply (5).

$$s = \bar{s} - \frac{1}{\theta}(i - \Delta \bar{s}^e) + \nu \tag{5}$$

where $\nu = -\varepsilon/\theta$. If (3) is common knowledge, then

$$\Delta \bar{s}^e = \pi \tag{6}$$

Using (6) to substitute for $\Delta \bar{s}^e$ in (5), we get an exchange rate model involving a kind of real interest differential.[3]

$$s = \bar{s} - \frac{1}{\theta}(i - \pi) + \nu \tag{7}$$

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[1] Somewhat more precisely, $i_t = \ln[(1 + I_t)/(1 + I_t^*)]$ where I_t and I_t^* are the domestic and foreign nominal interest rates, as an absolute rate of return from t to $t + 1$.

[2] For convenience in exposition, we set all constants to zero, including the long-run real exchange rate.

[3] As Frankel (1979) notes, it is not precisely a real interest differential, as we are subtracting expected equilibrium inflation rates from actual short-term interest rates.

For this reason, this model is often referred to as the "real-interest differential" model of exchange rate determination. Using (3) and (4), we get (8).

$$\bar{s}_t = \bar{m}_t - \phi \bar{y}_t + \lambda \pi_t \tag{8}$$

Finally, use (8) to substitute for $\bar{s}$, in (7).

$$s = \bar{m} - \phi \bar{y} + \left(\lambda + \frac{1}{\theta}\right) \pi - \frac{1}{\theta} i + \nu \tag{9}$$

2.2 Replication: Frankel (1979)

In this section, we replicate some key results of Frankel (1979). In order to implement (9) empirically, Frankel assumes that observed values of money and income equal their full-equilibrium values. This gives him (10), which we will refer to as the RID model of the spot rate.

$$s_t = m_t - \phi y_t + \left(\lambda + \frac{1}{\theta}\right) \pi_t - \frac{1}{\theta} i_t + \nu_t \tag{10}$$

Here m is the log of relative money supply and y is the log of relative real income. The RID model implies that the money supply coefficient is unity, that income and the interest rate have a negative coefficient, and that expected inflation has a positive coefficient. These predictions have been subject to a great deal of empirical scrutiny.

The results of Frankel's ordinary least squares estimations of (10) are reported in the OLS:f79 rows of Table 1. He finds all estimated coefficients have the correct sign and are of plausible size. We also see that his estimated coefficients appear significantly different from zero, excepting the interest rate coefficient. When Frankel restricts the coefficient on the relative money supply to unity, as implied by his theoretical model, the results are little changed.[4] The OLS:78 rows of Table 1 show that we are able to replicate the Frankel OLS results exactly.

However, there is strong evidence of serial correlation in the residuals. After correcting for serial correlation in the residuals Frankel find his OLS results little changed. The AR1:f79 rows of Table 1 report Frankel's iterated Cochrane-Orcutt (CORC) results. The AR1:78 rows of Table 1 show that we are able to replicate these results quite closely.[5] Frankel's results were seen as exciting initial support for the real interest rate differential model, and we find that his results are replicable.

2.3 Consistent Single Equation Estimation

Frankel (1979) focuses on the estimation of (10), but is this an appropriate regression equation? The answer depends on the stochastic properties of the regressors. Frankel offers two responses two this: he turns to instrumental variables to rectify possible defects (presumably measurement error) in his expected inflation variable, and he reports results with the unit coefficient on the money supply imposed as a response to possible money supply endogeneity. However, his reliance on the Dornbusch (1976) model and his concern about shocks to money demand rather naturally imply a concern with the endogeneity of the interest rate,

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[4] Frankel suggests that imposing this constraint addresses worries that central banks may vary money supplies in response to exchange rates, and may also improve the estimation if money demand shocks are important.

[5] Our CORC results can be produced with the iterative CORC procedure in the online GAUSS source code archive, setting the convergence criterion to .01 and the initial value of rho to zero. Our results differ noticeably from Frankel's only for a coefficient on the nominal interest differential. We estimate the coefficient as -2.61 while Frankel reports -0.259. Given that we are able to replicate his OLS results *exactly* and his other CORC results quite closely, we presume there is a typographical error in Frankel's table.

Frankel (1979) also "tests" for non-instantaneous adjustment in capital markets by including a lagged interest differential term. Our replication of this equation was also exact for OLS and very close for CORC. The equation is too *ad hoc* to report here, but our results are available upon request.

Finally, Frankel finds a significant sign on the interest rate only after turning to an instrumental variable procedure. Since the data set we obtained did not include his instruments, we were unable to replicate these results. The IV results reported in Table 1 are discussed in the next section.

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Table 1: Frankel (1979) Results, plus Replication and Extensions

	m_t	y_t	i_t	π_t	uni	R^2	D.W.	ρ
Equation 10 (dep. var.= s_t)								
OLS:f79	.87*	-.72*	-1.55	28.65*		.80	.76	
	(.17)	(.22)	(1.94)	(2.70)				
OLS:78	.87*	-.72*	-1.55	28.65*		.80	.76	
	(.17)	(.22)	(1.94)	(2.70)				
IV0:78	.90*	-.67*	-3.19	29.10*		.79	.73	
	(.19)	(.28)	(5.11)	(3.02)				
OLS:98	0.002	.66*	-6.40	-38.00*	-.13*	.50	.04	
	(.10)	(.16)	(7.63)	(9.46)	(0.05)			
IV0:98	-.10	.80*	-22.90	-27.41*	-.06	.49	.05	
	(.14)	(.21)	(18.19)	(14.24)	(.08)			
AR1:f79	.31	-.33*	-.259^a	7.72*		.91		.98
	(.25)	(.20)	(1.96)	(4.47)				
AR1:78	.30	-.32	-2.61	8.01*		.91	1.34	.98
	(.26)	(.21)	(2.06)	(4.62)				.07
IV1:78	.40	-.37*	-.71	5.90		.91	1.32	1.00*
	(.25)	(.21)	(1.37)	(4.43)				(0.06)
AR1:98	.14	0.00	-7.65*	-2.78	0.02	.98	1.34	.99*
	(.13)	(0.08)	(2.69)	(5.95)	(0.03)			(0.01)
IV1:98	.14	-0.02	.29	-9.70*	0.01	.98	1.37	.99*
	(.13)	(0.09)	(1.53)	(5.71)	(0.03)			(0.01)
Equation 10 with Constraint (dep. var.= $s_t - m_t$)								
OLS:f79		-0.69*	-1.77	30.17*		.92	.79	
		(0.21)	(1.91)	(1.68)				
OLS:78		-.69*	-1.77	30.17*		.92	.79	
		(.21)	(1.91)	(1.68)				
IV0:78		-.51*	-7.44	30.76*		.90	.64	
		(.28)	(5.26)	(1.92)				
OLS:98		.54*	31.21*	-65.47*	-.40*	.67	.07	
		(.18)	(7.74)	(10.53)	(0.05)			
IV0:98		.27	57.39*	-81.54*	-.49*	.66	.10	
		(.22)	(13.81)	(12.79)	(0.06)			
AR1:f79		-0.41*	-1.55	10.13*		.96		0.98
		(0.22)	(2.11)	(4.82)				
AR1:78		-.40*	-1.54	10.61*		.96	1.36	.97
		(.22)	(2.22)	(4.99)				0.05
IV1:78		-.29	-8.69*	30.72*		.94	1.72	.75*
		(.27)	(2.48)	(3.78)				(.14)
AR1:98		-0.01	-7.44*	-2.06	0.03	.99	1.32	1.00*
		(0.09)	(2.89)	(6.38)	(0.03)			(0.01)
IV1:98		-0.03	-.90	-7.80	0.02	.99	1.34	1.00*
		(0.09)	(1.62)	(6.08)	(0.03)			(0.01)

Notes:
Standard errors are in parentheses. Constants not reported.
*: |t-ratio|>1.65. **OLS**: ordinary least squares; **IV0**: instrumental variables (see text);
AR1: AR(1) correction (iterated Cochrane-Orcutt for replications); **IV1**: instrumental variables with AR1 correction.
f79: Frankel (1979, Tables 1 & 3); **78**: Frankel data, 1974.07–1978.02; **98**: *IFS* data, 1974.07–1998.11.
Data: monthly, Germany & U.S. (see appendix).
^{a} See footnote 5.

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as pointed out by Driskill and Sheffrin (1981). Furthermore, recall that Frankel (1979) assumes that the observed relative money supply is equal to its full-equilibrium level. This assumption can be justified, as in Driskill and Sheffrin (1981), but not if we treat the interest rate as exogenous. In short, the entire theoretical framework invoked by Frankel (1979) suggests that the interest rate is endogenous, raising concern that Frankel's estimated coefficients are biased and inconsistent.

We will illustrate the problem by turning to the rest of the Dornbusch (1976) model, following the discrete-time exposition of Driskill and Sheffrin (1981). Begin by considering money market equilibrium, as represented by (11).[6]

$$m_t - p_t = \phi y_t - \lambda i_t + v_{m,t} \tag{11}$$

Here $v_{m,t}$ is an error term representing money demand shocks, which is discussed in more detail in section 3. Solving for the interest rate yields (12).

$$i_t = -\frac{1}{\lambda}m_t + \frac{1}{\lambda}\phi y_t + \frac{1}{\lambda}p_t + \frac{1}{\lambda}v_{m,t} \tag{12}$$

This suggests we could approach the estimation of (10) in two stages, with m, y, and p as instruments for i.

In the basic Dornbusch (1976) model, it is natural to treat m, y, π, as exogenous. The proper treatment of p is a bit less evident. For example, let us follow Driskill and Sheffrin (1981) in representing the dynamic adjustment in the Dornbusch (1976) model by (13).[7]

$$p_t - p_{t-1} = \delta(s_{t-1} - p_{t-1}) + \pi_{t-1} + v_{p,t} \tag{13}$$

In this case p is a suitable instrument for i only if there is no correlation between the error in the price equation (v_p) and the error in the exchange rate equation (28). In the absence of such a restriction, we might use (13) to substitute for p in (12), yielding (14).

$$i_t = -\frac{1}{\lambda}m_t + \frac{1}{\lambda}\phi y_t + \frac{1}{\lambda}[(1-\delta)p_{t-1} + \delta s_{t-1} + \pi_{t-1} + v_{p,t}] + \frac{1}{\lambda}v_{m,t} \tag{14}$$

This suggests m, y, p_{t-1}, s_{t-1}, and π_{t-1} as instruments for i. Table 1 reports the implied instrumental variables estimates: the IV0:78 rows are reëstimates of the RID model without an AR1 correction for the autoregressive residuals, while the IV1:78 rows report the results with an AR1 correction. In brief, the results look much the same as before.

3   The RID Model with Rational Expectations

In this section we present the Driskill and Sheffrin (1981) version of the real-interest-differential model under rational expectations (RIDRE), and we attempt to replicate their empirical results. The Driskill and Sheffrin study is well known for two primary reasons: it offers a coherent, detailed attack on Frankel's classic RID model, and it contains an early empirical test of the parameter restrictions implied by the rational expectations hypothesis. The discussion below raises some questions about both of these contributions.

The structural model, which is just a discrete-time version of the Dornbusch (1976) model, comprises uncovered interest parity (1), money market equilibrium (11), and the price dynamics (13). Driskill and Sheffrin (1981) explicitly characterize the two random shocks: v_p is assumed to be white noise but v_m is allowed to be serially correlated.[8]

$$v_{m,t} = \rho_m v_{m,t-1} + \eta_t \tag{15}$$

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[6] Once we have equation (11), the RID model can be summarized as relating the *real* exchange rate to the same real interest differential: $s_t - p_t = -(\lambda + 1/\theta)(i - \pi_t) + \text{noise}$, which compares to Frankel (1979, equation A3).

[7] This a discrete time version of the price dynamics in the Dornbusch (1976) overshooting model, modified to include Frankel's secular inflation term. As in the Dornbusch model, the relative price level adjusts in response to relative excess demand in the goods market which, in turn, is indexed by the real exchange rate. An implication is that δ is the sum of the domestic and foreign real exchange rate coefficients. This price adjustment formulation is particularly popular because it conveniently treats the price level as predetermined.

[8] For the sake of "expository ease" (p.1069) their algebraic presentation assumes v_m to be white noise. Their empirical work includes a Cochrane Orcutt correction for serial correlation. This creates a problem that we address in section 3.3.

where η_t is white noise. In addition, Driskill and Sheffrin (1981) assume that expectations formation is rational in the sense of Lucas (1972).

$$s_{t+1}^e = \mathcal{E}_t s_{t+1} \tag{16}$$

Here $\mathcal{E}_t$ is the expectations operator conditional on the information available at time t, which includes the current and past values of all variables plus the structure of the model.

The Driskill and Sheffrin (1981) model specification is completed by an atheoretical characterization of the exogenous variables y, m, and π. (These are chosen to match the discussion in Frankel (1979).) Relative income is assumed to follow a random walk. The relative money supply is assumed to follow a random walk around a trend, π_t, which in turn follows a random walk. Driskill and Sheffrin follow Frankel in regarding π_t as the long-run growth rate of relative money that is known to the public. Equations (17), (18), and (19) characterize these stochastic processes, where $\eta_{y,t}$, $\eta_{m,t}$, and $\eta_{\pi,t}$ represent white noise.

$$y_t = y_{t-1} + \eta_{y,t} \tag{17}$$
$$m_t = m_{t-1} + \pi_t + \eta_{m,t} \tag{18}$$
$$\pi_t = \pi_{t-1} + \eta_{\pi,t} \tag{19}$$

3.1 Model Solution

The solution procedure leading to (20) is contained in the appendix.

$$s_t = (1 - c_2)m_t + c_2 p_t - \phi(1 - c_2)y_t + \lambda(1 - c_2)\pi_t$$
$$- [1/\lambda(1 - c_2\delta - \rho_m)]v_{m,t} \tag{20}$$

Here $c_2 = (1 - \sqrt{1 + 4/\lambda\delta})/2 < 0$. While Frankel (1979) focuses only on the determination of s, Driskill and Sheffrin (1981) consider the model's implied solutions for i and p as well. The Driskill and Sheffrin (1981) restricted model consists of equations (20), (12), and (13), and their corresponding unrestricted model is (21), (22), and (23).[9]

$$s_t = c_1 m_t + c_2 p_t + c_3 y_t + c_4 \pi_t + \epsilon_{s,t} \tag{21}$$
$$i_t = b_1 m_t + b_2 p_t + b_3 y_t + \epsilon_{i,t} \tag{22}$$
$$p_t = a_1 s_{t-1} + a_2 p_{t-1} + a_3 \pi_{t-1} + \epsilon_{p,t} \tag{23}$$

To move from the RID model to(21), we must drop the interest rate differential and add the relative price level to the regressors. The negative coefficient that the RID model predicts for the interest differential is now found on the price level. (We will return to this.)

The rational expectations solution of the model implies seven within-equation and cross-equation parameter constraints. They are shown in (24).[10]

$$\left. \begin{array}{llll} a_1 + a_2 = 1 & a_3 = 1 & b_1 + b_2 = 0 & c_1 + c_2 = 1 \\ & -c_3/(1 - c_2) = b_3/b_2 & (= \phi) & \\ & c_4/(1 - c_2) = 1/b_2 & (= \lambda) & \\ & \rho_s = \rho_m & & \end{array} \right\} \tag{24}$$

Note two small divergences between our theoretical presentation and that of Driskill and Sheffrin (1981). First, they treat the monetary shock, v_m, as serially correlated in their empirical discussion, while it is white noise in their algebra. To avoid the resulting inconsistency in exposition, we allow for serial correlation in our algebra. Second, they ignore the constraint in the price equation on the expected inflation variable coefficient ($a_3 = 1$), while we include it. In section 3.3, we deal with these issues and their implications in greater detail. But first we attempt to replicate the Driskill and Sheffrin (1981) results.

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[9] Without the shocks, the restricted model compares to equations (10)–(12) in Driskill and Sheffrin (1981): just set $\rho_m = 0$ in (20).

[10] Driskill and Sheffrin specify neither the restriction on $a_3 = 1$ nor the restriction $\rho_s = \rho_m$. Since they drop π in their final estimations, the first omission might be considered irrelevant to their empirics. (Also, note that they state the solution for p_t in terms of π_t by invoking (19).)

3.2 Replication: Driskill and Sheffrin (1981)

In this section we discuss our attempts to replicate key empirical results from the Driskill and Sheffrin (1981) study. Section 3.2.1 discusses some data issues that arise immediately. Section 3.2.2 presents some results using the original Frankel (1979) data. We find the empirical evidence apparently weighs against the Driskill and Sheffrin model. However our replication of the Driskill and Sheffrin (1981) study highlights further problems with their methods. Section 3.3 explains these problems and our attempts to resolve them. Correcting for these issues offers some improvement over our the Driskill and Sheffrin (1981) results.

3.2.1 Data Diagnostics

Driskill and Sheffrin (1981) do not discuss their data in any detail, simply noting that it is from Frankel. While the original Frankel data set does not allow an exact replication of the Driskill and Sheffrin (1981) data diagnostics, as seen in Table 2, the results are similar.[11]

Driskill and Sheffrin argue that the results in Table 2 indicate that relative money supply and relative income follow a random walk. Of course under the null hypothesis of a unit root, the distribution of t-ratio is non-standard for these regressions. Nevertheless, it is evident (as can be confirmed by augmented Dickey-Fuller regressions) that the levels of these variables contain a unit root. (That is, the sum of the coefficients on the lagged variables in Table 2 does not differ significantly from unity.) In addition, corresponding to the assumed data generating process for the exogenous variables, the coefficient on the one period lagged variable is relatively close to 1 (regressions 1–6 of Table 2), and the coefficients on the two-periods and three-periods lagged terms are always small enough to be insignificantly different from zero (regressions 2, 3, 5, and 6). Driskill and Sheffrin also argue (against Frankel) that relative money supply growth does not follow a random walk, citing the results for regression 7. The replications leave their qualitative conclusions intact.

However, in the context of a collection of tests of the stochastic specification of the exogenous variables, regression 7 of Table 2 offers a bit of a puzzle. Recall the RIDRE assumptions that relative money supply follows a random walk around a trend and that this trend in turn follows a random walk. This contrasts with regression 7 of Table 2, which neglects the term π in (18). We report results for (18) and (19) in Table 3. These results suggest that the expected inflation differential follows a random walk; regressing $\Delta \pi_t$ on π_{t-1} yields a coefficient estimate that is not significantly different from zero. However, the stochastic specification on relative money supply is not supported, in the sense that we can reject the null-hypothesis that the coefficient on π is unity in the DGP for the money supply.[12] However if we work with a longer data set, as reported in the bottom half of Table 3, we find this prediction is supported by the data.

3.2.2 Estimating the DS81 Empirical Model

In this section we attempt to replicate the RIDRE results reported by Driskill and Sheffrin (1981). Recall that they offer (21), (22), and (23) as their three unrestricted regression equations for relative price, nominal interest differential, and exchange rate. Driskill and Sheffrin initially estimate these using ordinary least squares. They discover the presence of serial correlation in both exchange rate and interest rate equations, based on a Durbin-Watson test, and they correct for this using a Cochrane-Orcutt adjustment.[13]

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[11] Note that we tried every conceivable start and end date in attempting this replication, to no avail. We contacted the authors, but they no longer have their data. Frankel also supplied his data to Haynes and Stone (1981) for their comment on his 1979 article. We are extremely grateful to Stephen Haynes for providing us with this data. Our results are obtained with a fixed final sample size of 44 (July 1974–Feb 1978, *after* adjusting for lags). Fixing the initial sample at July 1974–Feb 1978 and losing observations to due the lagged variables yields comparable results.

[12] Table 3 does not address another implication of the RIDRE model: that Δm is a random walk while $\Delta m - \pi$ is stationary. The existence of a second unit root in the money supply is notoriously controversial, and we sidestep that controversy in this paper. However, we note that despite apparent the unit root in π, $\Delta m - \pi$ appears stationary according to augmented Dickey-Fuller tests. To this extent we find evidence in favor of the stochastic specifications adopted in the RIDRE model.

[13] They also discover that the core inflation differential (π_t) is insignificant in both the exchange rate and price equations. On this basis, they drop π_t and re-estimate both equations. This has little effect on their results, so we report only the results based on the RIDRE model (which includes π where appropriate).

Table 2: DGPs: Diagnostic Autoregressions

Regression Equation						R^2	D.W.
dep. var. $= m_t$		c	m_{t-1}	m_{t-2}	m_{t-3}		
(1)	DS81	-0.108	0.96			0.96	2.25
		(0.02)	(0.03)				
	Frankel data	-0.022	0.95			0.95	2.38
		(0.02)	(0.03)				
(2)	DS81	-0.02	0.81	0.14		0.96	2.02
		(0.02)	(0.16)	(0.15)			
	Frankel data	-0.02	0.75	0.20		0.95	1.99
		(0.02)	(0.15)	(0.15)			
(3)	DS81	-0.03	0.76	-0.08	0.26	0.96	1.97
		(0.02)	(0.16)	(0.20)	(0.16)		
	Frankel data	-0.02	0.70	0.06	0.19	0.95	1.96
		(0.02)	(0.16)	(0.21)	(0.19)		
dep. var. $= y_t$		c	y_{t-1}	y_{t-2}	y_{t-3}		
(4)	DS81	-0.28	0.89			0.76	1.82
		(0.19)	(0.08)				
	Frankel data	-0.01	0.89			0.75	1.80
		(0.01)	(0.08)				
(5)	DS81	-0.27	0.87	0.02		0.76	1.78
		(0.19)	(0.17)	(0.16)			
	Frankel data	-0.010	0.87	0.02		0.75	1.76
		(0.007)	(0.17)	(0.17)			
(6)	DS81	-0.32	0.86	0.19	-0.17	0.77	1.73
		(0.20)	(0.17)	(0.23)	(0.17)		
	Frankel data	-0.01	0.85	0.23	-0.21	0.76	1.69
		(0.007)	(0.17)	(0.23)	(0.17)		
dep. var. $= \Delta m_t$		c	Δm_{t-1}				
(7)	DS81	0.005	-0.12			0.02	2.03
		(0.001)	(0.15)				
	Frankel data	0.006	-0.20			0.04	1.99
		(0.002)	(0.15)				

Notes:
OLS regressions, with standard errors in parentheses.
DS81: Driskill and Sheffrin (1981, table 1).
Frankel data: original Frankel data, 1974.07–1978.02.
Data: monthly, Germany & U.S. (see appendix).

Table 3: DGPs: Money and Expected Inflation

dep. var.	c	Δm_{t-1}	π_t	π_{t-1}	uni	$D.W.$
Original Sample (1974.07–1978.02):						
Δm_t	0.01*	-0.20	0.07			1.99
	(0.002)	(0.15)	(0.53)			
$\Delta \pi_t$	-0.0003*			-0.02		2.01
	(0.0001)			(0.03)		
Extended Sample (1974.07–1998.11):						
Δm_t	0.003*	.13*	1.05*		-0.0002	2.01
	(0.001)	(0.06)	(.56)		(0.002)	
$\Delta \pi_t$	-0.0001*			-0.04*	0.0001	1.69
	(0.00003)			(0.01)	(0.00004)	

Notes:

OLS regressions; OLS standard errors are in parentheses.

m: relative money supply; **π**: expected inflation differential.

Data: monthly, Germany & U.S. (see appendix).

Our attempts at replicating their single equation estimation results are presented in Table 4. The replication results are in general agreement with the original results reported by Driskill and Sheffrin (1981).

First consider the exchange rate equation. We reject the null hypothesis of no autocorrelation in the exchange rate equation error, based on a Durbin-Watson test (see the OLS:78 row of Table 4). Now consider the AR1:78 row of Table 4, which adds an AR(1) correction to the previous regression. The coefficients are correctly signed, but only the relative price level has a coefficient that is significant at the 5% level. At the 10% level the money supply coefficient is also significant but is also much smaller than predicted. Driskill and Sheffrin, on the other hand, find only relative income to be significant. The point estimate of the relative money supply coefficient is comparable to the Driskill and Sheffrin estimate, and it is significantly less that predicted by the overshooting theory. Similarly, the sum of the coefficients on the relative money supply and the relative price level is much smaller than the predicted value of unity.

Next consider the interest rate equation. As in Driskill and Sheffrin, we detect serial correlation in the residuals of the OLS regression (see the OLS:78 row of Table 4). Following Driskill and Sheffrin, we introduce an AR(1) correction, reported in the AR1:78 row, and also find only relative money supply to be significant at the 10% level.

Finally, consider the price equation. Like Driskill and Sheffrin, we find a highly significant coefficient on the lagged relative price variable (see the OLS:78 row of Table 4). The other variables are insignificant even at the 10% level, while Driskill and Sheffrin find the lagged exchange rate coefficient to be statistically significant. The theory predicts that the sum of the coefficients on the lagged relative price level and lagged exchange rate should be 1. Our estimates sum to 0.954 and is less than one standard deviation away from 1. The Driskill and Sheffrin estimates sum to 0.84 and this is significantly less than 1. In this modest respect, our RIDRE results are an improvement on Driskill and Sheffrin (1981).

Based on the single-equation estimation results of Table 4, our preliminary conclusion is that the data offer little support for the RIDRE model. Our attempted replications generally support the Driskill and Sheffrin (1981) conclusions, except for a marginal improvement in the price equation.[14]

Driskill and Sheffrin also estimate their three restricted equations simultaneously using non-linear least squares. (They refer to this as a full-information maximum-likelihood (FIML) method, but see item 5 in section 3.3.) Based on their tests of the exogenous processes and single-equation estimation results, they deem inclusion of the expected inflation differential "not appropriate" (p.1071, footnote 7). In their final

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[14] We observe that, in the case of the Driskill and Sheffrin exchange rate and price equations, our estimates of the coefficient on expected inflation differential (π_t) are quite different than theirs. In the case of the interest rate equation, *all* of our coefficient estimates are quite different from theirs. We transform all interest rates into absolute one-month rate-of-return terms (i.e. dividing the percent-per-annum rates by 1200) as implied by the model and the data frequency; apparently Driskill and Sheffrin did not. We will address this issue in section 3.3.

Table 4: RIDRE Model: Single Equation Estimation

Exchange Rate Equation		variable(predicted effect on s_t)					R^2	D.W.	ρ
	c	$m_t(>1)$	$p_t(<0)$	$y_t(<0)$	$\pi_t(>0)$	uni			
OLS:DS81	-4.66	0.80	1.92	-0.70	0.05		0.81	0.78	
	(-0.51)	(0.17)	(0.82)	(-0.19)	(0.009)				
OLS:78	0.49*	0.94*	2.03*	-1.22*	20.01*		0.86	1.21	
	(0.21)	(0.14)	(0.47)	(0.20)	(2.91)				
OLS:98	0.32*	-0.34*	0.82*	0.004	-54.85*	0.03	0.62	0.05	
	(0.06)	(0.09)	(0.09)	(0.14)	(7.21)	(0.04)			
AR1:DS81	-4.75	0.37	-0.66	-0.36	0.014		0.98	1.29	0.98
	(0.54)	(0.27)	(-0.88)	(-0.19)	(0.01)				(0.03)
AR1:78	1.95*	0.39*	-1.55*	-0.30	3.38		0.93	1.40	1.04*
	(0.46)	(0.22)	(0.75)	(0.19)	(3.86)				(0.03)
AR1:98	0.57*	0.14	0.35	-0.02	-9.49*	0.02	0.98	1.37	0.99*
	(0.20)	(0.13)	(0.47)	(0.08)	(5.55)	(0.03)			(0.01)
Interest Rate Equation		variable(predicted effect on i_t)					R^2	D.W.	ρ
	c	$m_t(<0)$	$p_t(>0)$	$y_t(>0)$		uni			
OLS:DS81	31.52	0.90	12.16	12.38			0.20	0.52	
	(17.71)	(4.58)	(18.54)	(6.79)					
OLS:78	0.004	-0.003	-0.01	0.05*			0.21	0.51	
	(0.01)	(0.01)	(0.04)	(0.02)					
OLS:98	-0.01*	-0.004*	-0.003*	0.02*		0.01*	0.66	0.34	
	(0.001)	(0.001)	(0.001)	(0.001)		(0.0003)			
AR1:DS81	1.27	-23.11	-1.78	6.40			0.71	2.10	0.82
	(16.49)	(7.53)	(26.35)	(6.09)					(9.41)
AR1:78	-0.03	-0.03*	0.04	0.02			0.70	2.07	0.80*
	(0.02)	(0.02)	(0.05)	(0.02)					(0.07)
AR1:98	-0.001	-0.0002	-0.01	0.003		0.0004	0.92	1.65	0.95*
	(0.002)	(0.002)	(0.01)	(0.002)		(0.001)			(0.02)
Price Equation		variable(predicted effect on p_t)					R^2	D.W.	
	c		$p_{t-1}(<1)$	$s_{t-1}(>0)$	$\pi_t(>0)$	uni			
OLS:DS81	0.08		0.81	0.03	0.0006		0.94	1.39	
	(0.05)		(0.09)	(0.01)	(0.0008)				
OLS:78	0.01		0.96*	0.003	0.08		0.97	1.35	
	(0.03)		(0.07)	(0.02)	(0.51)				
OLS:98	-0.01*		0.99*	0.01*	0.33*	0.002*	1.00	1.59	
	(0.001)		(0.002)	(0.001)	(0.18)	(0.001)			

Notes:

*: |t-ratio|>1.65.

OLS: Ordinary Least Squares; with estimated standard errors in parentheses.

AR1: AR1 correction; with estimated standard errors in parentheses.

DS81: Driskill and Sheffrin (1981, Table 2); **78**: Frankel data (1974.07–1978.02); **98**: IFS data (1974.07–1998.11).

Data: monthly, Germany & U.S. (see appendix).

estimations, they drop the expected inflation differential from the exchange rate and price equations.

Driskill and Sheffrin also report a likelihood-ratio test of the validity of the rational-expectations restrictions. The Driskill and Sheffrin restricted estimation calculates a total of 8 parameters: three structural parameters (δ, λ, and ϕ), three constants, and two autoregressive parameters.[15] The corresponding unrestricted model estimates a total of 13 parameters: eight coefficients, three constants, and two AR(1) parameters. The Driskill and Sheffrin results (DS81) and our attempted-replication results (DS81:78) are shown in Table 5.

Table 5: System Estimation of the DS81 and RIDRE Models

Model:	DS81	DS81:78	RIDRE:78	RIDRE:98	RIDRE:uni
π included?	no	no	yes	yes	yes
$\rho_s = \rho_m$?	no	no	yes	yes	yes
Parameter					
δ	0.04	0.02	0.03*	0.02*	0.02*
	(0.01)	(0.01)	(0.02)	(0.001)	(0.01)
λ	-0.06	360.64	56.27*	34.52*	64.22*
	(0.02)	(1071.54)	(11.38)	(2.72)	(12.56)
ϕ	0.21	0.32	-0.03	0.05	-0.09
	(0.14)	(0.20)	(0.20)	(0.08)	(0.13)
ρ_m	0.95	0.78*	0.70*	1.00*	0.99*
	(0.07)	(0.14)	(0.11)	(0.004)	(0.02)
ρ_s	1.01	1.00*	—	—	—
	(0.03)	(0.03)			
c_2	complex	-0.15	-0.45	-0.97	-0.47
$1 + \delta(c_2 - 1)$	complex	0.98	0.96	0.97	0.97
Likelihood-Ratio Test of RIDRE Restrictions:					
Parameters	8	8	7	10	7
Restrictions	5	5	11	11	11
LR Statistic	65.52	34.07	89.05	341.77	172.02
$\mathcal{X}^2_{df}(.01)$	15.09	15.09	24.72	24.72	24.72

Notes:
Asymptotic standard errors are in parentheses.
$\boldsymbol{\delta}$: price adjustment parameter; $\boldsymbol{\lambda}$: interest rate semi-elasticity; $\boldsymbol{\phi}$: income elasticity; $\boldsymbol{\rho_m}$: AR(1) parameter for the interest rate equation. $\boldsymbol{\rho_s}$: AR(1) parameter for the exchange rate equation (if not restricted to equal ρ_m).
DS81: Driskill and Sheffrin (1981, Table 3); **DS81:78**: DS81 model, Frankel data, 1974.07–1978.02; **RIDRE:78**: RIDRE model, IFS data, 1974.07–1978.02; **RIDRE:98**: RIDRE model, IFS data, 1974.07–1998.11; **RIDRE:uni**: RIDRE model, IFS data, 1991.01–1998.11.
Data: monthly, Germany & U.S. (see appendix).

We initially approach replication by estimating the DS81 model with the Frankel data. Our results in the DS81:78 column of Table 5 really lend no more support to the RIDRE model than the Driskill and Sheffrin (1981) results.[16] Driskill and Sheffrin find only δ to be significant and correctly signed; ϕ is correctly signed but insignificant, and λ is significant but incorrectly signed. We obtain correct signs on all coefficients, however we find none of the parameters of interest differ significantly from zero.

The parameter estimates for λ and δ reported in column DS81 of Table 5 imply a complex value for c_2 (the

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[15] Driskill and Sheffrin ignore the cross-equation equality restriction between the two autoregressive parameters. At this point, we do the same. We correct for this in section 3.3.

[16] However, it should be noted that these results are very fragile. For example, much better looking results can be obtained with an alternative interest rate series.

relative price level coefficient in the exchange rate equation). This conflicts with the saddle-path dynamics that are a core constituent of the RIDRE model. However, our attempted replication is more supportive, as shown in the DS81:78 column of Table 5. We find a negative computed value for c_2, as predicted. (This is the exchange rate overshooting condition, which is in fact assured by our positive estimates for λ and δ.) In addition, estimated parameters satisfy the condition $0 < 1 + \delta(c_2 - 1) < 1$, which assures the monotonic saddle-path dynamics generally presumed to characterize the overshooting model (Isaac 1996). (Equivalently, given our positive estimates for λ and δ, we find $\delta < \lambda/(1 + \lambda)$.)

That is the good news for the RIDRE model. However, like Driskill and Sheffrin, we find that a likelihood-ratio test easily rejects the overidentifying restrictions implied by the rational expectations hypothesis. In light of this, the results reported in column DS81:78 of Table 5, while differing from the Driskill and Sheffrin (1981) results reported in column DS81, support their basic contention that the RIDRE model is a poor fit to the data. However, in attempting this replication we ran into some issues which require further exploration. We address these in the next section, and we then discuss the "improved" estimates reported in the RIDRE:78 column of Table 5.

3.3 Estimating the RIDRE Model

In this section, we outline some problems we encountered as we attempted to replicate the Driskill and Sheffrin (1981) study. We fix these problems and report "corrected" empirical results in column RIDRE:78 of Table 5.

1. *Complex value for c_2.*

   Recall that c_2 is the coefficient on the relative price level in the exchange rate equation (20). In the RIDRE model, c_2 is given by

   $$c_2 = \left(1 - \sqrt{1 + 4/\lambda\delta}\right)/2 \tag{25}$$

   Since λ and δ should be positive, c_2 is predicted to be negative. Driskill and Sheffrin (1981, Table 3) report estimates of λ and δ that imply a negative discriminant, and they therefore report a complex value for c_2. As a result, the Driskill and Sheffrin estimates violate the saddle-path dynamics that are a core constituent of the RIDRE model.

   This result must be in error: a complex c_2 implies a complex value for the restricted likelihood function, or more generally for the generalized variance of the equation system. In the DS81:78 and RIDRE columns of Table 5 we report results that arise when this restriction is correctly imposed, and we find no such problem.

2. *Dropping the expected inflation differential variable (π) from exchange rate and price equations.*

   Based on their diagnostic autoregressions and single equation estimations, Driskill and Sheffrin drop the expected inflation differential variable from their joint estimation procedure. By doing so, they effectively adopt an alternative price adjustment mechanism and an alternative stochastic specification for the relative money supply.

   $$p_{t+1} - p_t = \delta(s_t - p_t) + v_{p,t+1} \tag{26}$$

   $$m_t = m_{t-1} + \eta_{m,t} \tag{27}$$

   In a comment upon Frankel (1979), the use of (26) and (27) is particularly odd, since Frankel places great emphasis on the role of the core-inflation differential. Thus, despite the title of their paper, the Driskill and Sheffrin study ceases to be a critique of Frankel's real interest differential theory of exchange rate determination. In the Table 5 RIDRE estimations, we retain the role of the expected inflation differential in the price and exchange rate equations. (As it turns out, however, this has little effect on the estimated values of the other coefficients.)

   Dropping the core-inflation terms from the regressions may be thought of as restricting the coefficients a_3 and c_4 to zero. However, as explained above, the parameter a_3 in the price equation (23) is restricted

to be unity: $a_3 = 1$. Similarly, $c_4 = \lambda(1 - c_2)$ is a RIDRE restriction. We therefore impose the constraints on a_3 and c_4 as over-identifying restrictions in our FIML estimations. (We note in passing that imposing this restriction on a_3 is appropriate only when using appropriately defined interest rates: absolute one-month terms, when using monthly data.)

3. *The definition of interest rates.*

   Recall the single-equation estimation results reported in Table 4: our estimated coefficients on expected inflation differential (π_t) are much larger than the Driskill and Sheffrin estimates. For the interest rate equation, our estimated coefficients are much *smaller* that those reported by Driskill and Sheffrin. This suggests a difference in interest rate scaling. Frankel (1979) divided interest rates by 400 to turn them from annual to quarterly rates of return: this shows up in the regressions using his data. For the extended sample regressions on Table 4, we use an absolute one-month rates of return, which we argue is the correct measure. The contrast in the coefficient sizes reported in Table 4 suggest that Driskill and Sheffrin (1981) failed to make either transformation.[17]

4. *Conflicting assumptions about the money market shock ($v_{m,t}$).*

   While solving the RIDRE model, Driskill and Sheffrin assume that the money market shock ($v_{m,t}$) is white noise. In their empirical work, however, they allow for the evident serial correlation in the errors of the interest rate and exchange rate equations. Under rational expectations, this implies important contradiction between the theory and their empirical implementation: allowing $v_{m,t}$ to be serially correlated affects the rational expectations solution, as shown in our appendix. Most importantly, there is a cross-equation restriction on first-order autoregressive parameters in exchange rate and interest rate equations: both error terms share the same autoregressive parameter. In the RIDRE columns of Table 5, we report results after imposing this cross-equation restriction (which in fact is not rejected by a Wald test).[18]

5. Endogeneity of p

   Driskill and Sheffrin (1981) offer endogeneity of the interest rate as a primary motivation for their reëvaluation of the RID model. Their point that this endogeneity undermines the standard RID estimates has been widely accepted. Ironically, their own formulation suffers from an identical problem: p_t is correlated with the error in the exchange rate equation unless we add an *ad hoc* stipulation that $\mathrm{corr}(v_{m,t}, v_{p,t}) = 0$. The RID model offers a quick way to see the point. Substituting (12) into (10) yields (28), which clearly links the RID and the RIDRE exchange rate equations. (Just set $\theta = -1/\lambda c_2$).

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[17]Naturally, this rescaling of the nominal interest rate affects all of the coefficients in the interest rate equation via the rescaling of the estimated interest rate semi-elasticity of money demand (λ). Similarly, rescaling the expected inflation term (π_t) would affect only its own coefficient in the price and exchange rate equations (a_3 and c_4 respectively); all other coefficients remain unchanged.

Our use of an absolute one-month definition for interest rates (dividing percent-per-annum rates by 1200) is based on our use of monthly data plus the following considerations.

- In the relative price adjustment equation (13), π_t is the core one-month change in relative prices.
- The money supply DGP (18) implies the expected one-month change in relative money will be π_t.
- The uncovered interest parity assumption (1) states that the nominal interest rate differential is given by the expected one-month change (depreciation rate) of the exchange rate.

[18]There is a related problem that we finesse in order to stay close to the original RIDRE model. As the model is laid out by Driskill and Sheffrin (1981), the error terms in the interest-rate and exchange-rate equations should be perfectly correlated. Driskill and Sheffrin simply ignore this implication, and we will essentially follow them in this. As a justification, we simply allow for unmodeled white noise to disturb the interest rate and exchange rate equations (*after* the AR(1) transformation).

We might also, very naturally, include random deviations from uncovered interest parity or turn to Muth-rational rather than Lucas-rational expectations. For example, addition of a white noise risk premium leads to a reduced form exchange rate equation where the error term involves the serially correlated error money market shock, $v_{m,t}$, and the risk premium while interest rate equation still involves $v_{m,t}$ only (Isaac 1998). We do not pursue this reasonable modification of the RIDRE model both out of fidelity to the original RIDRE project and to avoid an econometric complication: the transformation needed to deal with the serially correlated money market shock would introduce a moving average of the risk premium into the exchange rate equation.

And through this linkage, we return to the discussion of consistent estimation in section 2.3.

$$s_t = \left(1 + \frac{1}{\theta\lambda}\right)(m_t - \phi y_t + \lambda \pi_t) - \frac{1}{\theta\lambda}p_t + \nu_t - \frac{1}{\theta\lambda}\upsilon_{m,t} \tag{28}$$

Consider the exchange rate equation (28). The sign predictions of the model for m, y and π are unchanged from our earlier discussion, but we now expect a negative coefficient on p that was previously expected on i. In addition, the coefficient on m is now expected to be greater than unity. However, we have seen that (28) is not generally an appropriate regression equation. To get a true reduced form, we need to substitute (13) into (28). Consider the resulting exchange rate equation (29). The predictions of the model for m_t, π_t and y_t are unchanged from our earlier discussion, but the negative coefficient that was previously expected on p_t is now expected on s_{t-1}, p_{t-1}, π_{t-1}.

$$\begin{aligned} s_t = &\left(1 + \frac{1}{\theta\lambda}\right)(m_t - \phi y_t + \lambda \pi_t) - \frac{1}{\theta\lambda}[(1-\delta)p_{t-1} + \delta s_{t-1} + \pi_{t-1}] \\ &+ \nu_t - \frac{1}{\theta\lambda}(\upsilon_{m,t} + \upsilon_{p,t}) \end{aligned} \tag{29}$$

Making some concessions to minimizing notation, we can write the observable reduced form for the RIDRE model as (30), (31), and (32) subject to the same cross-equation restrictions as before.

$$\begin{aligned} s_t = &(1-c_2)m_t + c_2[\delta s_{t-1} + (1-\delta)p_{t-1} + \pi_{t-1}] - \phi(1-c_2)y_t + \lambda(1-c_2)\pi_t \\ &- [1/\lambda(1 - c_2\delta - \rho_m)]\upsilon_{m,t} + c_2\upsilon_{p,t} \end{aligned} \tag{30}$$

$$i_t = -(1/\lambda)m_t + (1/\lambda)[\delta s_{t-1} + (1-\delta)p_{t-1} + \pi_{t-1}] + (\phi/\lambda)y_t + (1/\lambda)(\upsilon_{m,t} + \upsilon_{p,t}) \tag{31}$$

$$p_t = \delta s_{t-1} + (1-\delta)p_{t-1} + \pi_{t-1} + \upsilon_{p,t} \tag{32}$$

In contrast with equation (20), which was used by Driskill and Sheffrin (1981), an unrestricted single-equation estimation of (30) provides consistent parameter estimates for a RIDRE economy.[19] Consider the exchange rate equation (30). The predictions of the model for m_t, π_t and y_t are unchanged from our earlier discussion, but the negative coefficient that was previously expected on p_t is now expected on s_{t-1}, p_{t-1}, π_{t-1}. Another way to look at this is to say that the instrumental variables results reported in Table 1 give us a better single equation approach to the RIDRE model than do the results in Table 4.

The RIDRE columns of Table 5 report a corrected estimation of the RIDRE model: interest rates are measured as absolute one-month rates, we correctly impose the implied restriction between c_2 and the structural parameters, we retain the core-inflation variable and correctly restrict its coefficient in the price equation and the exchange rate equation, we correctly model the serially correlated money market shock (which imposes equal autoregressive parameters on the exchange rate and interest rate equations), and we acknowledge the endogeneity of p in the RIDRE system. The results using the original sample are shown in the column RIDRE:78 of Table 5.[20]

The results from the RIDRE model produce correctly signed parameter estimates. We find δ and λ differ significantly from zero, with only ϕ insignificant at the 10% level. Our estimate of the price adjustment coefficient (δ) is close to that of Driskill and Sheffrin, but our estimate of λ—the interest rate semi-elasticity of money demand—is not of the same order of magnitude as theirs. In contrast with the Driskill and Sheffrin

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[19]Note that since the price equation fits pretty well, we might not expect this to make a large difference—and it does not. Note that a similar accommodation could be made for the 'exogenous' variables in the model, but since assuming block diagonality of the covariance matrix is a standard approach to the forcing variables, we do not pursue this point here. (Results allowing for this are available upon request.) Furthermore, in their discussion of the likelihood ratio test, Driskill and Sheffrin (1981) make it clear that they ignored the endogeneity of p when estimating the system (20), (12), and (13). Thus their "FIML" estimates simply minimize the generalized variance of that system, as it stands, which will generally yield inconsistent estimates.

[20]The Frankel (1979) data we obtained is unfortunately fully transformed, so to respond to the concerns we have enumerated we use *IFS* data even for this estimation over the original sample.

estimate, conversion to a per annum basis (roughly, division by 1200) shows our estimate of λ to be plausible. The final structural parameters also satisfy both the stability and overshooting conditions. Nevertheless, a likelihood-ratio test still rejects the overidentifying rational expectations restrictions. In short, our repairs to the RIDRE model lead to more favorable results than those discussed in section 3.2.2. Our overall conclusion is that the results reported in column RIDRE of Table 5 offer modestly better empirical support in favor of the RIDRE model than those reported by Driskill and Sheffrin (1981).

4   Estimation over an Extended Sample

In this section we update our empirical analyses and examine the recursive coefficient estimates.

4.1   The RID Model

Recall the first row of results in Table 1, which reports the rather promising OLS coefficient estimates reported by Frankel (1979). These results generated extensive empirical work on the RID model in the late 1970s. Initially this research in the late 1970s corroborated Frankel's encouraging results, but extension of the sample period past 1978 offered a dramatic contrast. Researchers began to report insignificant, negatively signed coefficients on relative money supplies.[21] As data accumulated, the support for the model deteriorated. As a representative example from the mid-1980s, consider the results of Baillie and Selover (1987). Using monthly German and U.S. data over the sample 1973.03–1983.12, they find a complete lack of support for the model. Most disturbing is their result that the OLS estimate of the coefficient on the relative money supply was significant but of the wrong sign. (They find similar problems for other countries.) Significant, perversely-signed coefficients on relative money supply are a common finding in later work—a startling problem for any "monetary" approach. Baillie and Selover (1987) note that correcting for serial correlation in the residuals eliminates the sign difficulties, but then all estimated coefficients appear insignificantly different from zero.

The OLS:98 and AR1:98 rows of Table 1 report results for the sample 1974.07–1998.11, indicating that serious difficulties continue plague the RID model even an extended data set.[22] The associated instrumental-variables regressions, reported in the IV0:98 and IV1:98 rows, are no better.

The difficulties with the basic RID model can be displayed even more dramatically. Figure 1 plots the recursive OLS estimates for the RID model. The parameter estimates meander, appearing upon causal inspection to follow a random walk. There is no evidence of parameter stability. (The "constrained version" of the model—with the coefficient on the money supply constrained to unity—yields similar results.)

Figure 1: RID Model, Recursive Coefficient Estimates (OLS)

[Figure 1 about here.   ]

Figure 2 displays the instrumental-variables recursive coefficient estimates for the RID model given an AR(1) correction. The situation is quite different: the parameter estimates no longer move dramatically over time. (One exception is the jump in the interest rate coefficient immediately following unification.) However, the spot rate now appears unrelated to the explanatory variables, with the short-term interest rate being a partial exception. From these two figures we must conclude the Frankel's empirical validation of the RID model was pure historical accident.

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[21] As we have seen, the theory predicts unity. Backus (1984) offers an exception to the rule: he supports this prediction for the CAD/USD exchange rate.

[22] Our analysis includes an intercept dummy for the German unification (uni), which takes on a value of 1 from January 1991 onwards. The German Economic, Monetary, and Social Union (GEMSU) between the former Federal Republic of Germany (FRG) and German Democratic Republic (GDR) came into effect on July 1, 1990. The deutsche mark became the sole currency in the GEMSU area, and customs borders were abolished. On October 3, 1990, the former GDR became part of the FRG. The money supply data series (m) jumps in January 1991 because from then on it includes data for the former GDR. The other data series are unaffected.

Figure 2: RID Model, Recursive Coefficient Estimates (IV1)

[Figure 2 about here.   ]

4.2 The RIDRE Model

We also re-estimate the RIDRE model over the extended sample. As with the RID model, our analysis includes an intercept dummy for the German unification (*uni*), which takes on a value of 1 from January 1991 onwards. Single equation estimates for the three reduced form equations are shown in Table 4. The results for the extended sample (:98) are in general agreement with the replication results (:78): only the price equation looks at all promising, and that is driven largely by the inertia in the relative price level.

First consider the exchange rate equation. The OLS results, reported in the OLS:98 row of Table 4, finds almost all the estimated coefficients significantly different from zero (even at the 1% level), but unfortunately they are all incorrectly signed. When we include an adjustment for serial correlation, sign problems remain and most of the estimated coefficients differ insignificantly from zero (even at the 10% level).

The interest rate equation is a little better, with sign reversals only for p. OLS estimation of the RIDRE interest rate equation also finds all the variables to differ significantly from zero. The coefficients on relative money and relative income are correctly signed, but the sign on the coefficient on relative price is incorrect. This time, the introduction of an AR(1) correction retains the same pattern in the sign of the coefficients. Additionally, the relative price variable loses its significance in the regression.

Finally, consider the price equation. The coefficients on lagged relative price and lagged exchange rate in the Driskill and Sheffrin (1981) price equation are highly significant and correctly signed. The results reported in the OLS:98 row are similar.

It is worth taking a quick look at the recursive OLS coefficient estimates for the Driskill and Sheffrin (1981) version of the RIDRE exchange rate equation. The OLS results are presented in Figure 3. There is some suggestion that the parameter estimates are slowly settling down over time, although they are subject to a great deal of variation. For the RIDRE model, however, the results are frustrating, since the sign pattern of the estimated coefficients appears to be converging to the precise opposite of that predicted by the model.

Figure 3: RIDRE Model, Recursive Coefficient Estimates (OLS)

[Figure 3 about here.   ]

However, as we have seen, there is reason to suspect that Driskill and Sheffrin (1981) did not consider a valid regression equation. For this reason, Figure 2 is more interesting as an assessment of the RIDRE model. Unfortunately, it is not any more encouraging.[23]

Surprisingly, the results of the FIML estimation of the simultaneous equation system implied by the RIDRE model are a bit more promising. For the extended sample, these results are shown in the RIDRE:98 column of Table 5. Qualitatively, the results match those from the RIDRE:78 column. Both δ and λ are correctly signed and significant, but ϕ differs insignificantly from zero. Recalling that we are working with monthly absolute rates of return, our estimate for the interest rate semi-elasticity of money demand (λ) is of reasonable magnitude. (In all three equations, we also obtain for the unification dummy significant coefficients, which are not reported.) The conditions for exchange rate overshooting and monotonic saddle-path dynamics are satisfied. However, as with the shorter sample, there is no support for the RIDRE restrictions implied by the rational expectations hypothesis. Using a likelihood ratio test, the restrictions are overwhelmingly rejected. Once again, we do not find much evidence to support the RIDRE model, although our results remain more favorable than those reported by Driskill and Sheffrin (1981). As in the

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[23]As an aside, note that addition of an energy price regressor eliminates some parameter instability without altering the qualitative conclusions. (Results available upon request.)

replication study, we find some support from the FIML estimations, while the single equation estimates are thoroughly discouraging.

Table 5 also reports RIDRE results for a post-unification sample. Looking at column RIDRE:uni, we find that the choice of sample makes little difference to the outcome. The results are almost identical to the RIDRE:78 and RIDRE:98 results.

5   Conclusion

As the empirical performance of the simple monetary approach to the determination of flexible exchange rates deteriorated, interest in the Dornbusch (1976) "sticky price" model of exchange rate overshooting grew apace. Frankel (1979) offers the classic empirical formulation of this model: his real interest differential (RID) model. He supports the Dornbusch model against the simple monetary approach model. However, Driskill and Sheffrin (1981) show that Frankel's coefficient estimates are inconsistent. They also stress that Frankel ignores the possibility of testing the overidentifying restrictions imposed upon the model by the rational expectations assumption. Driskill and Sheffrin develop an empirical rational-expectations version of the Frankel model: the RIDRE model.

In this paper, we attempt a replication and update of the Frankel (1979) and Driskill and Sheffrin (1981) results. We successfully replicate the Frankel (1979) results, but upon extending the sample we find all support for the RID model vanishes. Replication of the Driskill and Sheffrin (1981) results proved more problematic, and we provide a discussion of the problems. We find the data cut less decisively against the RIDRE model than suggested by Driskill and Sheffrin (1981), but it is still the case that its performance is poor. The RIDRE model also proves more stable than the RID model, but the over-identifying restrictions implies by the rational expectations hypothesis prove increasingly unacceptable. It does not appear to be a good model of exchange rate determination.

Many researchers will find the decisive lack of support for the very attractive real interest differential model disappointing and puzzling. We stress that we have reviewed the difficulties of only the best known and most popular empirical approaches to the overshooting model—the simple RID model of Frankel (1979) and the RIDRE model of Driskill and Sheffrin (1981). Our negative results for these empirical models is intended to highlight the futility of relying of them for anything beyond introductory pedagogy, but we intend this as a spur to the elaboration of more fruitful empirical implementation of the Dornbusch (1976) approach to exchange rate determination. Our negative results are indeed stark, and it is worth emphasizing that we have focused only on the *in-sample* properties of these models.

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Appendix

Model Solution Details under Rational Expectations

The model solution presented in this appendix follows the solution presented in Driskill and Sheffrin's appendix very closely. Rewrite (16) as

$$s_t = \mathcal{E}_t s_{t+1} - i_t \tag{33}$$

and substitute the solution for i_t from (11) to get (34).

$$s_t = \mathcal{E}_t s_{t+1} + \frac{1}{\lambda}(m_t - p_t - \phi y_t - v_{m,t}) \tag{34}$$

Using the method of undetermined coefficients, we "guess" that the final solution for s_t will be of the form (35). (We can include p since it is predetermined; see Isaac (1998) for an alternative solution procedure.)

$$s_t = c_1 m_t + c_2 p_t + c_3 y_t + c_4 \pi_t + c_5 v_{m,t} \tag{35}$$

Take expectations of the implied s_{t+1} at time t to get (36).

$$\mathcal{E}_t s_{t+1} = c_1 \mathcal{E}_t m_{t+1} + c_2 \mathcal{E}_t p_{t+1} + c_3 \mathcal{E}_t y_{t+1} + c_4 \mathcal{E}_t \pi_{t+1} + c_5 \mathcal{E}_t v_{m,t+1} \tag{36}$$

This compares with (A3) in the Driskill and Sheffrin appendix. (Note that in their solution algebra, Driskill and Sheffrin assume $\mathcal{E}_t v_{m,t+1} = 0$; the last term in (36) thereby disappears.)

Recalling that η_π, η_m, η_y, v_p, and η_z are assumed to be white noise, equations (13) through (19) imply

$$\mathcal{E}_t \pi_{t+1} = \pi_t$$
$$\mathcal{E}_t m_{t+1} = m_t + \pi_t$$
$$\mathcal{E}_t y_{t+1} = y_t$$
$$\mathcal{E}_t p_{t+1} = \delta s_t + (1 - \delta)p_t + \pi_t$$
$$\mathcal{E}_t v_{m,t+1} = \rho_m v_{m,t}$$

Substituting these into equation (36) gives an expression for the current period expectation of the one-period ahead exchange rate.

$$\begin{aligned} \mathcal{E}_t s_{t+1} &= c_1(m_t + \pi_t) + c_2[\delta s_t + (1 - \delta)p_t + \pi_t] + c_3 y_t + c_4 \pi_t + c_5 \rho_m v_{m,t} \\ &= c_1 m_t + c_2(1 - \delta)p_t + c_3 y_t + (c_1 + c_2 + c_4)\pi_t + c_2 \delta s_t + c_5 \rho_m v_{m,t} \end{aligned} \tag{37}$$

Substituting (37) into equation (34) we get,

$$\begin{aligned} s_t &= c_1 m_t + c_2(1 - \delta)p_t + c_3 y_t + (c_1 + c_2 + c_4)\pi_t + c_2 \delta s_t + c_5 \rho_m v_{m,t} \\ &\quad + \frac{1}{\lambda}(m_t - p_t - \phi y_t - v_{m,t}) \end{aligned} \tag{38}$$

and rearranging, yields (39).

$$\begin{aligned} (1 - c_2 \delta)s_t &= \left(c_1 + \frac{1}{\lambda}\right)m_t + \left[c_2(1 - \delta) - \frac{1}{\lambda}\right]p_t + \left(c_3 - \frac{\phi}{\lambda}\right)y_t \\ &\quad + (c_1 + c_2 + c_4)\pi_t + \left(c_5 \rho_m - \frac{1}{\lambda}\right)v_{m,t} \end{aligned} \tag{39}$$

Equation (39) matches (A5) in the Driskill and Sheffrin (1981) appendix (setting $\rho_m = 0$), except for the sign on ϕ (which is correct in (39)).

Both (35) and (39) express the exchange rate in terms of the same variables. Comparing coefficients across equations we get,

$$c_1 = \frac{1}{(1 - c_2\delta)}\left(c_1 + \frac{1}{\lambda}\right) \tag{40}$$

$$c_2 = \frac{1}{(1 - c_2\delta)}\left[c_2(1 - \delta) - \frac{1}{\lambda}\right] \tag{41}$$

$$c_3 = \frac{1}{(1 - c_2\delta)}\left(c_3 - \frac{\phi}{\lambda}\right) \tag{42}$$

$$c_4 = \frac{1}{(1 - c_2\delta)}(c_1 + c_2 + c_4) \tag{43}$$

$$c_5 = \frac{1}{(1 - c_2\delta)}\left(c_5\rho_m - \frac{1}{\lambda}\right) \tag{44}$$

Equations (40)–(44) match (A6)–(A10) in the Driskill and Sheffrin appendix (setting $\rho_m = 0$ in (44)), except that Driskill and Sheffrin repeat their sign error on ϕ and this is corrected in (42) above. Equation (41) implies (45).

$$c_2^2 - c_2 - \frac{1}{\lambda\delta} = 0 \tag{45}$$

which gives the solutions for c_2 as

$$c_{2,1} = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{\lambda\delta}} \qquad c_{2,2} = \frac{1}{2} - \frac{1}{2}\sqrt{1 + \frac{4}{\lambda\delta}} \tag{46}$$

Given positive λ and δ, the condition for real roots, $1 + 4/\lambda\delta > 0$, is obviously satisfied.

We note that $c_{2,1}$ is positive and greater than one while $c_{2,2}$ is negative. Driskill and Sheffrin eliminate $c_{2,1}$ as a possible solution, by requiring that the price equation be stable. Substituting the exchange rate specification (35) into the price adjustment equation (13), we find

$$p_{t+1} = p_t + \delta(c_1m_t + c_2p_t + c_3y_t + c_4\pi_t + c_5\upsilon_{m,t} - p_t) + \pi_t + \upsilon_{p,t+1} \tag{47}$$

Rearranging yields (48), which compares with (A12) in the Driskill and Sheffrin appendix.

$$p_{t+1} = \delta c_1 m_t + [1 + \delta(c_2 - 1)]p_t + \delta c_3 y_t + (\delta c_4 + 1)\pi_t + \delta c_5 \upsilon_{m,t} + \upsilon_{p,t+1} \tag{48}$$

For the first difference of price level to be stationary, the parameter restriction $|1 + \delta(c_2 - 1)| < 1$ needs to be satisfied. It is clear that $c_{2,1}$ (> 1) will not meet this requirement. Hence, Driskill and Sheffrin rule out this solution, and take $c_2 = c_{2,2} < 0$ as the only possible solution.[24]

Recall that Frankel (1979) develops his model under the regressive expectations hypothesis: expected exchange rate depreciation is equal to the expected long run inflation differential, π_t, plus some proportion θ of the gap between the long run equilibrium exchange rate, $\bar{s}_t$, and current exchange rate, s_t.

$$s_{t+1}^e - s_t = \theta(\bar{s}_t - s_t) + \pi_t \tag{49}$$

This matches equation (2) in Frankel and, together with the uncovered interest parity condition, compares with (A14) in the Driskill and Sheffrin appendix. For this regressive expectations model to be consistent with the Driskill and Sheffrin rational expectations model, Driskill and Sheffrin show that the coefficient of adjustment, θ, should be equal to $-1/\lambda c_2$. This condition is (A18) in the Driskill and Sheffrin appendix. For expectations to regress to a long run equilibrium value, we need θ to be positive. $c_2 = c_{2,2} < 0$ satisfies this requirement.

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[24] The price stability condition requires that $1 > c_2 > 1 - (2/\delta)$. This becomes an implicit restriction on $c_{2,2}$ and thereby on the relative sizes of λ and δ. Since λ and δ are positive, the first inequality is satisfied automatically (i.e., $c_2 < 0$). We usually also impose monotonic saddle-path dynamics—in this case, requiring $\delta < \lambda/(1 + \lambda)$ (Isaac 1996).

For reasons that will become apparent shortly, we note that (45) implies

$$1 - c_2 = -\frac{1}{\delta\lambda c_2} \tag{50}$$

Since we have c_2 in terms of the structural parameters (in (46)), getting solutions for the other coefficients in terms of c_2 would suffice. From (40), and using the algebraic result in (50), we have

$$
\begin{aligned}
c_1 &= -\frac{1}{\delta\lambda c_2} \\
&= 1 - c_2 \qquad > 1
\end{aligned}
\tag{51}
$$

From (42), and using the result in (50), we have

$$
\begin{aligned}
c_3 &= \frac{\phi}{\delta\lambda c_2} \\
&= -\phi(1 - c_2) \qquad < 0
\end{aligned}
\tag{52}
$$

From (43), and using the results in (50) and (51), we have

$$
\begin{aligned}
c_4 &= -\frac{1}{\delta c_2} \\
&= \lambda(1 - c_2) \qquad > 0
\end{aligned}
\tag{53}
$$

From (44) we have

$$c_5 = -\frac{1}{\lambda(1 - c_2\delta - \rho_m)} \tag{54}$$

Plugging these solutions into equation (35) gives the exchange rate solution as (55).

$$
\begin{aligned}
s_t =&(1 - c_2)m_t + c_2 p_t - \phi(1 - c_2)y_t + \lambda(1 - c_2)\pi_t \\
&- \frac{1}{\lambda(1 - c_2\delta - \rho_m)}v_{m,t}
\end{aligned}
\tag{55}
$$

Setting $\rho_m = 0$, this "reduced form" expression for exchange rate is the same as equation (6) in Driskill and Sheffrin. (At this point their sign error disappears; the coefficient on y_t is correctly signed.)

Equation (11) gives the "reduced form" equation for the interest rate, repeated here as (56).

$$i_t = -(1/\lambda)m_t + (1/\lambda)p_t + (\phi/\lambda)y_t + (1/\lambda)v_{m,t} \tag{56}$$

Equation (13) gives the reduced form equation for the price level, repeated here as (57).

$$p_t = \delta s_{t-1} + (1 - \delta)p_{t-1} + \pi_{t-1} + v_{p,t} \tag{57}$$

The Driskill and Sheffrin reduced form price equation is in terms of π_t rather than π_{t-1}. In order to match their specification we can, alternatively, substitute for π_{t-1} from equation (19) to get (58).

$$p_t = \delta s_{t-1} + (1 - \delta)p_{t-1} + \pi_t + v_{p,t} - \eta_{\pi,t} \tag{58}$$

The system of estimable reduced form equations can thus be written as (20), (12), and (13) in the text.

Data

IFS Data

Monthly data for the period 1974.07 to 1998.11 are take from the June 1999 CD of the *International Financial Statistics*. For the U.S., the bond-equivalent T-bill rate is used as the short rate, but the *IFS* series starts at 1974.09 and so two observations are taken from the very similar T-bill series. The data were transformed as follows. Money, income, and prices are transformed into logs of the ratios of German to U.S. data. Interest rates, which the *IFS* reports in annual percentage terms, are divided by 1200 to get one-month absolute returns. Interest differentials are formed as the log of the ratio of the gross monthly rates. Following Frankel (1979) and Driskill and Sheffrin (1981), π is the long-rate differential.

Series	IFS Code
German Data	
Spot Rate	134..RF.ZF...
M1	13439MACZF...
Industrial Production	13466..CZF...
CPI	13464...ZF...
Short (call) rate	13460B..ZF...
Long (bond) rate	13461...ZF...
U.S. Data	
M1	11159MACZF...
Industrial Production	11166..CZF...
CPI	11164...ZF...
Short (bill) rate	11160C..ZF...
Short (bill, bond equiv.) rate	11160CS.ZF...
Long (bond) rate	11161...ZF...

Frankel (1979) Data

For the convenience of the reader, the original Frankel data set is provided here.[25]

```
Variable names and definitions
ECENDM : US cents per DM exchange rate, monthly average.
EDMOOL : ln(DM per US$ exchange rate)
         i.e. EDMOOL=ln[1/(ECENDM/100)]
GRM1   : ln(German M1/US M1)
         German : end of month, seasonally adjusted, billions of marks.
         US : averages of daily figures, seasonally adjusted, billions $.
GRPRD  : ln(German Industrial Production/US Industrial Production)
         German : index, seasonally adjusted.
         US : index, seasonally adjusted.
GR3MR  : Short run interest rate differential: ln[(1+i)/(1+i*)]
         German and US : Representative bond equivalent yields on
         major 3-4 month money mkt instruments, excluding T-Bills.
GRLGR  : Long term interest rate differential
         German and US : Long term govt bond yield, at or near
         end of month.
GRINC  : CPI inflation differential.
         Average logarithmic rate of change over preceding year.
         German: Cost of Living, seasonally adjusted.
         US : Urban dwellers and clerical workers.
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[25] These data were provided to us by Steve Haynes as a photocopy of the photocopy he received from Jeffrey Frankel. The "NOTES" were on this photocopy. Data entry was double-keyed. Legibility was difficult for a few entries, for which we consulted Steve Haynes.

| MONTH	ECENDM	EDMOOL	GRM1	GRPRD	GR3MR	GRLGR	GRINC
7401	35.529	1.03482	-0.705403	-0.0290141	0.00716584	0.00531212	-0.004626
7402	36.844	0.998478	-0.700128	-0.022614	0.00603159	0.00728513	-0.00555666
7403	38.211	0.962047	-0.706315	-0.0323052	0.00433751	0.00659773	-0.00696727
7404	39.594	0.926493	-0.703321	-0.0250189	-0.00343709	0.00635195	-0.00671987
7405	40.635	0.900541	-0.703285	-0.0314035	-0.00593088	0.00705627	-0.00807256
7406	39.603	0.926266	-0.698878	-0.0500564	-0.0068417	0.00712959	-0.00946475
7407	39.174	0.937157	-0.687457	-0.0387001	-0.00538289	0.00713	-0.0106479
7408	38.197	0.962414	-0.682861	-0.0726745	-0.00649652	0.00527149	-0.00903268
7409	37.58	0.978699	-0.678647	-0.0666476	-0.00319306	0.00583462	-0.0107211
7410	38.571	0.95267	-0.681847	-0.0694745	0.00102617	0.00688706	-0.011359
7411	39.836	0.920399	-0.661905	-0.0536235	-0.00100262	0.00552243	-0.0129882
7412	40.816	0.896097	-0.63908	-0.0557512	-0.003179	0.00391237	-0.014606
7501	42.292	0.860572	-0.636935	-0.0271721	0.00194107	0.00315779	-0.01282
7502	42.981	0.844413	-0.631584	-0.0148937	-0.000369944	0.0025715	-0.0122385
7503	43.12	0.841183	-0.621506	-0.00598106	-0.00162696	0.00188483	-0.0100461
7504	42.092	0.865312	-0.611944	-0.014006	-0.00300767	0.000733901	-0.0094186
7505	42.546	0.854585	-0.608514	-0.0433361	-0.00160451	0.000196166	-0.00784676
7506	42.726	0.850363	-0.606938	-0.0668053	-0.00332784	0.000881035	-0.00679852
7507	40.469	0.904634	-0.605622	-0.0739891	-0.00537686	0.00068545	-0.0080178
7508	38.857	0.945283	-0.593259	-0.0859548	-0.00673348	0.00124803	-0.00622968
7509	38.191	0.962571	-0.57023	-0.0950046	-0.00730053	0.00073462	-0.00415864
7510	38.737	0.948375	-0.567142	-0.076592	-0.00481411	0.00173784	-0.00400194
7511	38.619	0.951426	-0.562665	-0.0776953	-0.00446927	0.00127356	-0.00439498
7512	38.144	0.963802	-0.547561	-0.0755666	-0.00422103	0.00166547	-0.00371816
7601	38.425	0.956462	-0.547406	-0.0766602	-0.00304033	0.000734638	-0.00342034
7602	39.034	0.940737	-0.552476	-0.0709593	-0.00388087	-0.000171147	-0.00213181
7603	39.064	0.939969	-0.552262	-0.0863566	-0.00388087	-0.000638012	-0.00183525
7604	39.402	0.931354	-0.555894	-0.0705184	-0.00378296	-0.000343263	-0.00187819
7605	39.035	0.940712	-0.546603	-0.0798158	-0.00521153	9.81204E-05	-0.00304452
7606	38.797	0.946828	-0.529804	-0.0836664	-0.0039259	0.000539277	-0.00330287
7607	38.842	0.945668	-0.54214	-0.0882677	-0.00254304	0.000758857	-0.00323899
7608	39.538	0.927908	-0.535831	-0.0928478	-0.00219775	0.00083312	-0.00242852
7609	40.169	0.912075	-0.537179	-0.0711749	-0.0019507	0.000122536	-0.00332925
7610	41.165	0.887582	-0.541304	-0.068112	-0.00130874	-9.82359E-05	-0.00413946
7611	41.443	0.880852	-0.541023	-0.0795496	-0.0006673	-0.000270631	-0.00296455
7612	41.965	0.868334	-0.567005	-0.0863497	0.000123456	0.000196714	-0.00217253
7701	41.792	0.872465	-0.539816	-0.072254	-0.000296839	-0.0016696	-0.0026383
7702	41.582	0.877503	-0.530119	-0.0805466	-0.00042031	-0.00191503	-0.00480856
7703	41.812	0.871987	-0.532405	-0.0787976	-0.0006673	-0.00277501	-0.00579993
7704	42.119	0.864672	-0.54306	-0.102085	-0.0006673	-0.00373558	-0.00728356
7705	42.394	0.858164	-0.535128	-0.117486	-0.00358083	-0.00366149	-0.0068723
7706	42.453	0.856773	-0.534082	-0.114498	-0.00330902	-0.00358809	-0.00688643
7707	43.827	0.824921	-0.53446	-0.129819	-0.00358222	-0.00444961	-0.00565952
7708	43.168	0.840071	-0.530506	-0.116673	-0.00481411	-0.00435328	-0.00671564
7709	43.034	0.84318	-0.526279	-0.119565	-0.00565108	-0.00511573	-0.00682872
7710	43.904	0.823165	-0.527039	-0.121729	-0.00653736	-0.0052616	-0.00576209
7711	44.633	0.806697	-0.511927	-0.116591	-0.00629003	-0.00528567	-0.00701683
7712	46.499	0.76574	-0.531105	-0.101501	-0.00799526	-0.00602296	-0.0077475
7801	47.22	0.750353	-0.483731	-0.0850789	-0.00853824	-0.00678419	0
7802	48.142	0.731015	-0.472505	-0.124607	-0.00878691	-0.00710474	0

Figure 1: RID Model, Recursive Coefficient Estimates (OLS)
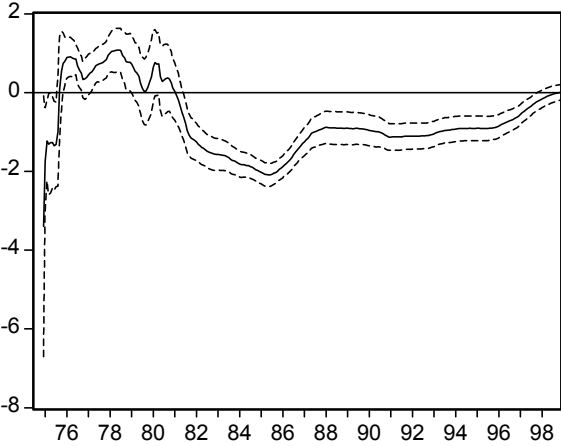
Coefficient on M
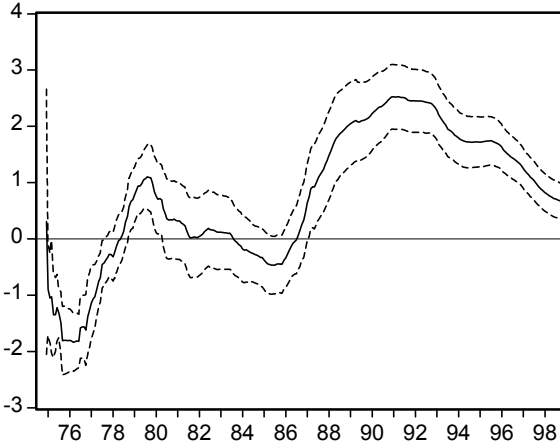
Coefficient on Y
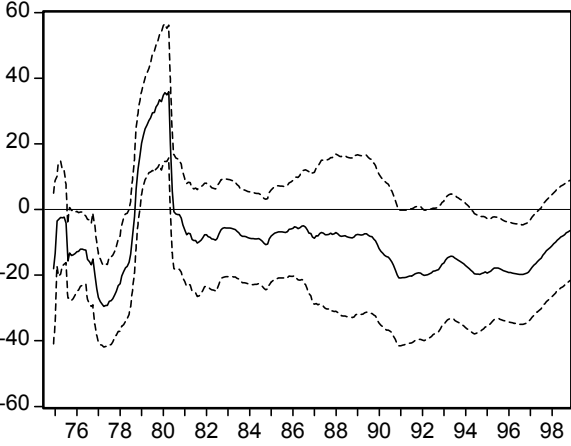
Coefficient on I

Coefficient on ILR

Figure 2: RID Model, Recursive Coefficient Estimates (IV1)
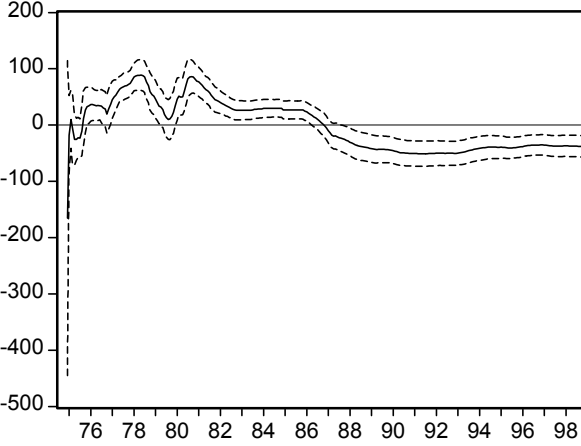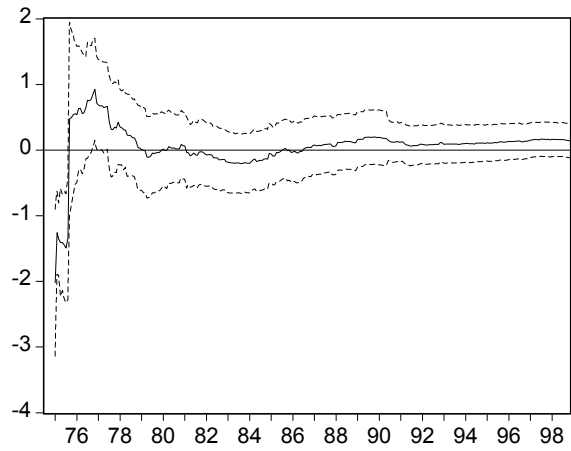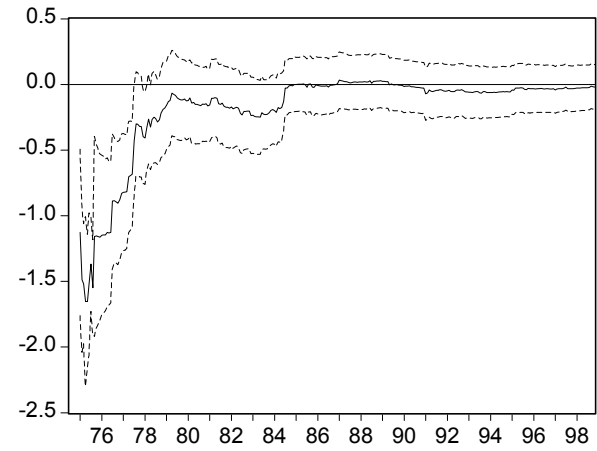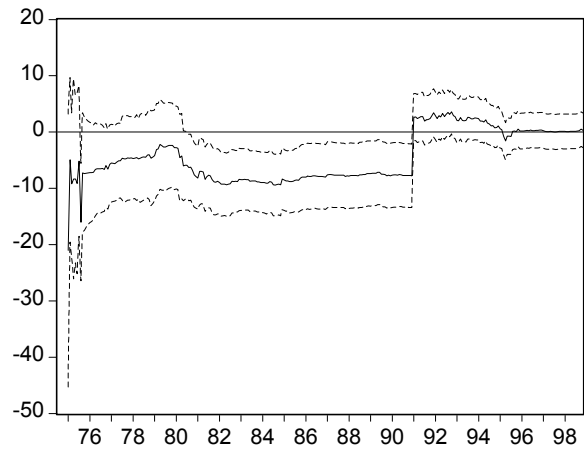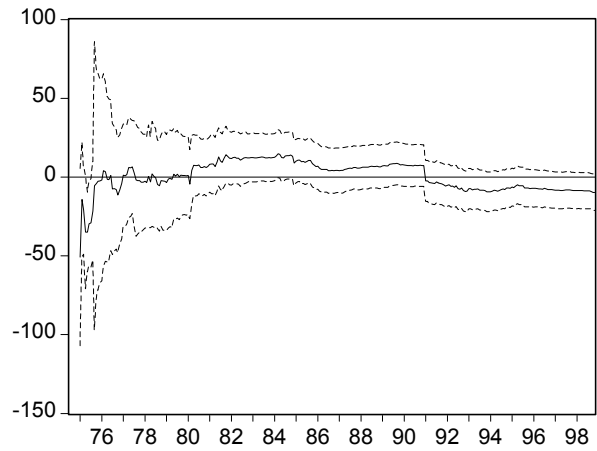
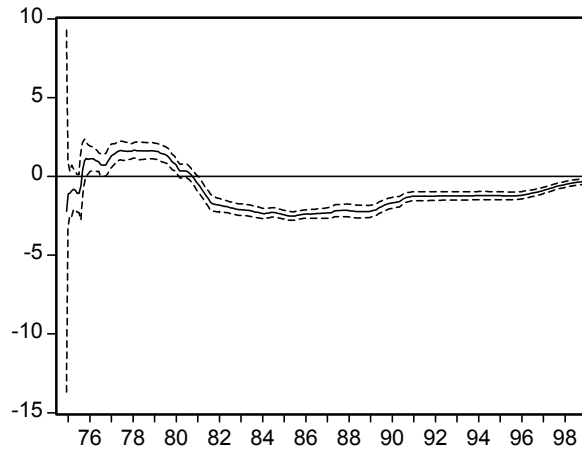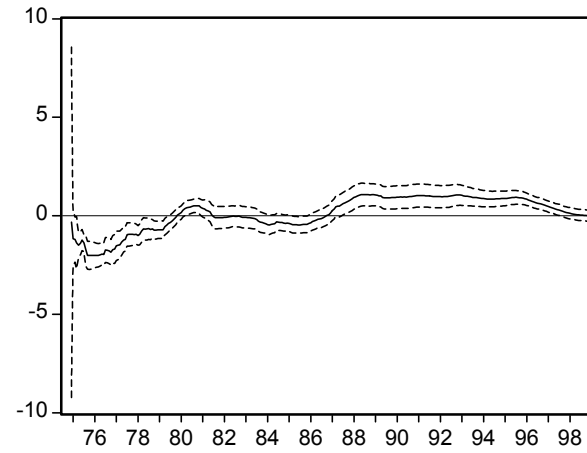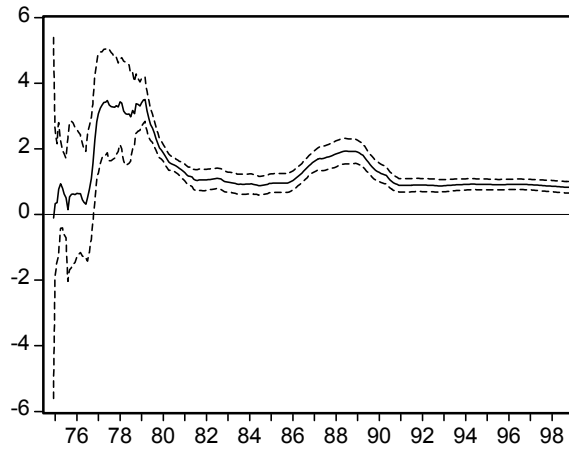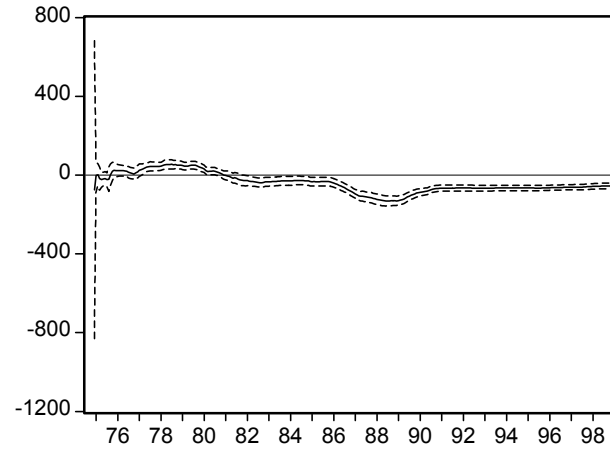Coefficient on M

Coefficient on Y

Coefficient on I

Coefficient on ILR

Figure 3: RIDRE Model, Recursive Coefficient Estimates (OLS)